Fiber Optics
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Lecture – 14
Electromagnetic Analysis of Waveguides – IV

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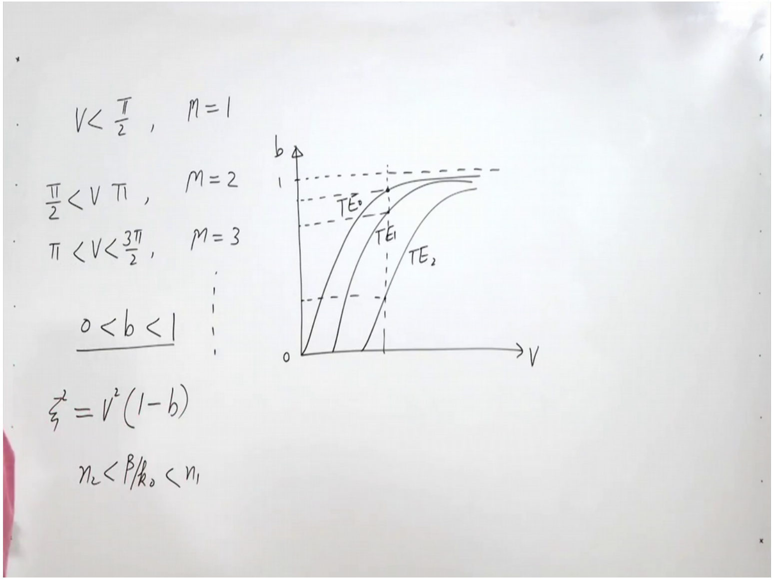

In the last lecture we had seen that if for a planar waveguide V is less than pi by 2, then the number of modes is one. It supports only one mode and when V lies between pi by 2 and pi there are 2 modes and if it is between pi and 3 pi by 2, then there are 3 modes and so on. If I keep on increasing the value of V then the number of modes would increase, but I also see that the propagation constants can be approximated by certain expression, and we can find out the propagation constants even without solving the transcendental equation for first few modes. So, let us see how.

So, if it is a highly multimode waveguide where V is much larger than 1 typically V is more than 10 or so then What I see if I now again plot these transcendental equations this is this is eta is equal to psi tan psi and the blue one is eta is equal to minus psi cot psi.

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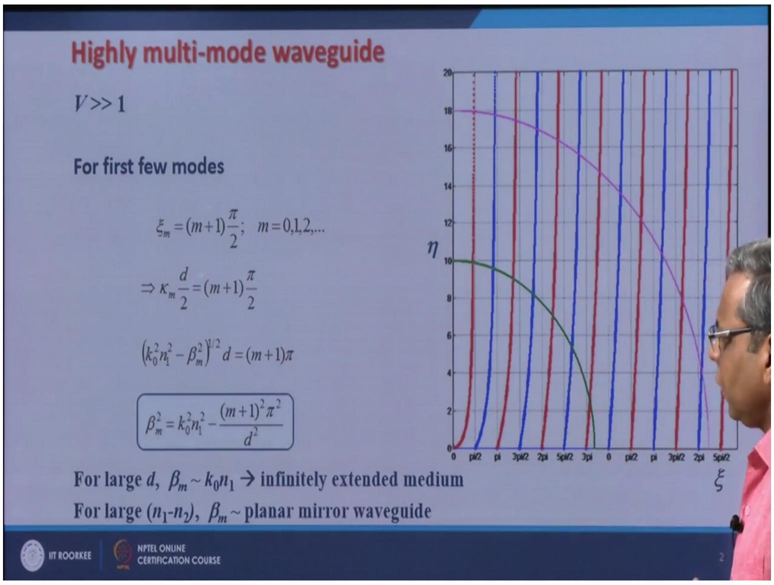

Now if I plot the sections of circles for large values of V let us say V is equal to 10 or V is equal to 18 or 20, what do I see there for the first few modes the points of cross sections are somewhere here.

So, for first mode that is m is equal to 0 mode the point of intersection is close to pi by 2. And for m is equal to 1 the point of intersection is close to pi and so on. So, and these are only for first few modes, if you go to higher order modes than these points of intersections are much further from these values. So, what I can say that for first few modes, the points of intersections can be given psi m is equal to m plus 1 pi by 2, where m is equal to 0 1 or 2. So, for m is equal to 0 it is pi by 2 for m is equal to 1 it is pi and so on.

So, in this case in this case I can find out the beta in this way, I know psi m is equal to kappa m d by 2. So, it is equal to m plus 1 pi by 2 and kappa m is k naught square n 1 square minus beta m square is square root. So, so this gives me beta m square is equal to k naught square n 1 square minus m plus 1 square pi square by d square. Now I can increase the value of V by 2 ways I can have large value of V by 2 ways - one is that if I have very large value of d. If I have large value of d then you can see there that is beta m will approach to k naught n 1 beta m will approach to k naught n 1. And it is obvious it if I start if a start from a very thin waveguide then there are discrete modes and they have propagation constants which lie between k naught n 2 and k naught n 1.

