Fiber Optics
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Lecture – 15
Electromagnetic Analysis of Waveguides – V

Let us continue our discussion on the modes of a planar symmetric waveguide. In the last lecture we had obtained the modal fields and propagation constants of symmetric and antisymmetric modes of asymmetric dielectric planar waveguide. Let us now look more carefully at these modes and understand what do they actually represent. What are the modal fields actually and what do the propagation constants represent.

So, for that let me write down the field of for example, TE 0 mode or any symmetric mode.

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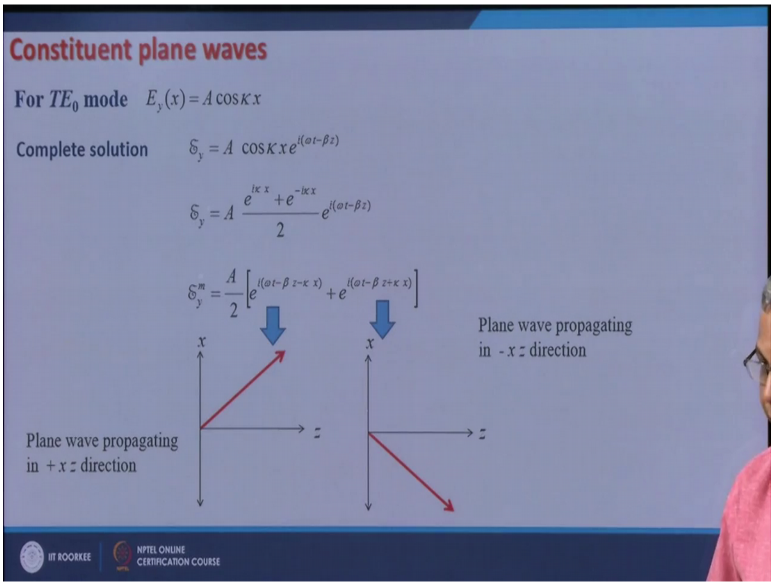

So, it is given in the core or in the guiding film it is given by E y of x is equal to a cosine kappa x. If I write down the complete solution that is the z part and t part also, then it would be a cosine kappa x E to the power i omega t minus beta z.

Let me expand this cosine kappa x it is in the same way as I had done for planar mirror waveguide. So, it is E to the power I kappa x plus E to the power minus I kappa x divided by 2, times E to the power i omega t minus beta z. So, let me arrange the terms in a particular passion. And then what I see that this term has E to the power i omega t minus beta z minus kappa x and this has E to the power i omega t minus beta z plus kappa x. So, this is nothing but a plane wave propagating in plus x z direction, making certain angle with z axis and this is another plane wave which is propagating in minus x z direction making certain angle from z axis.

So, I have this field comes out to be the superposition of 2 plane waves one going in plus x z direction another going in minus x z direction. So, these are the 2 constituent plane waves.

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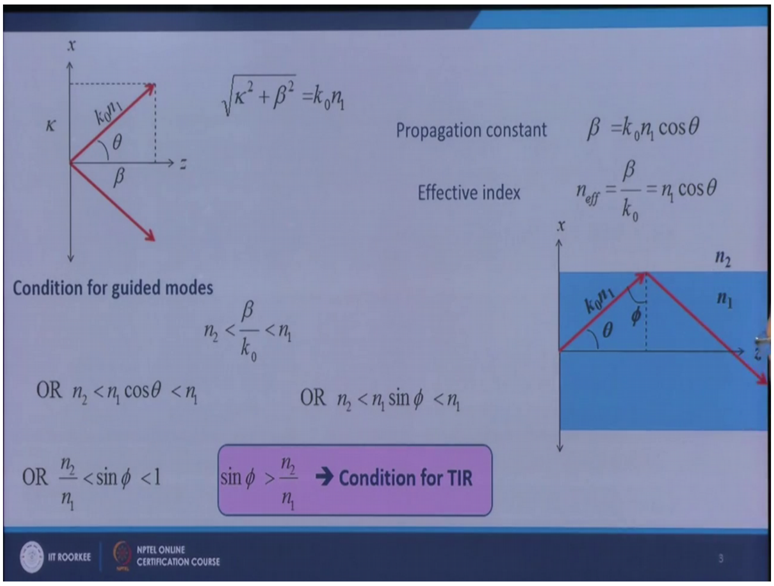

And what happens is if I do this kappa square plus beta square it comes out to be k naught n 1 and k naught, n 1 is nothing but the propagation constant with which a plane wave propagates in an infinitely extended medium of refractive index n 1.

So, kappa is nothing but, kappa is nothing but the component of this along x. So, if I resolve this in x and z direction then k naught n 1 sin theta will give me kappa. So, it is the component of k naught n 1 along x. And beta is the component of this k naught n 1 along z. So, in x direction I will get from here and here I will get 2 counter propagating waves, and these 2 counter propagating waves gives me standing wave in x direction. So, the energy stands in x direction it does not flow out.

However this standing wave pattern flows in z direction with propagation constant beta, exactly in the same way as planar mirror waveguide. So, these modal fields these modal fields are the standing wave patterns in x direction, because I have E y of x E y of x and this is standing wave pattern.

