Fiber Optics
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Lecture – 12
Electromagnetic Analysis of Waveguides- II

So, in this lecture, let me continue with the previous lecture where I was doing the analysis of planar mirror waveguide.

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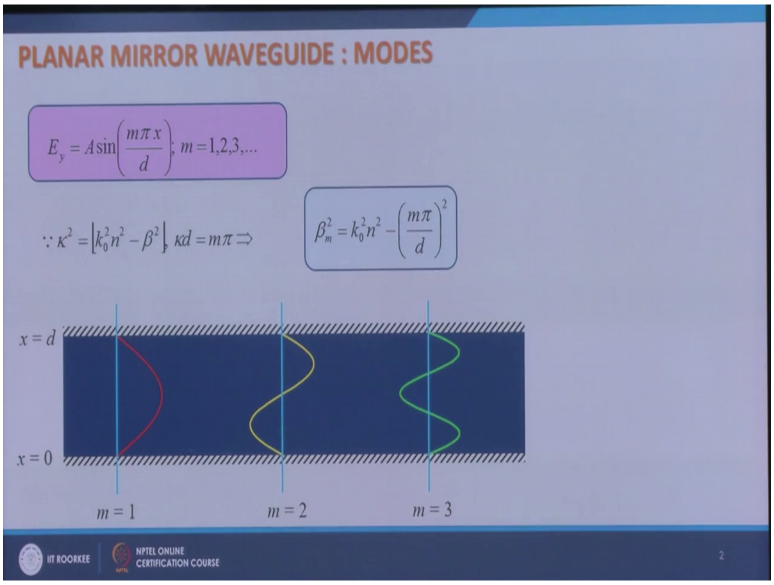

So, what I was doing was to find out the modes of a planar mirror waveguide and what I did that I took the planar mirror waveguide which has refractive index n and metal deposited at x is equal to 0 and x is equal to d, and the modes of this waveguide I found out as E y is equal to A sin m pi x over d and corresponding beta are given by beta m square is equal to k naught square n square minus m pi over d square.

So, what I got that I got this kind of variation of electric field E y with respect to x or this or this and these fields propagate in the waveguide and sustain their shape as they propagate. They also propagate with certain propagation constants beta 1, beta 2 beta 3 and so on. Let me understand what they are.

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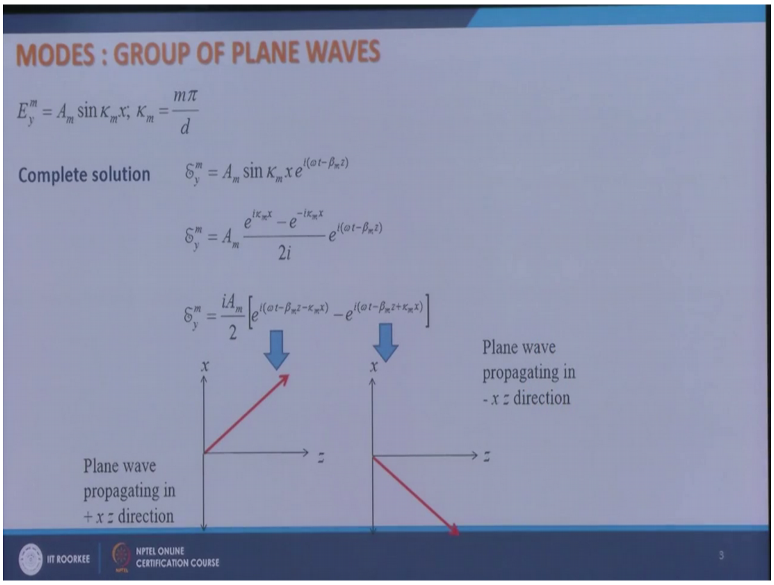

So, the field is A m sin kappa m x where kappa m is equal to m pi over d if I write the complete solution this is only E y m of x, x part of the solution remember that there is t and z parts also. So, if I write down the complete solution there it is some A m sin kappa m x, e to the power i omega t minus beta m z. Let me write this sin kappa m x in the form of e to the power plus minus I kappa m x.