But when I increase this when I increased this width of the waveguide and the width is very large as compared to lambda then it is as good as infinitely extended medium. So, so the light will propagate with propagation constant k naught n 1 where n 1 is the refractive index of the medium. So, I will approach towards infinitely extended medium if d is very large. Now if I obtain the large value of V by having very large index contrast, that is very large difference between n 1 and n 2, then what happens is there is a huge index contrast and this very large index contrast will give very high reflection a very high reflection ok.

So, it is it would be difficult for the wave to penetrate into n 2 region. So, it would be something like it would be very close to what is planar mirror waveguide and in that case you can see the expression for these beta closely resemble to the expression of beta for planar mirror waveguide. Let us work out some more examples here.

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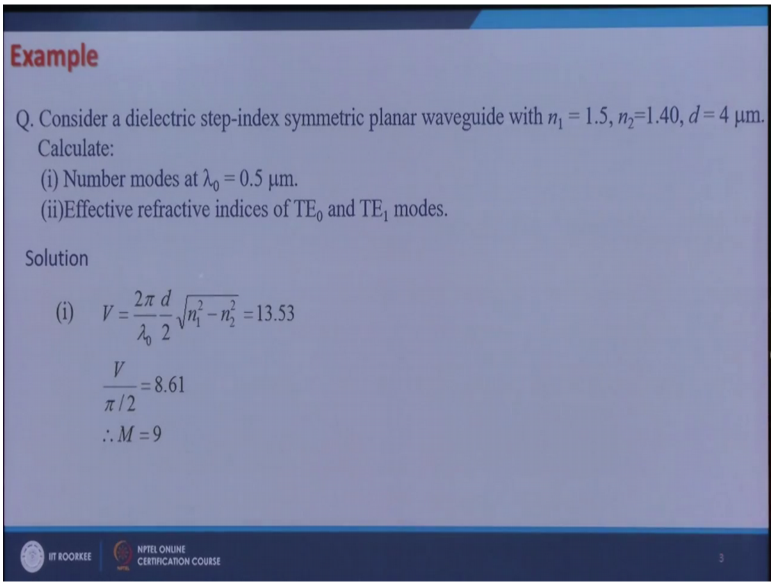

I consider a step index planar symmetric waveguide with n 1 is equal to 1.5, n 2 is equal to 1.4 and d is equal to 4 micro meter. And let us calculate the number of modes at 0.5 micro meter and effective refractive indices or effective index of modes of TE 0 and TE 1 modes.

So, again to find out the number of modes it is the same I find out the value of V. So, when I find out the value of V. It comes out to be 13.53 at lambda naught is equal to 0.5 micrometer if I divide this value of V by pi by 2 This number comes out be 8.61.

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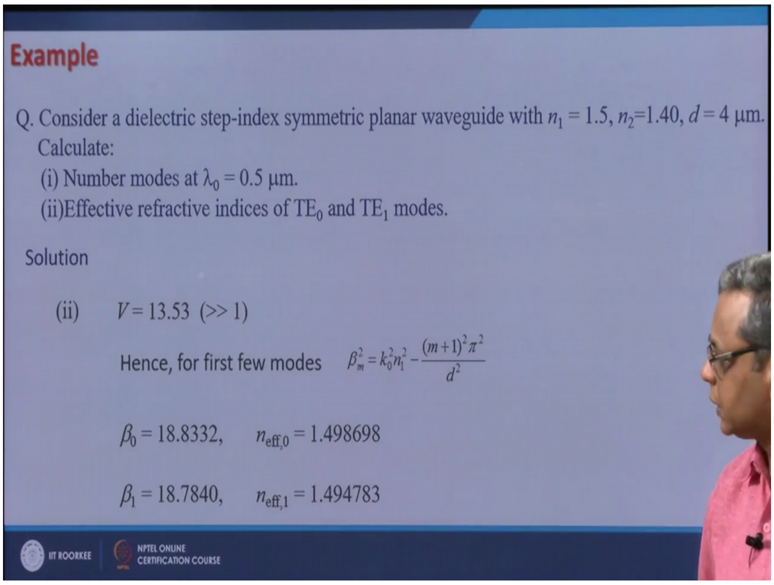

So, the number of modes supported would be 9 I can see this value of V is very large. So, it would be now easier for me to find out the propagation constants of TE 0 and TE 1 mode, even without solving the transcendental equation.

Since it is much larger than 1. So, for the first few modes the values of beta would be given by this, and for TE 0 mode m is equal to 0 and for TE 1 mode m is equal to 1 and in this way if I find out the values of beta, then they come out to be 18.83 and 18.78 correspondingly the effective index of TE 0 mode comes out to be 1.498698, and for TE 1 mode 1.494783.

