Fiber Optics
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Lecture – 13
Electromagnetic Analysis of Waveguides – III

Let us continue our analysis on planar dielectric optical waveguides. If you remember that we were doing the analysis for confinement in x direction and propagation in z direction.

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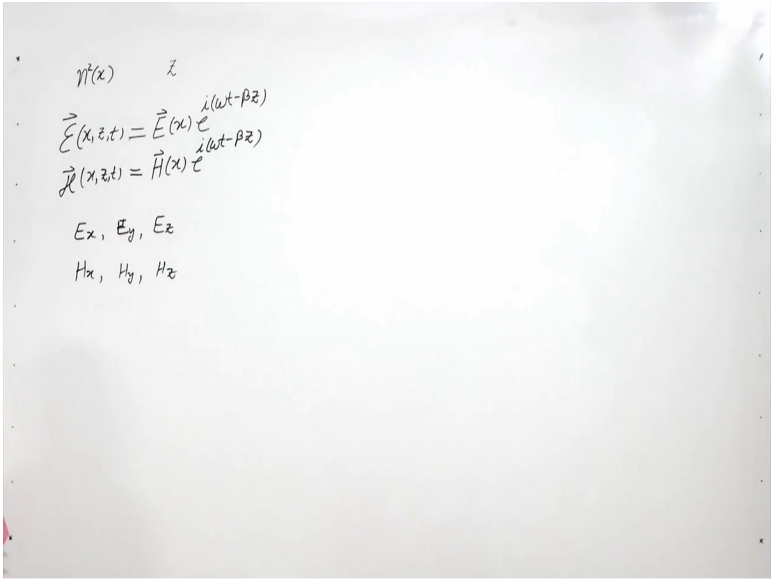

And we had seen that if the confinement is in x direction and the propagation is in z direction. Then the electric and magnetic fields associated with the light wave can be given as E x y z t. In fact, here you need not to take why because I am taking the planar waveguide. So, so it is simply E x z t vector is equal to now it is E e 0 or E of x e to the power i omega t minus beta z.

Similarly, H is x z t is equal to H of x this is H of x e to the power i omega t minus beta z. So, they represent the modes they represent the modes E of x and H of x and we know that since this is this is propagating in z direction. Then what we have is basically E x, E E y, E z all these components will be there and H x H y H z all these components of E and H would be there. This is this is a more propagating inside direction and you should remember that these E and H are not constant. So, these are not plane waves, these are not plane waves that is why I will have all the components here ok.

What I want to find out now is what are the values of beta and what are the corresponding E and H, they define the modes. And we were doing the analysis of this; we were doing the analysis for symmetric step index planar waveguide where this n of x is defined like this. And this waveguide is symmetric about x is equal to 0, it has high refractive index region n 1 and low refractive index regions n 2 and the width of the high index region is t.

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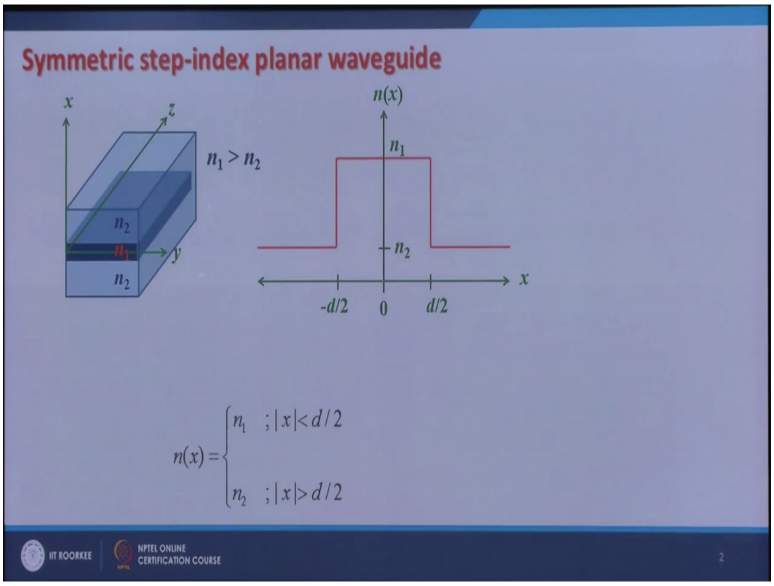

So, what we did we categorize the modes of this waveguide using the symmetry of the structure into symmetric and antisymmetric modes and for symmetric modes.