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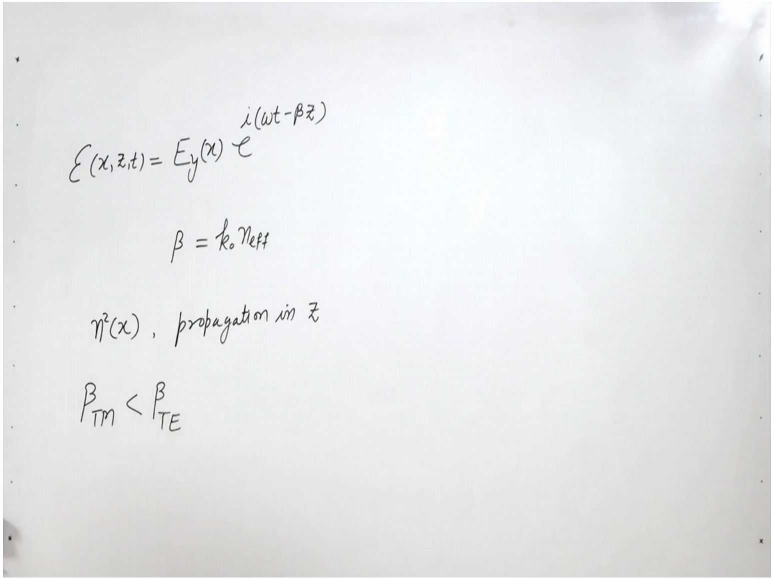

And this E y of x is propagating with propagation constant beta. So, if you look at it again this E x z t is equal to E y of x E to the power i omega t minus beta z. Beta is k naught n effective. Which basically tells with what velocity this particular field this is standing wave will propagate. If you remember that in one of the previous lectures I had said that intuitively I can I can understand the effective index of the mode as effective refractive index felt by a particular field distribution because the field is now distributed in n 1 and n 2 regions. So, the effective refractive index would be somewhere between n 2 and n 1. And that was intuitively correct and that was not rigorously correct. And I see that rigorously, rigorously n effective is nothing but n effective is nothing but the velocity with which it will it will define the velocity with which a particular field pattern will travel ok.

Rigorously I cannot get I cannot get from the distribution of power in n 1 and n 2 regions. So, this n effective is nothing but n 1 cos theta. So, it is the component of the plane wave k naught n 1 in z direction. And this distribution the effective refractive index as the distribution of field into 2 regions is also not rigorously correct in the sense that

that I cannot get the effective index from that field distribution in that way. And also if you look at planar mirror waveguide then you will say that the field is always in n 1 region then why it should have different effective indices for different modes and why it should be different from n 1 or refractive index n.

Similarly, if you talk about radiation mode in radiation modes the energy is distributed in n 2 region it goes up to infinity, and the effective index for radiation modes as we will see later it is it is less than n 2. So, rigorously effective index of the mode is nothing but the component of this along z direction. And it gives me the velocity with which a particular standing wave pattern will travel in z direction. And since different different modes are different in standing wave patterns and constitute different plane waves which are at different angles from z axis. So, n effective would be different.

If I look at the condition for guided mode then it is beta over k naught lies between n 2 and n 1. And beta over k naught is equal to n 2 gives me the cutoff of a particular guided mode. From here what I get beta over k naught is nothing but n 1 cos theta. So, if I replace this beta over k naught by n 1 cos theta here. And if I look at it this cos theta is nothing but sin phi if phi is the angle which the plane wave bakes with the normal to the interface of n 1 and n 2 regions than this is n 2 less than n 1 sin phi is less than n 1.

Let me divide the whole thing by n 1. So, this gives me n 2 over n 1 is smaller than sin phi is smaller than 1 or sin phi is greater than n 2 over n 1. This is nothing but the condition for total internal reflection. So, I automatically get that that the cutoff condition translates to the condition for total internal reflection. If this angle phi is smaller than if this angle phi is smaller than sin inverse of n 2 over n 1, then this wave will not undergo total internal reflection at this interface and it would be refracted. And the corresponding mode would be radiated out it would not be guided anymore. So, this is how I can understand the mode of a waveguide.

Let us work out again some examples. Let me consider a waveguide with n 1 is equal to 1.5 n 2 is equal to 1.46 d is equal to 2 micrometer.