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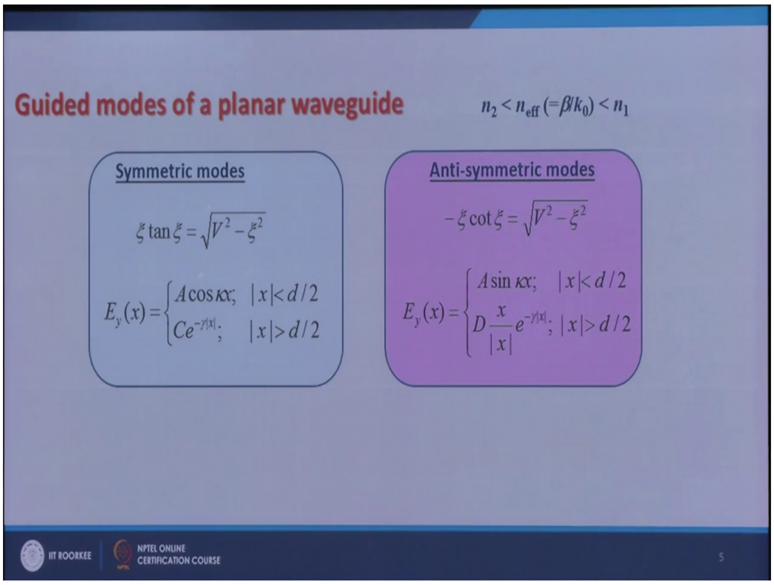

You can see the very close to the refractive index of the guiding film which is 1.5 ok.

Now, let us let us do extend all this analysis by using normalized parameters. So, we had symmetric modes antisymmetric modes.

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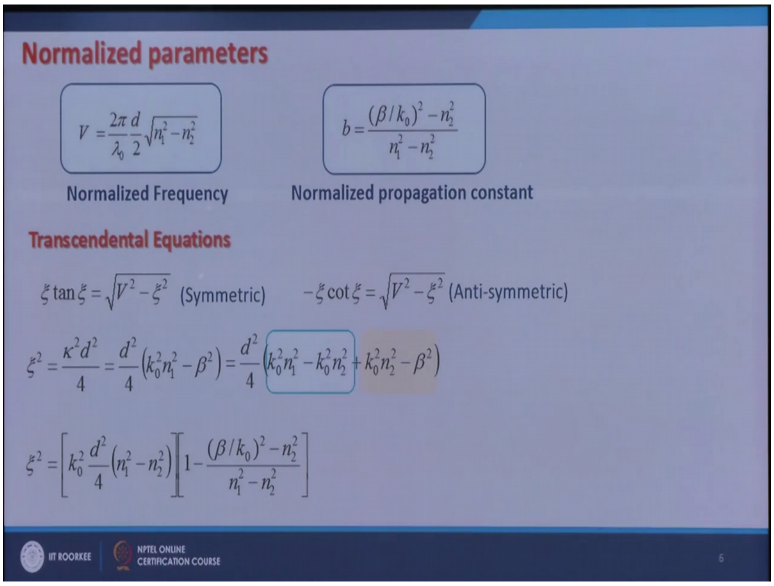

And the transcendental equations were in terms of psi and V. Now I already have one normalized parameter which is normalized frequency V which contains all the waveguide parameters and wavelength. Let me also define normalized propagation constant, that is let me normalized beta and how do I normalized beta yet I take beta over k naught square minus n 2 square divided by n 1 square minus n 2 square. If I normalized beta in this way then for guided modes the value of b lies between 0 and 1.

So, for guided modes the value of b lies between 0 and 1. So, this is now normalized propagation constant. Now my task is to replace that psi with b and V. So, how do I do this? So, this is the transcendental equation for symmetric mode this is the transcendental equation for antisymmetric mode. Let me find out how can I how can I get psi in terms of b and V. So, psi square is equal to kappa square d square by 4. So, this is nothing but d square by 4 and kappa square is k naught square n 1 square minus beta square. I do a little mathematical manipulation I add and subtract k naught square in 2 square here, and then I regroup these 2 terms and these 2 terms.

So, if I group these 2 terms and these and also I take this term outside. Why I am doing? So, because I can immediately identify that if I club this term with this I get d square. So, I do that. So, this becomes psi square is equal to k naught square d square by 4 n 1 square minus n 2 square times one and from here I take minus outside and I write it down as beta over k naught square minus n 2 square divided by n 1 square minus n 2 squares. So, this is nothing but b. So, this is b this is V square.

So, what I have got now from here psi square is equal to V square and this is 1 minus b. So, this gives me psi is equal to V times square root 1 minus b. So, as soon as I get the psi in terms of V and b I put it in these equations and I get the transcendental equation in terms of b and V only.