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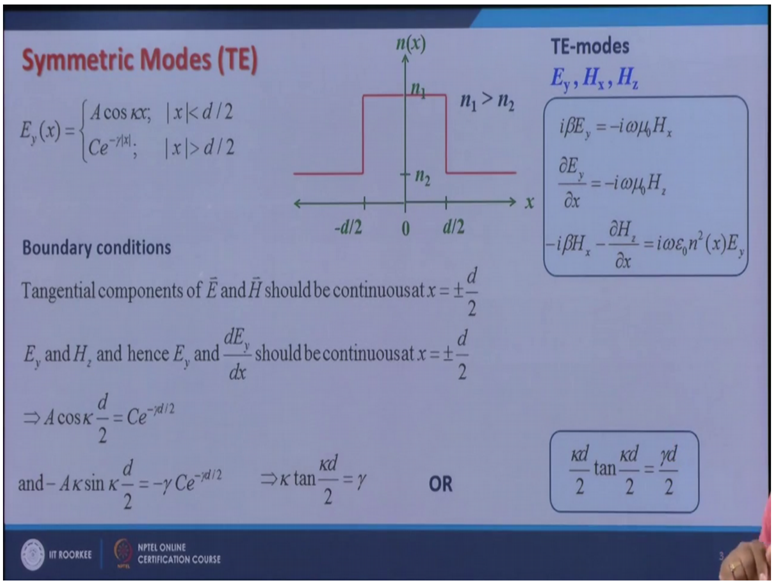

The field the electric field we were doing the analysis of T E modes. So, for T E modes the field is given by this, A cosine kappa x for mod x less than d by 2 that is in the guiding film. And C e to the power minus gamma mod x for mod x greater than d by 2, which is in these 2 regions since it is t modes. So, the non vanishing components of E and H r E y H x and H z and E y H x and H z are related by these 3 equations. So, we apply the boundary conditions at the interfaces x is equal to plus minus d by 2, and the boundary conditions are the tangential components of E and H should be continuous at x is equal to plus minus d by 2, and the tangential components at x is equal to constant r E y and H z.

So, and since H z is related to E y if some constant d E y over d x that is why E y and d E y over d x should be continuous at x is equal to plus minus d by 2. So, when we applied this then we got the transcendental equation kappa d by 2 10 kappa d by 2 is equal to gamma d by 2. Now our task is now our task is to solve this equation, let us see how we can solve this equation and get physical insight into the modes of a waveguide.

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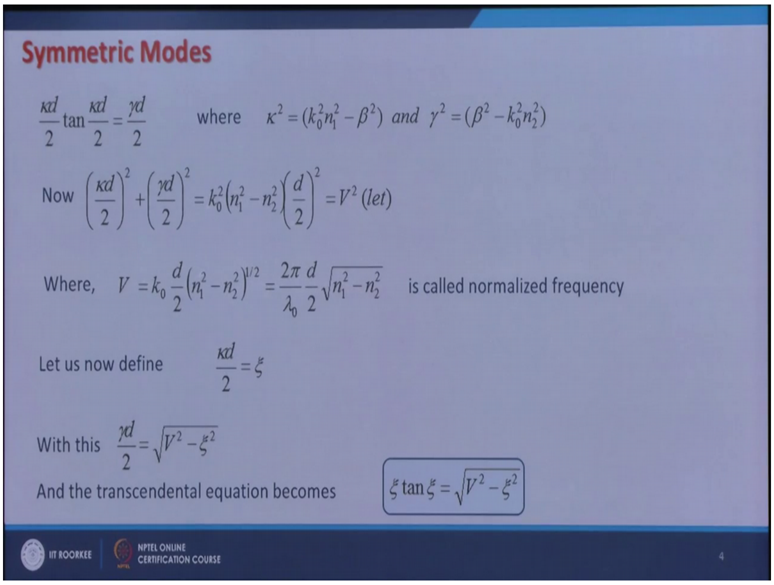

So, to do that what we do when we define we what we do we take this transcendental equation kappa d by 2 tan kappa d by 2 is equal to gamma d by 2. And here we see there are 2 terms appearing which contain kappa d by 2 and gamma d by 2 where kappa square is given by this and gamma square is given by this.