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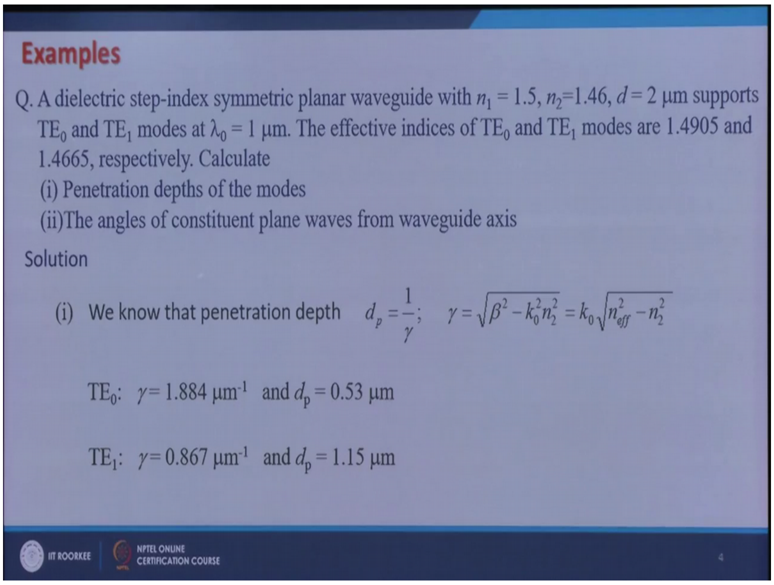

And it supports 2 modes TE 0 and TE 1 mode at lambda is equal to 1 micrometer. And if I find out the effective indices of these 2 modes they come out to be 1.4905 and 1.4665 respectively.

Now, let me calculate the penetration depths of these modes. And the angles of constituent plane waves from the waveguide axis. I know that penetration depth is given by 1 over gamma, where gamma is equal to beta square minus k naught square n 2 square. Or k naught times square root of n effective square minus n 2 square. So, k naught is 2 pi over lambda naught lambda naught is given n effective and n 2 are given. So, if I now calculate these gamma for both the modes, then for TE 0 mode gamma is equal to 1.884 micrometer inverse and correspondingly penetration depth is 0.53 micrometer.

For TE 1 mode gamma comes out to be 0.867 micrometer inverse. And correspondingly the penetration depth comes out to be 1.15 micrometer. What are the angles of constituent plane waves from the waveguide axis? Well I know that these angles are given by n effective is equal to n 1 cos theta. So, for these values of n effective I can immediately find out the values of theta, because I know the value of n 1 also.

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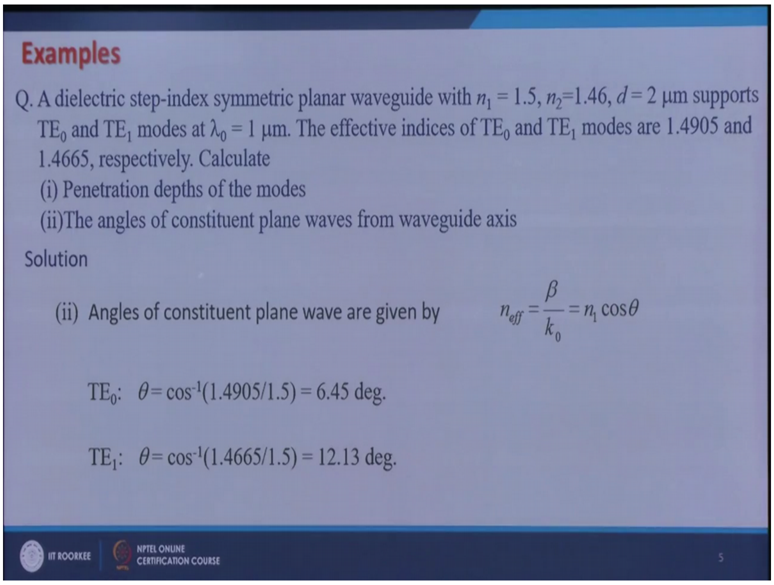

So, for TE 0 mode this theta would be cos inverse n effective over n 1. So, it would be 6.45 degrees and for TE 1 it would be 12.13 degrees. So, TE 0 mode is nothing but is nothing but the superposition of 2 plane waves, making angles making angles plus minus 6.45 degrees from the waveguide axis. And TE 1 mode is the super position of 2 plane waves which make angles plus minus 12.13 degrees from the waveguide axis.

So, after doing the analysis of the waveguide for TE modes, now in the same manner we can do the analysis for TM modes transfers magnetic modes. If I go back and see that for a waveguide which has confinement in x direction, that is refractive index variation in x direction and propagation in z direction, then the non vanishing components of E and H are H y E x and E z.

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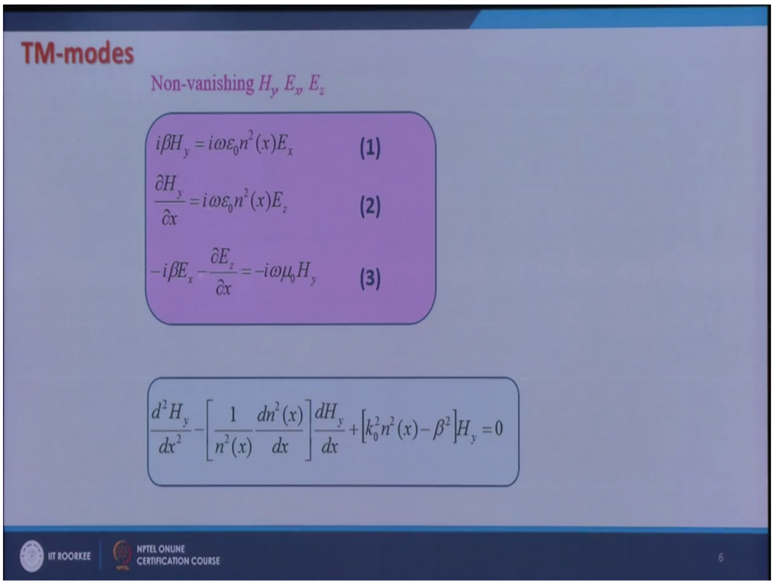

And these 3 components are related by these 3 relations. The procedure to obtain the modes and modal fields for t a modes is exactly the same as we had done in the case of TE polarization. So, what I need to do? I need to find out a differential equation for example, in H y and solve it.