So, if I do that I get this and then let me put it in this form. So, what I have got from here, I have got it e to the power i omega t minus b time z minus kappa m x and then e to the power i omega t minus b time z plus kappa m x what they are and remember you see now outside I have a constant it is not a function of x, the x dependence I have included in the exponential now, the x dependence which was earlier here I have included in the exponential and when I see this now this is nothing, but the plane wave because outside is a constant.

So, and this is the plane wave which is moving in this direction, moving in x z plane making certain angle from z axis. And this is a plane wave which is moving in plus z minus x plus z minus x direction k. So, what I have got that this mode, this mode is nothing but the superposition of 2 plane waves, this plane wave and this plane wave with some phases. So, it is the superposition of 2 plane waves 1 propagating in this direction another propagating in this direction. If you look at the waveguide if you look at the waveguide this is x this is z. So, this is x z plane this is x this is z. So, one wave is moving like this another wave is moving like this and this is what you have, that if you launch wave like this then it will get reflected then reflected from here, reflected from here.

So, all the time you will have 2 waves one going in this direction and another going in this direction. Let me put them together here in x z plane.

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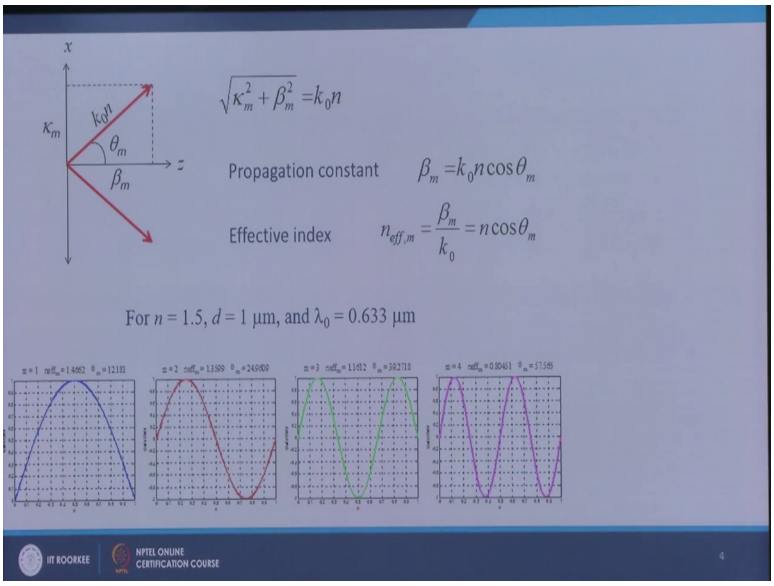

So, one is moving an angle theta m with z axis and another one is moving minus theta m angle with z axis and this has to be the propagation constant in the medium of a plane wave these are now plane waves plane waves we will have propagation constant k naught n. So, this is the component of propagation constant in x direction this is the component of propagation constant in z direction. So, if I now do kappa m square plus beta m square is square root then it comes out to be k naught n and it comes out to be k naught n because you remember that kappa m square is equal to k naught square n square minus beta m square which means that this is indeed k naught n this is indeed k naught n.

So, the propagation constant in z direction is k naught n cos theta m. Let us understand it. If I take the components in x direction I have 1 wave going in positive x another wave going in negative x, which means that in x direction I have 2 counter propagating waves and when I have 2 counter propagating waves they give you a standing wave solution they give you standing wave. So, in x direction I have a standing wave. In z direction if I see this also gives me propagation in positive z this also gives me propagation in positive z.

So, I have a standing wave in x which propagates in z. The energy does not flow out in x direction the wave stands in x direction, but that standing wave pattern flows in z direction this is the mode. So, these modes are nothing, but the standing wave patterns these modes are nothing, but the standing wave patterns and they are made out of 2 waves 1 going in this direction another going in this direction.

I can now find out what is the effective refractive index of a given mode or effective index of the mode by beta m over k naught which is nothing, but k n cos theta m. So, for every mode I have different beta m and therefore, the angles that the constituent plane waves make with z axis would be different and these angles can be found out from here. For example, for n is equal to 1.5 d is equal to 1 micrometer and lambda naught is equal to 0.633 micrometer helium neon wavelength. I find out that there are these modes four modes. Let me look at them closely.