So, if I add these 2 up kappa d by 2 square plus gamma d by 2 square then I get k naught square n 1 square minus n 2 square times d by 2 square. What does it have? It has all the waveguide parameters and the wavelength because k naught is 2 pi over lambda naught. So, so by adding these 2 up I get something I get something which contains only the waveguide parameter waveguide parameters, and the wavelength which are the inputs basically.

So, let me define this since everywhere there is a square. So, let me define this as V square and where V is k naught d by 2 square root of n 1 square minus n 2 square or 2 pi over lambda naught times d by 2 square root of n 1 square minus n 2 square. And this I call normalized frequency, because it contains lambda in the denominator. So, I call this normalized frequency or generally I also call it V parameter. So now, in order to in order to solve the transcendental equation graphically what I do I define this kappa d by 2 this kappa d by 2 as some parameters psi, some variable psi.

Because this is the transcendental equation in beta kappa contains beta the only unknown here is beta. So, let me define kappa d by 2 is equal to psi and therefore, if I put this kappa d by 2 is equal to psi here then gamma d by 2 automatically becomes a square root of V square minus psi square. So, if I put these 2 here in this equation the then the transcendental equation becomes psi tan psi is equal to V square minus psi squares square root.

So, in this equation I have all the input parameters in the form of V. So, if I have if I have all the waveguide parameters with me that is n 1 n 2 and d. And I have the wavelength at which I am operating my waveguide then I can calculate V from here and for that value of V I can solve this transcendental equation and get the values of psi and hence the values of beta. Similarly if I if I do the analysis for antisymmetric modes in the same way then it leads to the transcendental equation Which is given by minor psi cot psi is equal to square root of V square minus psi square.

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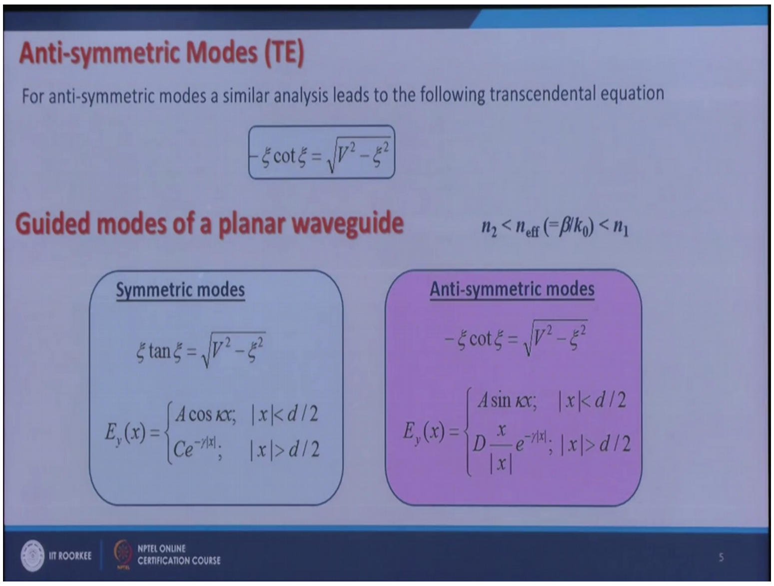

So, the guided modes of a planar waveguide where a symmetric or antisymmetric there it fall in the in the range of an effective line from n 2 to n 1, and they are given by E y of x is equal to A cosine kappa x for mod x less than d by 2 and C e to the power minus gamma x for mod x greater than d by 2 for symmetric modes this is the symmetric solution. And the values of beta which are allowed are given by psi tan psi is equal to square root of V square minus psi square.