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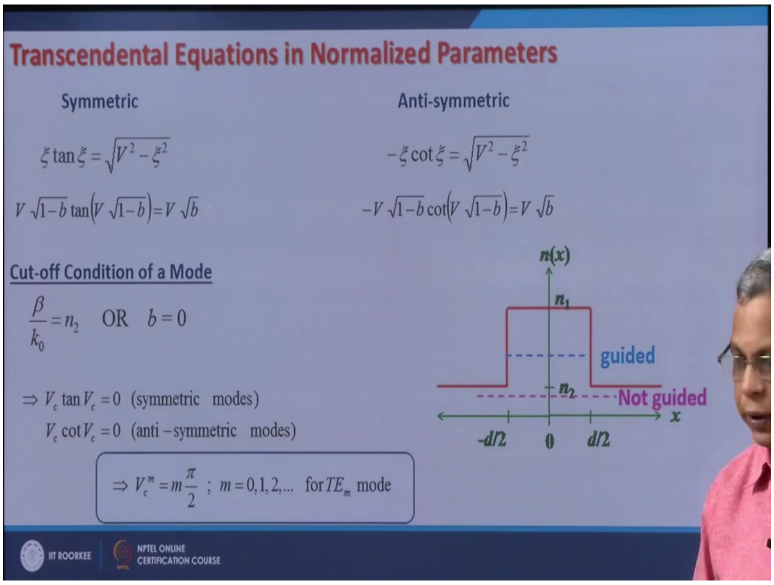

So, for symmetric mode this transcendental equation will transform to V times square root 1 minus b tan V is square root of 1 minus b is equal to V times square root of b ok.

Because from here V square minus psi square V square minus psi square would be V times square root of b. And for antisymmetric mode this equation will become this. From here I can find out the cut off condition of a mode. A mode is said to be cut off when it is propagation constant approaches k naught n 2. I know that if an effective for guided modes I know for guided modes an effective or beta over k naught beta over k naught beta over k naught or an effective lies between n 2 and n 1.

So, if this beta over k naught approaches n 2 then a particular mode is cut off. So, mode is guided when the when the effective index of the mode lies between these 2, but as soon as the effective index goes below this then it is no more guided. So, this value of an effective will give me the cutoff. This beta over k naught is equal to n 2 gives me b is equal to 0. So, this is the cut off condition in terms of b. So, from here I can immediately find out for what values of V for what values of normalized frequency a mode would be cut off. So, I simply put b is equal to 0 here then the solutions of the equation that I get will give me the cutoffs of the modes.

So, this will give me if I put V is equal to 0 sorry b is equal to 0 and V is equal to V c which symbolizes the cutoff frequency cutoff normalized frequency, then it gives me V c tan V c is equal to 0 for symmetric modes. And for antisymmetric modes it will be V c cot V c is equal to 0. So, these 2 together will give me the cutoff of mth mode is m pi by 2 where m is equal to 0 1 2 and so on.

So, TE 0 mode will have no cut off m is equal to 0, TE 1 mode will have a cutoff of pi by 2 TE 2 mode will have cut off pi and so on. So, this is how I get the cutoffs of the modes.

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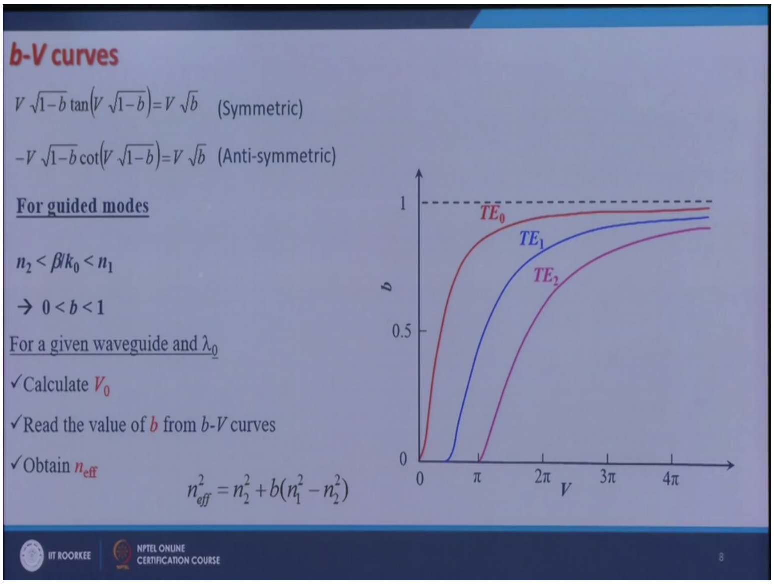

Now very beautiful thing about these normalized parameters is that they are independent of waveguide parameters. I get these 2 equations and these 2 are purely mathematical equations 2, purely mathematical transcendental equations. What I can do? I can solve these equations for I can solve these equations for different values of V. And I find out the roots of these and plot these roots as a function of V.