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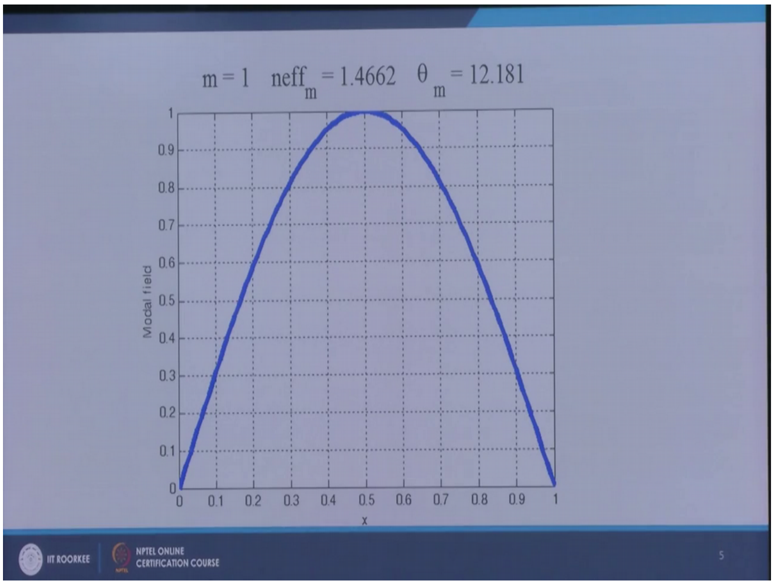

The first one has effective index 1.4662 and the angles of constituent plane waves with z axis is z axis r plus minus 12.18 degrees.

So, there are 2 plane waves one making plus 12 degrees and other making minus 12 degree with z axis and this is the a standing wave pattern in x direction corresponding to those waves.

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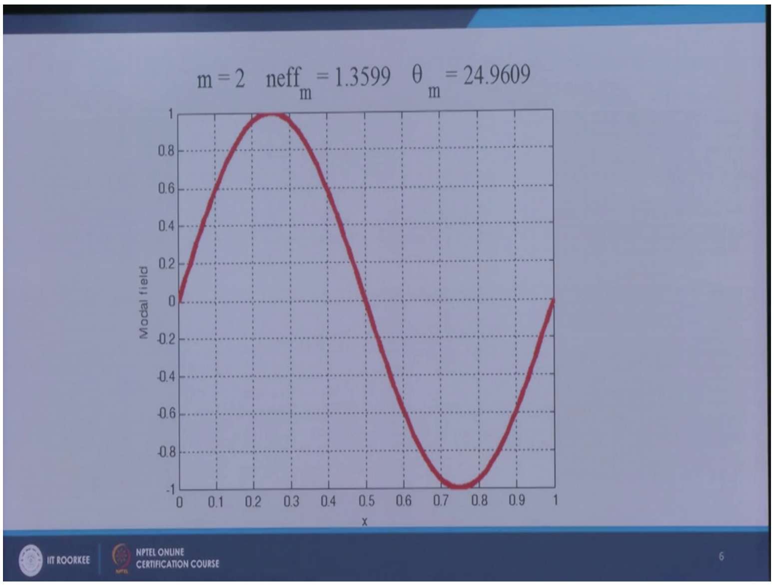

The second one is with n effective is equal to 1.3599 with angels plus minus approximately 25 degrees. Yet another one is this with the angles plus minus 39 degrees and this one plus minus 57.565 degrees.