So, these are symmetric modes, and similarly the antisymmetric modes satisfy the equation minus psi cot psi is equal to square root of V square minus psi square and the model field is given by in terms of sin function sin kappa x in the guiding film and again exponentially decaying function in the in the n 2 region or lower index surrounding. So, by solving these equation first I need to get the values of beta, only certain values of beta or certain values of psi are allowed, I first need to find out those values of beta and from those values of beta I can find out kappa and gamma those values I put here in the field and then I get the model field that is how I will get the complete solution for the modes of planar dielectric waveguide.

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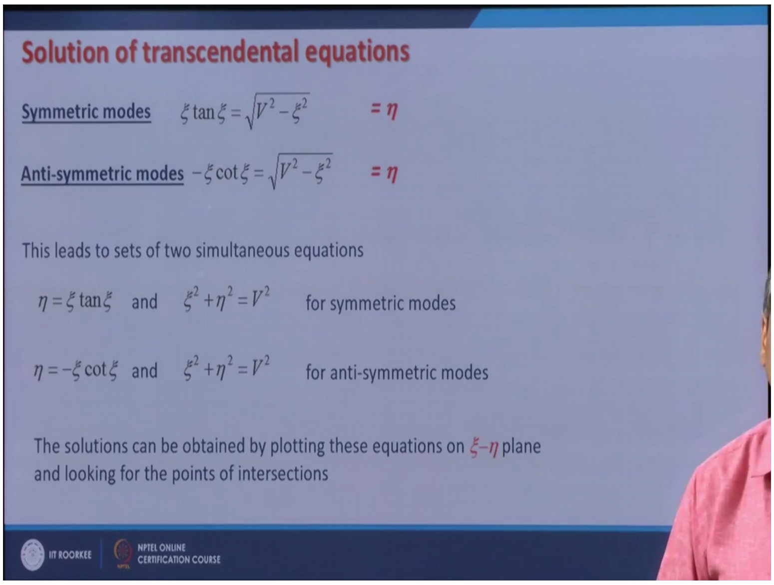

So, let us solve it. So, I have again symmetric modes antisymmetric modes in order to solve these 2 equations what I do I equate both the l H s and r H s of these equations to some variable eta. So, in this way I will have sets of 2 simultaneous equations for symmetric modes as well as for antisymmetric modes, and these are eta is equal to psi tan psi from here and if I equate this to eta then it gives me psi square plus eta square is equal to V square. These 2 equations are for symmetric modes and similarly for antisymmetric modes I get the equations eta is equal to minus psi cot psi and psi square plus eta square is equal to V square.

Now, I can obtain the solutions of these equations by graphical method. So, what I need to do I just plot these 2 equations on psi eta plane and look for the points of intersection. Those point of intersection will give me the values allowed values of psi and hence the allowed values of beta ok.

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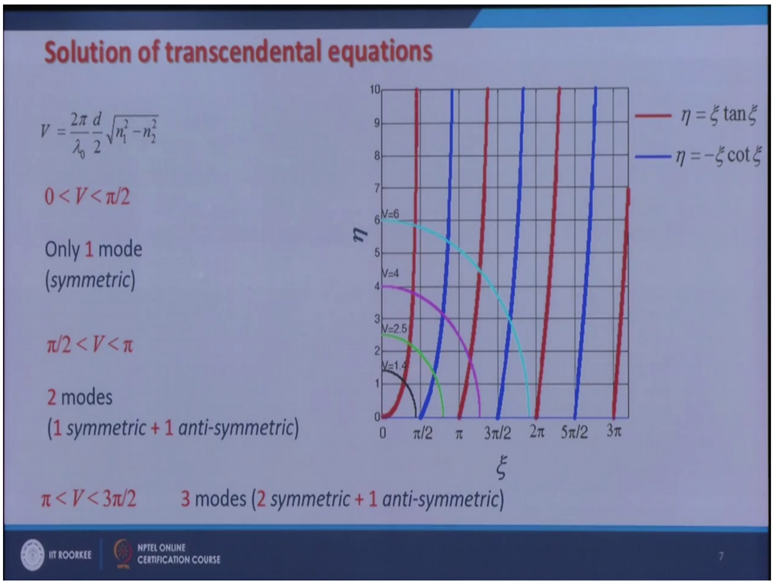