So, to find out the differential equation in H y, I substitute for E x and E z from these 2 equations into this equation, and rearrange the terms. Then I get a differential equation in H y as d 2 H y over d x square minus 1 over n square d n square over d x d H y over d x plus k naught square n square minus beta square H y is equal to 0. You may see that you may notice that this equation is different from the wave equation that we had got for the TE case. In the TE case this term was not there, excuse me in the TE case this term was not there, but here I have this term. So, let us see how do we take care of this now we need to solve this equation for given n square of x to find the modes. So, this is the waveguide which we are considering again symmetric dielectric planar waveguide where n of x is given like this.

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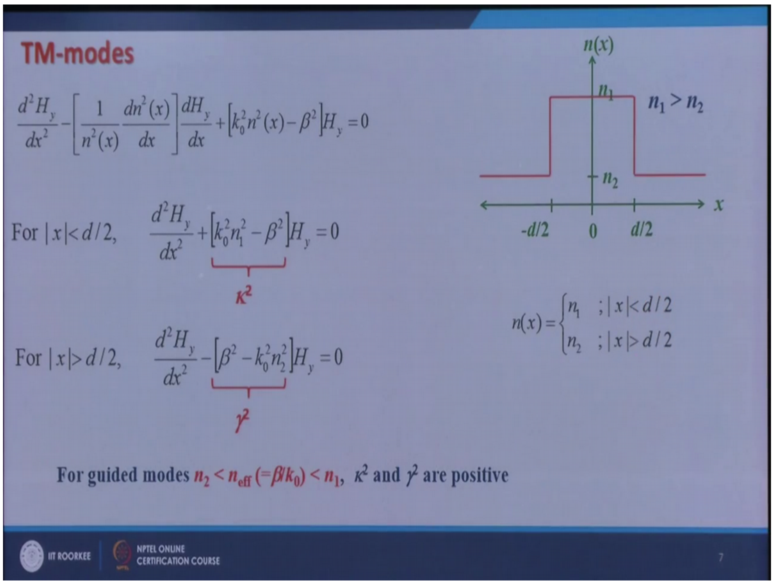

Procedure is again the same that I need to write this equation in this region and in this region. So, I do this for mode x less than d by 2 I get this equation. Why? Although I have this term in general for a variation n square of x, but when I write this equation in the individual regions in this kind of a step index waveguide the refractive index is uniform and when the effective index as uniform in this region. So, this term goes off. So, I have got this equation for mode x less than d by 2, and this equation for mode x greater than d by 2 and you can see these equations are exactly the same as we had obtained in the case of TE polarization.

So, again I define this is kappa square and this is gamma square, and since I am interested in guided modes. So, kappa square and gamma square are positive.

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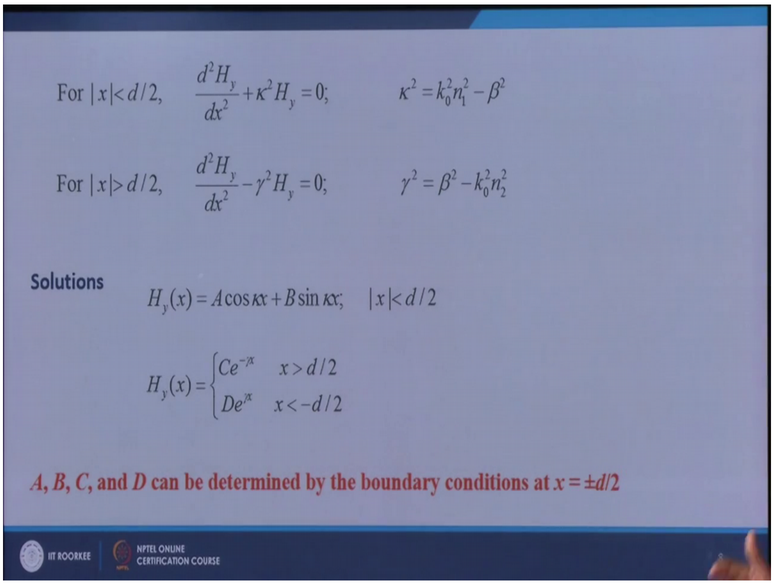

So, these are the equations and the solutions are again the same exactly the same, H y is equal to a cosine kappa x plus b sin kappa x for mode x less than d by 2 and the decaying solutions for mode x greater than d by 2. And again A B C and D can be determined with the help of boundary conditions.