So, I start with I start with a very small value of V. When I do this as long as the value of V is less than pi by 2, then what I see that there is no root coming out from this equation and there is only one root coming out of this equation. I find out that root for different values of V and plot it, it go something like this. As soon as the value of V crosses pi by 2 1 root starts one root starts appearing from this equation. And the value of V up to pi I will get one root from here, one root from here I keep plotting them. And then I further increase more and more roots start appearing and I plot them ok.

The very first root which comes from here which is closer to which is closer to 1 the first root which is closer to 1 is TE 0 mode. The first root from here is TE 1 mode the second root from here is TE 2 mode and so on. So, I plot them, once I have plotted them then these curves are these curves are universal curves. So, once I have done this exercise once I have done this exercise then it is forever these are universal curves, now what I can do using these curves?

If there is a given waveguide and wavelength then I simply calculate the value of V this should be V naught and there should not be any 0 here. So, so I just calculate the value of V, and then I go I go to these curves and read the value of b corresponding to that value of b for example, if I get this value of V let us say V is equal to 1.4 then I draw a vertical line corresponding to V is equal to 1.4 and read the value of b corresponding to this from b V curves. Once I get the value of b then I can find out the effective index of the mode from here, this is coming out from the very definition of the normalized propagation constant.

If I am somewhere here, if I am somewhere here if the value of V is let us say 3.5. So, it will go like this I will have 3 points of intersection and corresponding to that I can read the value of b. So, I will get the propagation constants of all the 3 modes. So, in this way I can find out the propagation constants for any symmetric planar waveguide. Let us do some examples. So, if I have a waveguide with n 1 is equal to 1.5 n 2 is equal to 1.48 d is equal to 2 micrometer and I operate it lambda naught is equal to 1.5 micrometer.

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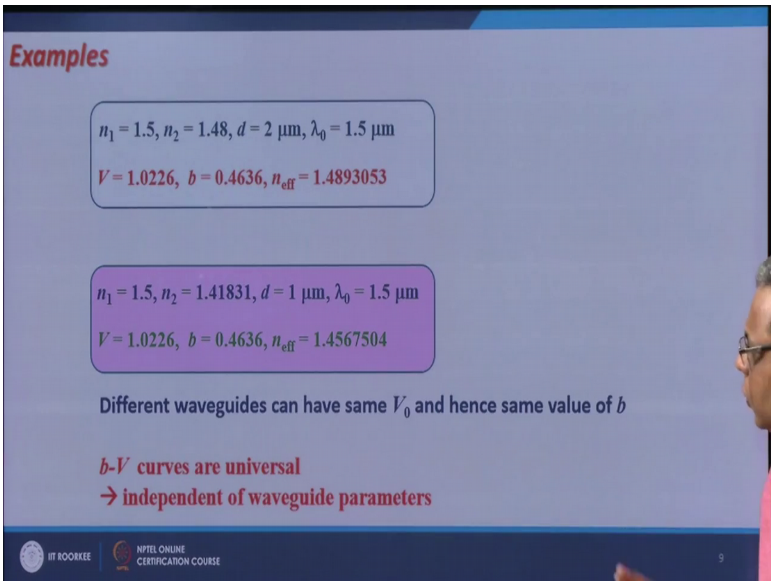

If I calculate the value of V from here it comes out to be 1.0226. I go back to those b V curves and read the value of b from here corresponding to this value of V it comes out to be 0.4636. And then I find out the value of an effective, which comes out to be 1.4893053 ok.

Now, I change my waveguide parameters a little. So, I now have n 1 is equal to 1.5 n 2 I have change to 0.418. D 2 1 micrometer and lambda naught is equal to 1.5. I have artificially change these parameters for achieving the same value of V actually. What I want to demonstrate is that, if my waveguide is different d one if my waveguide is different, but it gives the same value of V then the value of b is always the same. For same value of V even if the waveguide is different the value of b normalized propagation constant is the same, but the effective index would be different now, because n 1 and n 2 are different n 1 and n 2 are different. So, in this case the effective indexes this while in this case the effective index would be this.

So, different waveguides can have same value of V and hence the same value of b which means that these b V curves are universal and they do not depend upon waveguide parameters. Let us now look at modal fields, how do the modal fields look like. Let us plot the field for TE 0 mode TE 0 is symmetric mode.