So, let us plot these. So, here the red line, the red line shows the equation eta is equal to psi tan psi, which corresponds to symmetric modes. And these blue lines they show the plot for eta is equal to minus psi cot psi they correspond to antisymmetric modes. And these circles for different values of V are basically psi square plus eta square is equal to V square.

So, for a given V for a given value of V, I take the circles. And see the points of intersections of these circles with these blue and red curves and they will give me the allowed values of psi. So, let us examine these let us examine these. So, V is this. So, for a given waveguide that is n 1 n 2 and d and given wavelength that is lambda or lambda naught, V is known. If this V lies between 0 and pi by 2 if this V lies between 0 and pi by 2 we can see that there would only be one point of intersection. And that would be with the red curve that is corresponding to symmetric modes.

So, this point of intersection will give me the value of psi for this symmetric mode. So, in this range if V lies in the range 0 2 pi by 2, then there is only one mode, which is possible and this mode is symmetric mode. Because it is obtained from the intersection of red curve with the circle and red curve corresponds to symmetric modes. Now if V lies between pi by 2 and pi for example, if V is equal to 2 pint 5 I see 2 points of intersection. One is with red and one is with blue. So, one is with symmetric and another is with antisymmetric. So, I get 2 modes one symmetric and one antisymmetric. Then if I look in the range pi 2 3 pi by 2 if V lies from pi 2 3 pi by 2 for example, V is equal to 4 then I get 2 symmetric modes and one antisymmetric mode ok.

So, in this wave for any given value of V I can find out I can find out the number of modes the number of symmetric modes the number of antisymmetric modes and their propagation constants.

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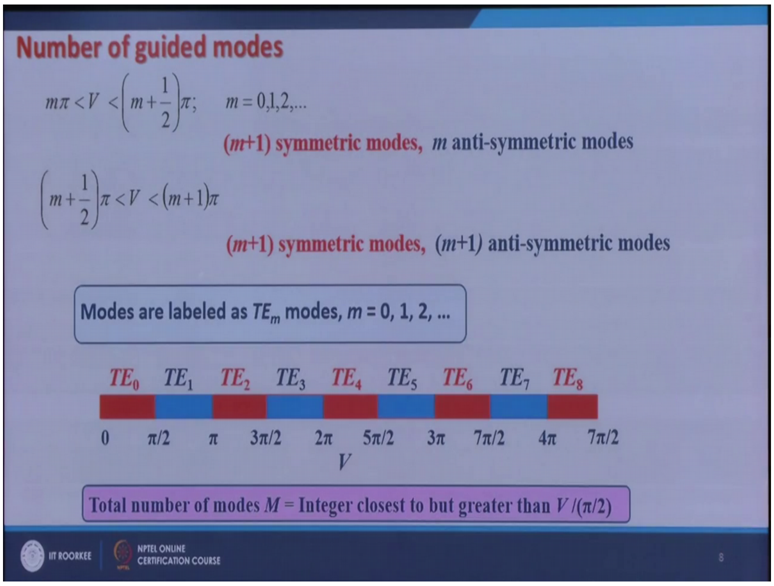

So, in general the number of guided modes I can find by looking at the value of V if V lies between m pi and m plus half pi where m is an integer 0 1 2 and so on, then I will get m plus 1 symmetric modes and m antisymmetric modes. On the other hand if V lies between m plus half pi and m plus 1 pi then I will get m plus 1 symmetric modes and m plus 1 antisymmetric modes.