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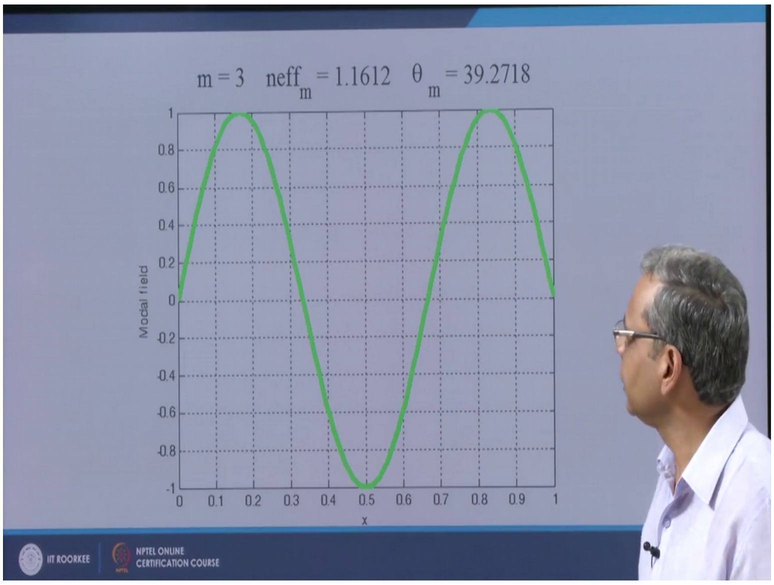

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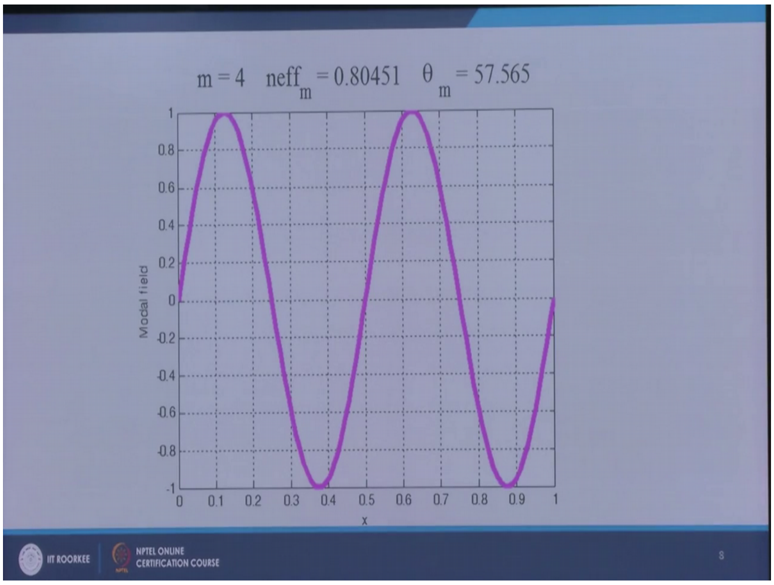

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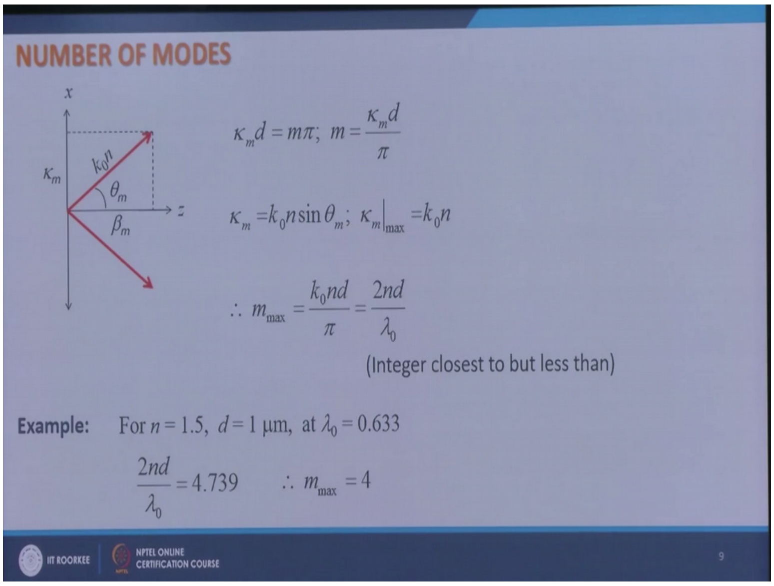

So, these are the modes how many modes are there for a given waveguide. Now let me look at this picture again and from here I know that kappa m d is equal to m pi. So, this m would be kappa m d over pi and kappa m is nothing, but k naught n sin theta m and the maximum value of this would be k naught n because the maximum value of sin theta is 1.