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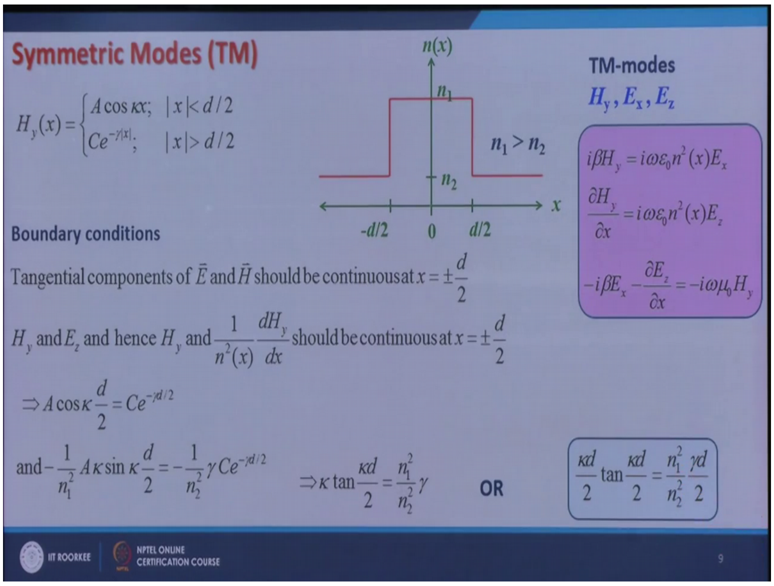

In this case also I can make use of the symmetry of the problem and divide these solutions into symmetric and antisymmetric. So, for symmetric modes my H y would be a cosine kappa x in the guiding film, and Ce to the power minus gamma mode x for mode x greater than d by 2 that is in these regions. So, what next? What I should do? Now I should apply the boundary conditions, to this kind of modal field. And what are the boundary conditions the boundary conditions are again the same the tangential components of E and H should be continuous at x is equal to plus minus d by 2.

What are the tangential components here? Well if I look at TM modes, the non vanishing field components are H y E x and E z. So, the components which are tangential to x is equal to plus minus d by 2 planes are H y and E z. So, H y and E z should be continuous. How E z is related to H y? We will look at this. So, E z is related to H y with some constant times 1 over n square d H y over d x. So, this because this n square depends upon depends on x. So, I should take it to that side. So now, now the boundary conditions give me H y and 1 over n square d H y over d x should be continuous. So, this is the difference with the TE case.

In TE case I had the boundary conditions the boundary condition led to E y and d E y over d x should be continuous at the boundaries, but here I have H y and 1 over n square d H y over d x should be continuous at the boundary. So, I apply these so first I apply that the field should be continuous H y is continuous at x is equal to let us say plus d by 2. So, a cosine kappa d by 2 is equal to C E to the power minus gamma d by 2, and then I apply this 1 over n square d H y over d x should be continuous at x is equal to plus d by 2, then it is on this side I have 1 over n square minus a sin kappa d by 2 is equal to minus 1 over n 2 square gamma C E to the power minus gamma d by 2.

So, this gives me the transcendental equation, if I divide this by this. Kappa tan kappa d by 2 is equal to n 1 square over n 2 square times gamma or by multiplying with d by 2 on both the sides I get kappa d by 2 tan kappa d by 2 is equal to n 1 square over n 2 square times gamma d by 2. So, this is the transcendental equation the difference is this factor. I have this factor n 1 square over n 2 square extra in the case of TM polarization.

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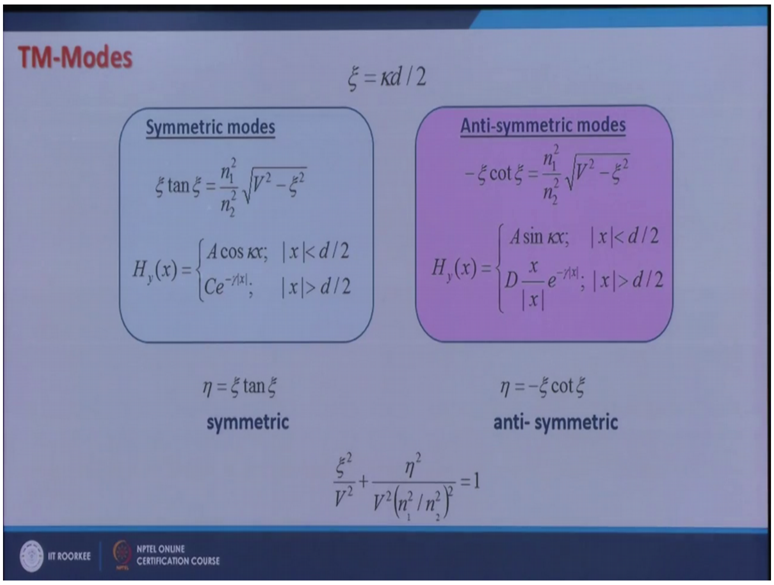

So, for symmetric modes, if I now convert this equation into psi and V then I know psi is nothing but kappa d by 2, then in exactly the same way as I had done for TE case for symmetric modes I will have the equation as psi tan psi is equal to n 1 square over n 2 square V square minus psi square and this is the field. Similarly for antisymmetric modes the equation would be transformed to minus psi cot psi is equal to n 1 square over n 2 square, square root of V square minus psi square and this is the field.