In general I can get the modes for any value of V and I labeled these modes as T E m mode where m is equal to 0 1 2 and so on. So, the very first mode is T E 0 mode than there is T E 1 T E 2 T E 3 and so on. So, that is how the modes are labelled. Now the number of modes I can also find out by looking them at V line. So, if I have V line and I divided into the sections of pi by 2, then what I see that from 0 to pi by 2 there would be T E 0 mode if V lies somewhere here than T E 0 and T E 1 if V lies somewhere here than T E 0, T E 1, T E 2 and so on.

So, every pi by 2 every pi by 2 I am adding one mode. So, in general if I know the value of V then the total number of modes can be found out by dividing this V by pi by 2. So, I divide this V by pi by 2 and get a number then the number of modes would be an integer closest to, but greater than that number closest to, but greater than that number. So, if this V over pi by 2 comes out to be 2.1, then the number of modes would be 3. Now what is the single mode waveguide?

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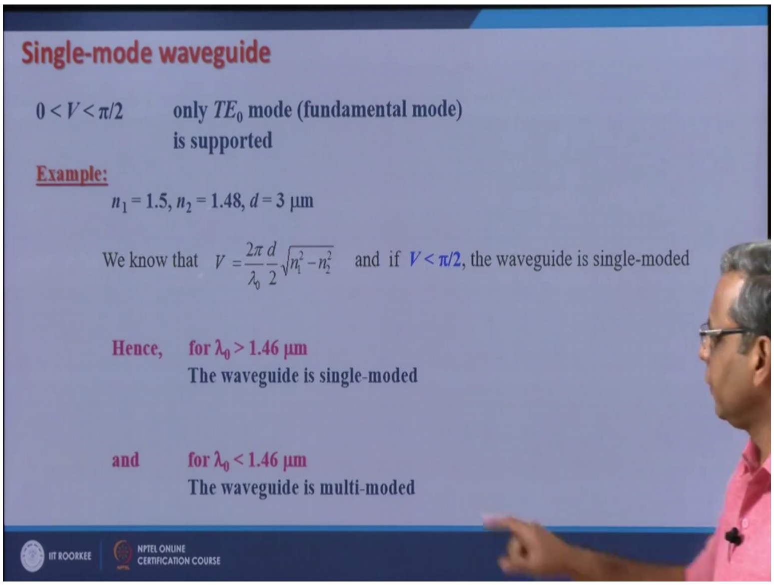

If V lies between 0 and pi by 2 that is V is less than pi by 2, then there is only one mode supported that is T E 0 mode and this T E 0 mode is the fundamental mode of the waveguide, I give you an example, if n 1 is equal to 1.5, n 2 is equal to 1.48 and d is equal to 3, then this waveguide can be single moded or multimoded it depends upon the wavelength I am using because V contains wavelength also apart from these 3 parameters.

So, so by just looking at waveguide parameters I cannot say whether the waveguide a single moded or multimoded I will have to say it is single moded at this wavelength or it is single moded in this range of wavelengths. So, I can find out that range. I know that V is equal to 2 pi over lambda naught times t by 2 times square root of n 1 square minus n 2 square and if V is less than pi by 2 then the waveguide a single moded than this gives me that for these parameters of waveguide if lambda naught is greater than 1.46 micrometer then the waveguide is single moded. That is for all the wavelengths longer than 1.46

micrometer the waveguide is single moded and for all the wavelengths which are smaller than 1.46 micrometer the waveguide is multimoded.

For all the wavelength shorter than 1.46 micrometer the waveguide is multimoded. Let us do few more examples. So, let us consider a dielectric step index symmetric planar waveguide, with n 1 is equal to 1.5 n 2 is equal to 1.46 and d is equal to 4 micrometer. For this waveguide let us calculate the number of symmetric and antisymmetric modes at wavelength lambda naught is equal to 1 micrometer.