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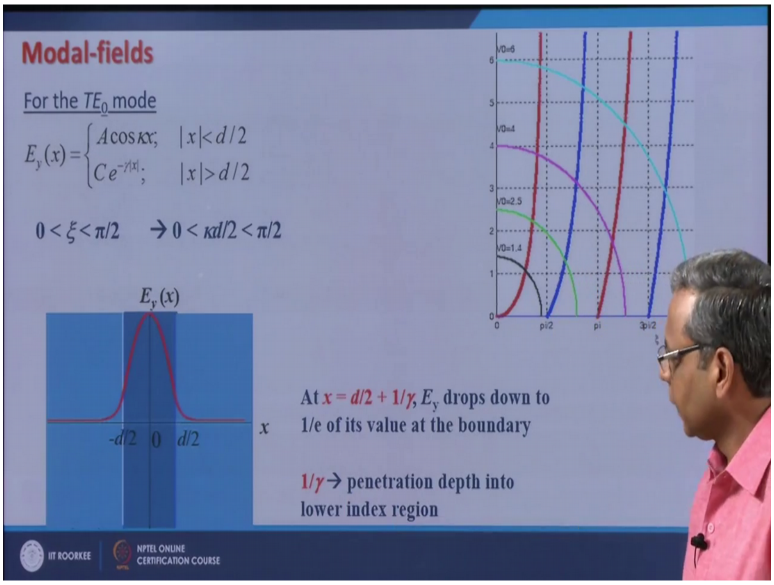

So, the modal field will be given by E y of x is equal to A cosine kappa x in the region mode x less than d by 2, and Ce to the power minus gamma mode x for mode x greater than d by 2. I know for TE 0 mode psi would lie between 0 and pi by 2, because V would lie between 0 and pi by 2. So, the point of intersection the value of psi for point of intersection will always lie between 0 and pi by 2.

It will not exceed pi by 2. What is psi? Psi is kappa d by 2. So, kappa d by 2 will lie between 0 and pi by 2 now let us look at the waveguide. And the modal filed in the region mode x less than d by 2 the solution is cosine function. So, I will have cosine kappa x what would be the value at the boundary? What would be the value at the boundary? At the boundary I have kappa x is equal to kappa d by 2 and minus kappa d by 2, but kappa d by 2 is always less than pi by 2, if kappa d by 2 is always less than pi by 2 then cosine kappa d by 2 will never cross any 0. The first 0 of cosine function will occur at kappa x is equal to pi by 2, but I have even at the boundary kappa x is always less than pi by 2. So, I would not have any 0 crossing.

So, e y in the guiding film will be like this. And then in the lower refractive index region the exponential decaying part exponentially decaying solution will take over and it will go like this. So, I will have the modal field which would look like this all right. What I see that there is oscillatory solution here and decaying solution here. The field extends field extends to lower indexed region also. How much does the how much does this extend? Into this region I can I can quantify this, I find that I find that at x is equal to d by 2 plus 1 over gamma because this field is going as e to the power minus gamma times x. So, at x is equal to plus x is equal to 1 over gamma this field will decay to 1 over e of it is value at the boundary.

So, if I go if I start from here and mover distance 1 over gamma from the boundary then the field will decay to 1 over e of it is value at the boundary. Then this is how I can quantify this distance up to which this field extends into the lower index region. So, it is defined by 1 over gamma, and it is known as penetration depth into lower index region. So, 1 over gamma is penetration depth into lower indexed region let us plot the field now for TE 1 mode. TE 1 mode is antisymmetric mode and the solutions would be given by sin function in the in the guiding film and exponential decaying function again in the lower index medium.

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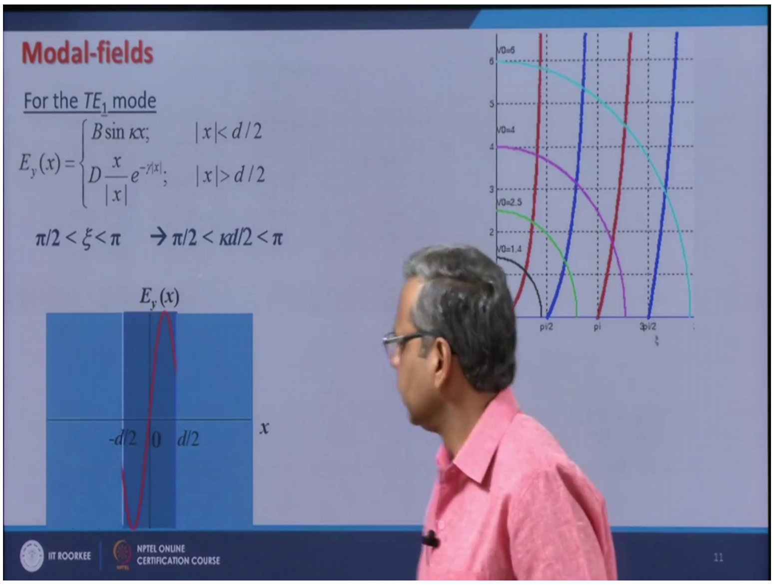

Now for TE 1 mode, psi would lie between pi by 2 and pi psi would lie between pi by 2 and pi, because this the green curve will cut the blue curve only in this region. Now if I which means that kappa d by 2 would lie between pi by 2 and pi, and now in the same way if I plot the field in the guiding film first. So, there would be a sin function ok.