So, the transcendental equation is this. So, if I use the graphical solution to solve this. So, I should equate this on the right hand side and left hand side both to eta in this case as well as in this case. So, for symmetric mode I will have eta is equal to psi tan psi and for antisymmetric is as well as eta is equal to minus psi cot psi and from the right hand side I will get psi square over V square plus eta square over V square times n 1 square over n 2 square whole square is equal to 1.

In the case of TE modes this factor was not there and I had the circles here psi square plus eta square is equal to V square. But now, but now I no more have circles, but this is the equation of an ellipse. So, I have ellipse here, and what is the major axis of the ellipse it is along eta. So, what I will have to do now? To have graphical solutions I will have to plot these 2 together for symmetric modes and these 2 together for antisymmetric modes and look for points of intersections.

So, I do this. So, again these red curves show eta is equal to n 1 square over n 2 square psi tan psi.

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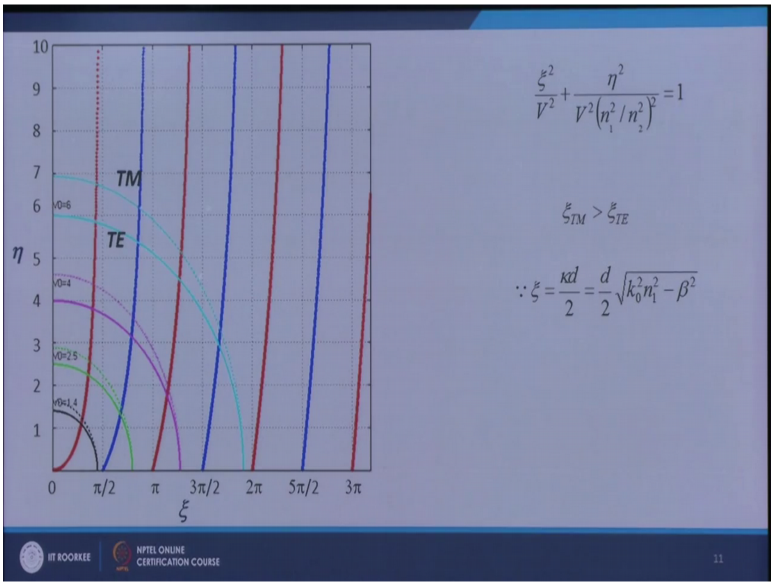

And so, this is for symmetric modes this is for antisymmetric modes. So, I this is eta is equal to psi tan psi it is eta is equal to minus psi cot psi. Now the solid lines the solid lines represent the circles, psi square plus eta square is equal to V square. And correspond to TE modes while these ellipse which are represented by dotted lines they are the ellipse corresponding to the TM modes. So, these are the ellipse corresponding to TM modes.

What I see here if I take a particular value of V, if I take a particular value of V and see and see the points of intersection of these TE and TM modes, then it should not be V naught it should be V. So, for example, for V is equal to 6, for V is equal to 6 the point of intersection corresponding to TE mode is somewhere here. And for TM mode it is at slightly larger value of psi. So, what I have for a given for a given mode whether it is m is equal to 0 or m is equal to 1 or m is equal to 2, psi for TM mode is always greater than psi for TE mode. And since psi is equal to kappa d by 2 which is d by 2 square root of k naught square n 1 square minus beta square, then beta for TM is always less than beta of TE.

In terms of normalized parameters in the same way as I had done for TE case I can also write down the transcendental equation for TM modes, symmetric and antisymmetric.

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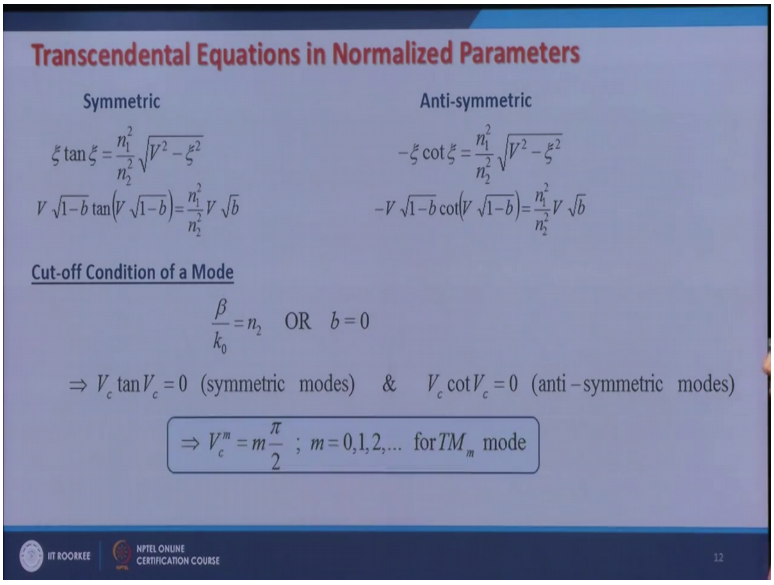

So, equations are similar, only thing is this extra factor of n 1 square over n 2 square in both the cases symmetric and antisymmetric cases.