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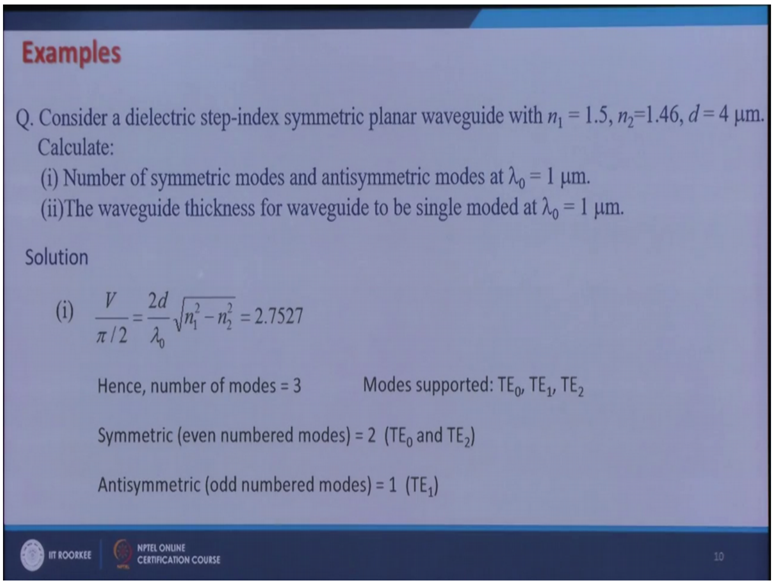

First part and second part the waveguide thickness for waveguide to be single moded at lambda naught is equal to 1 micrometer.

Well, so let us first solve the first part, I know for the number of modes what I should do? I should find out the value of V and divided by pi by 2. So, I should calculate V over pi by 2. V over pi by 2 comes out to be 2 d over lambda naught times square root of n 1 square minus n 2 square, because we know V is equal to V is equal to 2 pi over lambda naught timesd by 2 times square root of n 1 square minus n 2 square. So now, I put the value of d lambda naught n 1 n 2 here and I find out that this V over pi by 2 comes out to be 2.7527.

Which means that the number of modes should be an integer closest to, but greater than this and there it is why the number of modes would b 3. If there are 3 modes what modes are supported? Well we start from 0. So, T E 0 then T E 1 and T E 2; T E 0, T E 1 and T E 2 and we see that all the even number modes are symmetric modes and odd number modes are antisymmetric modes that is how we got the solutions also. So, so here we have 2 even number modes and one odd number modes. So, so we have 2 symmetric modes and one antisymmetric mode.

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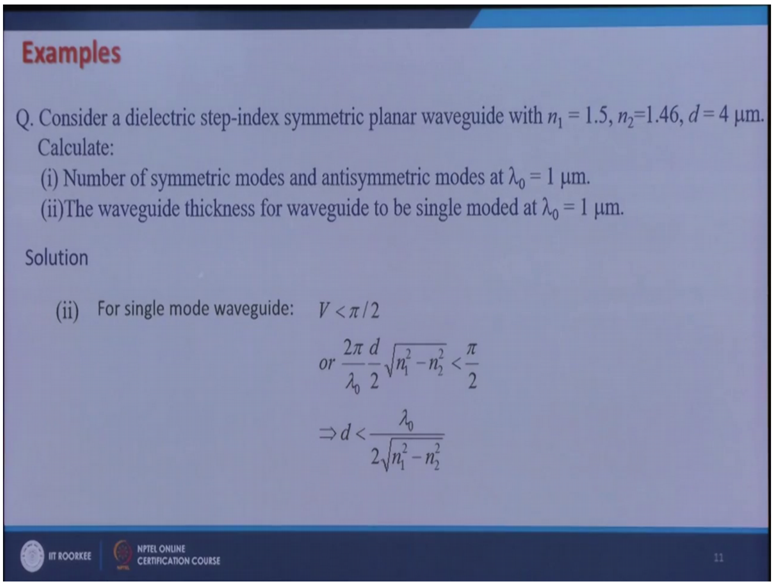

Let us do the second part, waveguide thickness for waveguide to be single moded. We had seen that that if the waveguide thickness is 4 micrometer, then at lambda is equal to 1 micrometer the waveguide supported 3 modes. Now if I still want to operate at lambda naught is equal to 1 micrometer and want my waveguide to be single moded, then what should be the value of d? That is what I need to find out. So, I know again for single mode waveguide V should be less than pi by 2, which means 2 pi over lambda naught times d by 2 times square root of n 1 square minus n 2 square should be less than pi by 2 or d should be less than lambda naught divided by 2 square root of n 1 square minus n 2 square.