But at the boundary it has to be less than pi. So, kappa d by 2 has to be less than pi. So, it would not cross another 0 there would only be 1 0 which is at x is equal to 0. So, in the guiding film the solution would be like this and in the lower index region the exponential decaying term will take over and this is how the modal field would look like. So, I can see that if I go back then here for TE 0 mode there was no 0 in the modal field there was no 0 crossing for TE 1 mode there is only one 0 crossing.

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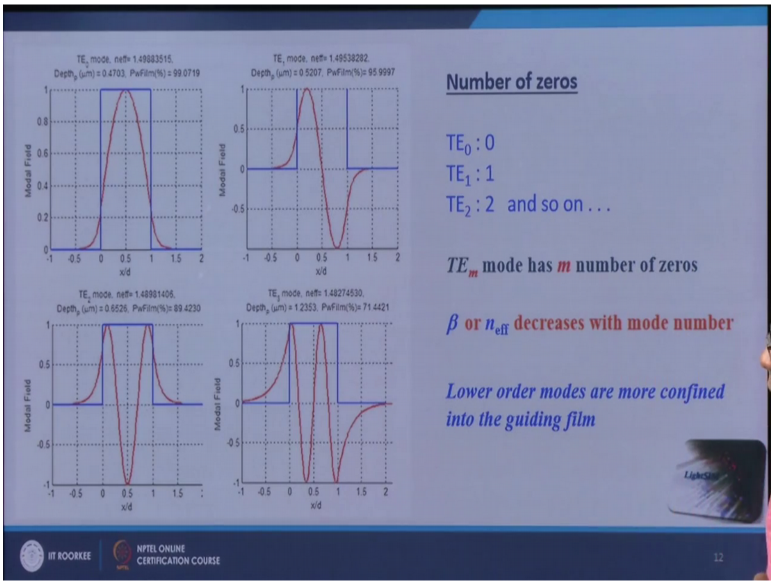

And if I go on then TE 2 mode has 2 0 crossings TE 3 mode has 3 0 crossings which means that if it is TE m mode then it will have m number of 0s ok. It will have m 0 crossings, this is one thing another thing that I see is when I solve this, that with the number of modes the propagation constant decreases right, if you remember the b V curves.

So, you have this is V which is normalized frequency this is b which is normalized propagation constant, and b always lies between 0 and one for guided modes. I had seen that TE 0 mode goes something like this. This is TE 0 mode TE 1 mode it goes something like this, this is TE 1; TE 2 mode like this. So, so if I have a value of V somewhere here, then the propagation constant of TE 0 mode is here, propagation constant of TE 1 mode is here, propagation constant of TE 2 mode is here. So, the propagation constant decreases as the ode number increases.

I also see that the lower order modes are more confined into guiding film. So, you can see that TE 0 mode it has more power here. And as I and the and the tail into n 2 region is a smaller, but as I go to higher order modes this tail extends more and more. So, if the number of mode increases if the number of if the number of ode increases then the corresponding confinement in the code decreases.

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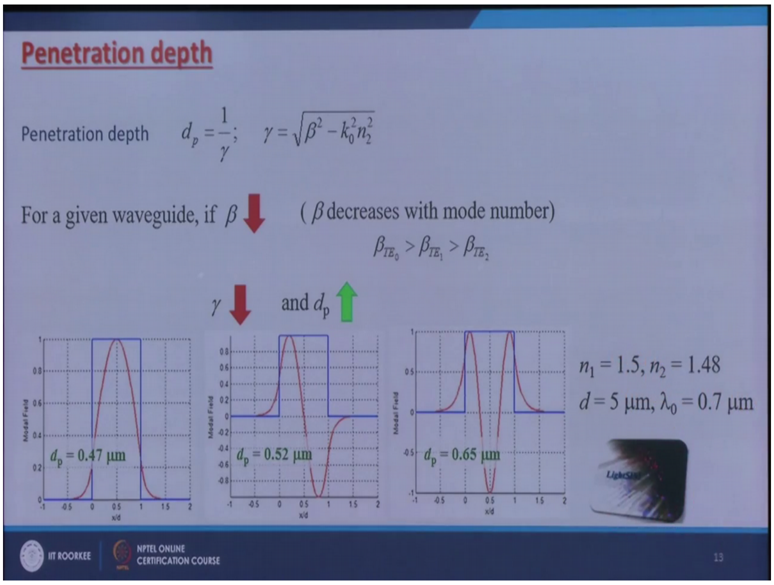

I can see the effect of this penetration depth I can see this penetration depth. So, for a given waveguide if beta decreases because penetration depth is defined by 1 over gamma and gamma is equal to beta square minus k naught square n 2 square it is a square root.