Cut off conditions. What are the cut off conditions? Well beta over k naught should be equal to n 2, which means b should be equal to 0, if I put b is equal to 0 here and here I will get the cut off conditions for symmetric and antisymmetric modes. And you can see that since b has to be 0 if I put b is equal to 0 then this factor is absorbed here in 0. So, it would not have any effect it would not have any effect on the cut offs.

So, for symmetric modes the cut off condition remains V c tan V c is equal to 0, and for antisymmetric mode it remains V c cot V c is equal to 0. So, in general I have the cut offs for TM m modes as V c m is equal to m pi by 2, which is the same as in the case of TE modes ok.

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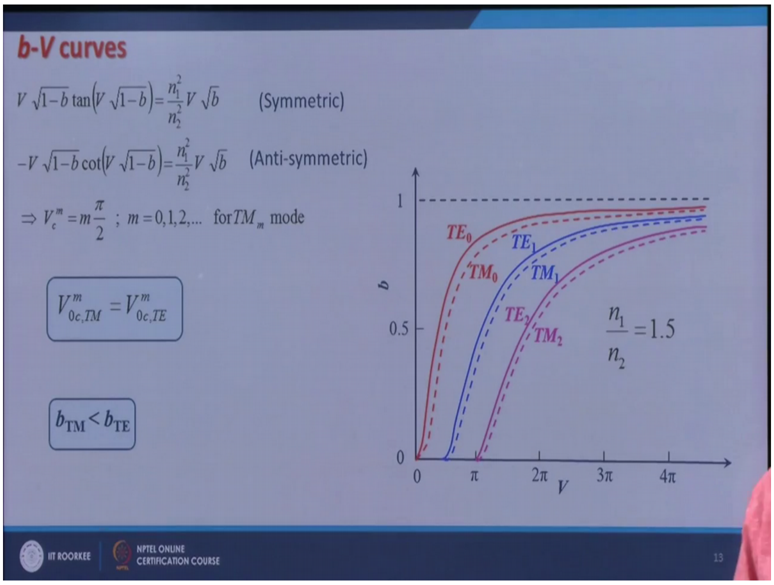

Let us now plot b V curves. So, so that I will have to plot for a given value of n 1 over n 2, let me see how does it look like schematically for n 1 over n 2 is equal to 1.5. So, the first thing is the cut offs cut offs are the same. So, the plots will start from the same value for TE and TM. So, if I plot it for TE 0 mode then for TE 0 mode it both will start from 0 because they have the same cut off. So, TE 0 mode goes like this and TM 0 will also start from here, but whether this curve would be below this or above this. And I see that since the propagation constants of TM modes are smaller than the propagation constants of TE modes. So, the curve corresponding to TM mode would lie below the curve corresponding to TE modes. So, it will go like this, for m is equal to 1, So this would be TE 1 and this is TM 1 similarly TE 2 TM 2.

Again these curves are universal for given value of n 1 over n 2 now. So, here I should also take care what is this ratio n 1 over n 2. Modal fields, if you look at modal fields then again in the same way the modal field for TM 0 would look like this. This is for low index contrast waveguide and for TM 1 mode it looks like this. What I should take care here is and why it I should pay attention to is that in case of TE polarization E y and d E y over d x was continuous.

So, the field as well as it is slope was continuous at the boundaries x is equal to plus minus d by 2, but you remember that in TM polarization H y and 1 over n square d H y over d x are continuous at the boundaries.

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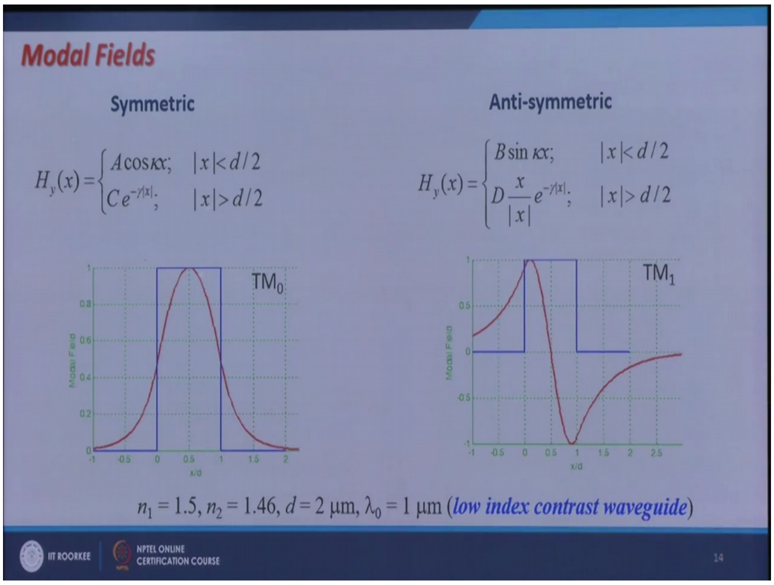

So, the slope is not continuous. Although here I see that the slope is also continuous at the boundary, but the, but rigorously it is not continuous. It is the discontinuity in the slope is. So, small that it does not show up because if you look at n 1 and n 2 n 1 and n 2 are very close. So, if it is weakly guiding if the waveguide is weekly guiding the n 1 the values of n 1 and n 2 are very close to each other than that would not show up here.