Now, I just put the values of lambda naught and n 1 n 2 it gives me for all the values of d is smaller than 1.5431 micrometer this waveguide would be single moded at lambda naught is equal to 1 micrometer. I can also find out the number of modes with wavelength. So, if I have a given waveguide let us say defined by n 1 is equal to 1.5 n 2 is equal to 1.46 and d is equal to 4 micrometer.

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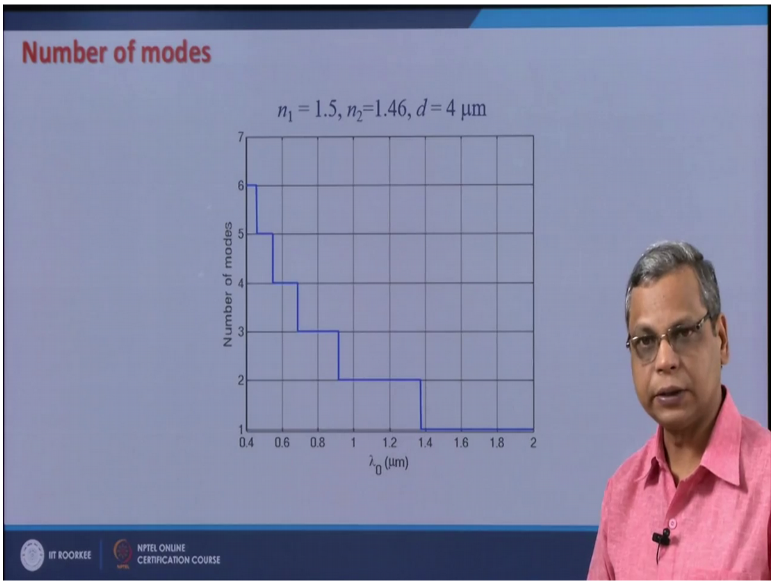

Then, then by if I change the wavelength if I change the wavelength the number of modes will change. If I start if I start from a wavelength which is say 2 micrometer then there would be only one mode supported, let us say and that would be T E 0 mode. If I decrease the wavelength now gradually then up to about little less than 1.4 micrometer there would be only one mode supported, that is T E 0 mode.

As soon as I cross this then T E 1 mode will also come into the picture. So, here I will have T E 0 and T E 1 up to this point let us this is about 0.9 micrometer as I further decrease the wavelength below 0.9 micrometer, then T E 2 mode will also come into picture now here you will have T E 0 T E 1 and T E 2 mode up to about let us say it is little less than 0.7 micrometer. And then we I will have T E 3 mode and so on. So, so as I decrease the wavelength as I decrease the wavelength of operation in a given waveguide the number of modes will change.

So, I go to shorter wavelengths I increase the number of modes. So, this is how I can find out how many modes would be supported and how the number of modes will change with wavelength, how the number of modes will change with waveguide parameters. What is still remains to find out is the values of beta, the values of propagation constant, the values of the model fields the expression for model field the field plots. So, for that what I need to do now I have already got the value of beta, but approximate value approximate value of beta. This approximate value of beta I can refine by using numerical techniques. I can I can solve the transcendental equation by various numerical techniques and I can find out the much more accurate value of beta and from those values of beta I can find out the model field ok.

So, in the next lecture in the next lecture we would look into the model fields and before that. In fact, I would also like to define these transcendental equations in terms of normalized parameters. We have already defined one normalized parameter which is normalized frequency V. I would like to also define a normalized para meter with respect to propagation constant. So, normalized propagation constant and the idea of idea of defining these normalized parameters is to have certain universal curves certain universal solutions which do not depend upon waveguide parameters or the wavelength. So, so in the next lecture we will we will do extend all this analysis by using the normalized parameters and then we will look into the model fields also.

Thank you.