So, if beta decreases then, then if beta decreases then gamma decreases which means penetration depth increase and beta decreases with mode number. So, that is why the penetration depth increases. So, if I have this waveguide and this wavelength and I see the mode profiles of different modes. So, this is TE 0 mode it has penetration depth which is about 0.47 micrometer for TE 1 mode this is 0.52 micrometer and for TE 2 mode it is 0.65 micrometer.

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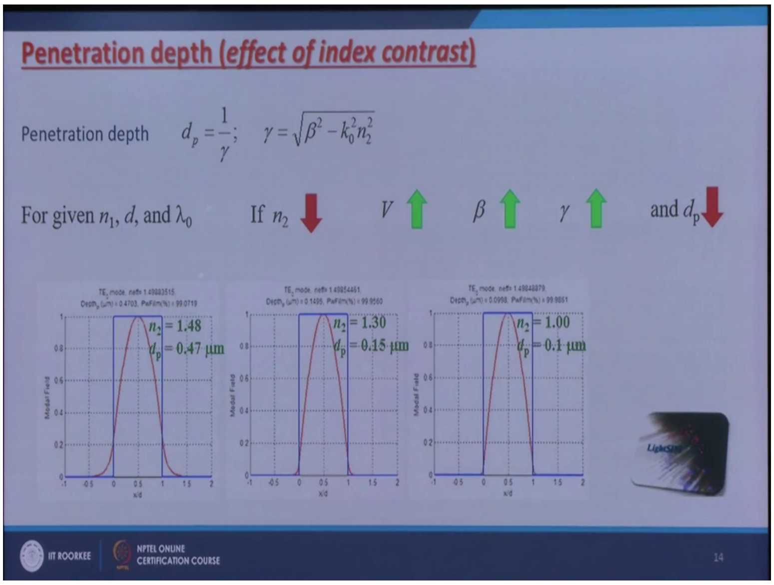

What is the effect of index contrast on penetration depth? Well again, the penetration depth is given by this, so for a given n 1 d and lambda naught. Now if I change the value of n 2 and hence the index contrast, for example, if I decrease n 2 if I decrease n 2 then the value of V increases. The value of V increases because n 1 square minus n 2 square increases. If V increases the propagation constant if V increases the propagation constant increases, beta increases if beta increases gamma increases and penetration depth decreases.

So, this is how it looks like. So, if I have n 2 is equal to 1.48. Then penetration depth is 0.47 micrometer. For TE 0 mode for the same mode if I change the refractive index n 2 from one point 48 to 1.3 then you can see the penetration depth decreases the field is pushed more towards the guiding film. And if I change to one then it is even more confined into the high index region and penetration depth decreases to 0.1.

What is the effect of wavelength if for a given waveguide if I decrease the wavelength? If I decrease the wavelength then I am increasing the value of V which means I am increasing the value of propagation constant ok.

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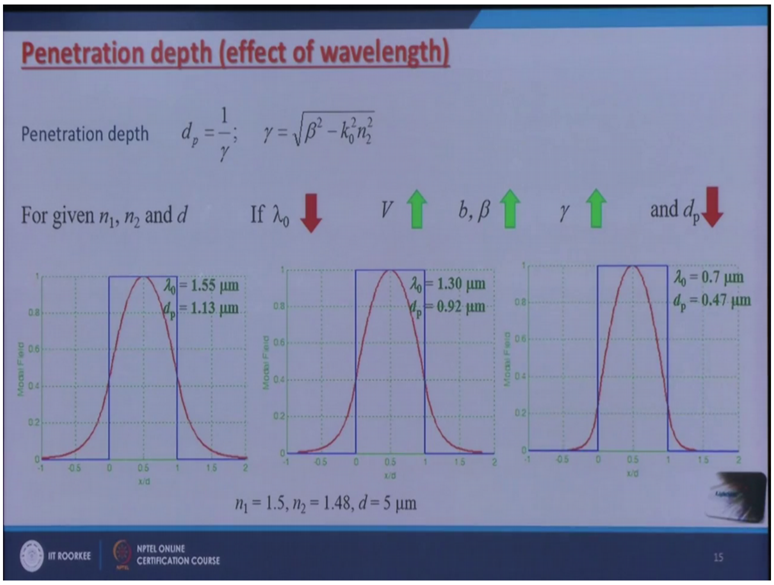

If I am increasing the value of b that is beta square minus k naught square n 2 square increases gamma increases. So, penetration depth decreases. So, this is the field of a given waveguide when operated at 1.55 micrometer you can see how much the field extends outside and when I decrease the wavelength to 1.3 the field is pushed more towards n 1 region and when it is 0.7 micrometer than the penetration depth is much smaller.

So, this is the; this is how this is how the modal field would vary with different parameters of the waveguide. And wavelength in the next lecture we will have more inside into the modes and we will understand what these modes basically are. We will have physical understanding of modes.

Thank you.