If on the other hand if I plot this for high index contrast waveguide you now look at n the values of n 1 and n 2, the factor n 1 over n 2 is now 3.5.

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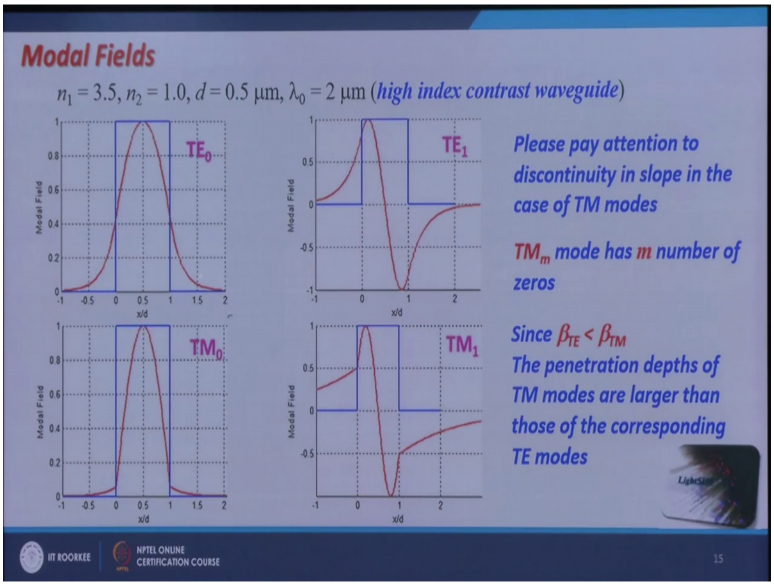

Now, if I plot the modal fields the modal fields for TE 0 mode are like this there is no discontinuity in the slope while here this discontinuity shows up, because the factor n 1 over n 2 is very large.

So, this is the first thing I will see, similarly in the case of TM 1 mode there would be huge discontinuity. Again just as in the case of TE modes in TM modes also mth TM mode will have m number of 0s, and since the propagation constant of TM mode this is wrong it should be it should be beta TM is less than beta TE please make correction. So, the penetration depth of TM modes are larger, you can see the penetration depth of TM modes are larger than the penetration depths of TE modes.

I can look at some examples if I consider this waveguide with n 1 is equal to 1.5 n 2 is equal to 1.46 d is equal to 2 micrometer.

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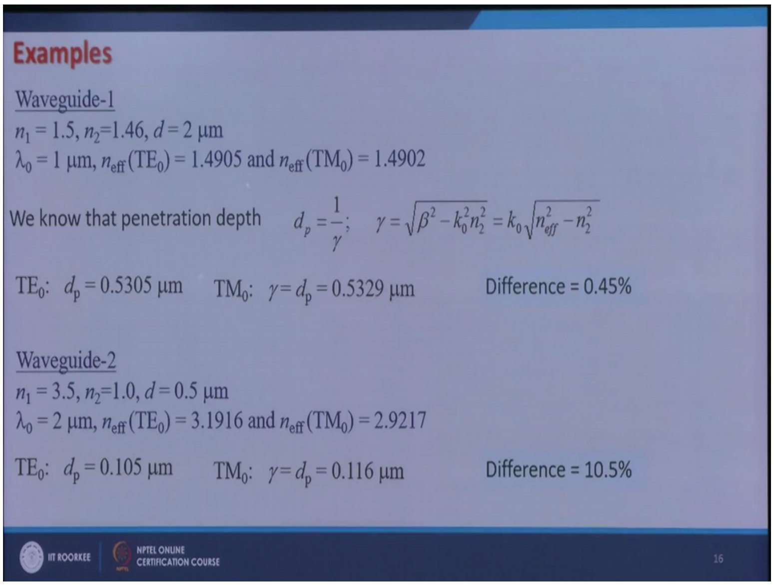

At lambda is equal to 1 micrometer n effective of TE 0 mode comes out to be this and of TM 0 mode comes out to be this. So, you can see if the index difference is very small, TE 0 and TM 0 modes have nearly the same propagation constants, very close to each other. And if I now calculate the penetration depths for the 2 modes the penetration depths are also very close to each other 0.5305, 0.5329 differences only about 0.45 percent, but if I take high index contrast waveguide where n 1 is 3.5 n 2 is 1. Then n effective of TE 0 mode is 3.1916 and that of TM 0 mode is 2.92. So, which is very different there is huge difference between the propagation constants. Now, if I calculate the penetration depths then the difference is about 10 and half percent.

With this I have done the analysis of dielectric planar waveguide for symmetric modes as well as antisymmetric modes. I have calculated the propagation constants, I have found out the modal fields, their behaviors, penetration depth. Now only thing that remains is how much energy do they carry, how much energy how much power these modes carry. We will go into that in the next lecture.

Thank you.