So, the maximum value of k m is k naught n and this will give me what is the maximum value of m. So, I put maximum value of k m here which is k naught n. So, m max would be k naught n by pi and k naught is equal to 2 pi over lambda naught. So, this would be 2 nd over lambda naught. So, the maximum number of modes that can propagate in a planar mirror waveguide would be a number and integer which is closest to, but less than this, it is closest to but less than this. For example, if I have n is equal to 1.5 d is equal to 1 micrometer and lambda is equal to 0.633 micrometer there should be micrometer here then 2nd over lambda naught comes out to be 4.739 and this gives me the maximum number of modes as 4. So, there would be 4 mode supported.

So, this is the analysis of planar mirror waveguide which gives me insight into the modes of a waveguide which is I structure which is quite intuitive and light guidance and data structure is also quite intuitive, but this is a structure planner mirror waveguide is of very little practical use for 1 it is not feasible to make this waveguide that you have one micrometer film of certain material and you deposit metal on top and bottom of that and use it. Second since it involves metal coatings and metal is highly absorbing material. So, as these molds propagate they will attenuate very quickly. So, they have very high loss they have very high loss. So, this kind of waveguide is of little practical use although it gives a good understanding of mode propagation.

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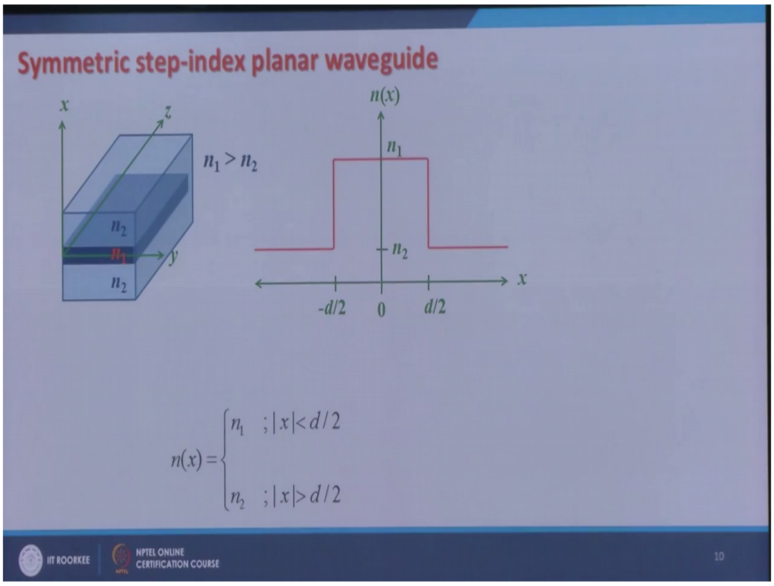

So, now what we are going to do is to look at more practical waveguide which is asymmetric step index planar waveguide dielectric waveguide planar dielectric waveguide which is obtained by sandwiching high index lab between 2 lower refractive index labs. So, I have n 1 n 2, n 2 here and n 1 is greater than n 2. Again if you look there is refractive index discontinuity only in x direction, y direction it is infinitely extended in z direction also there is no indexed discontinuity and it is infinitely extended.

If you look at the refractive index profile of this waveguide then it looks like this and let me put my x is equal to 0 access in the middle of the waveguide so that I can make use of the symmetry of the problem let it have of it d the width d. So, this is x is equal to d by 2 this is x is equal to minus d by 2. I can write down the refractive index variation with respect to x in this fashion.

So, this region is mod x less than d by 2 while these 2 reasons can be represented by mod x greater than d by 2. So, in mod x less than the d by 2 I have refractive index n 1, in mod x greater than d by 2 I have refractive index n 2 and now my problem is to find out E y and corresponding beta for this given n of x. So, how do I do? This again go back to the wave equation corresponding to t e votes. Let me first solve the problem for TE-modes.

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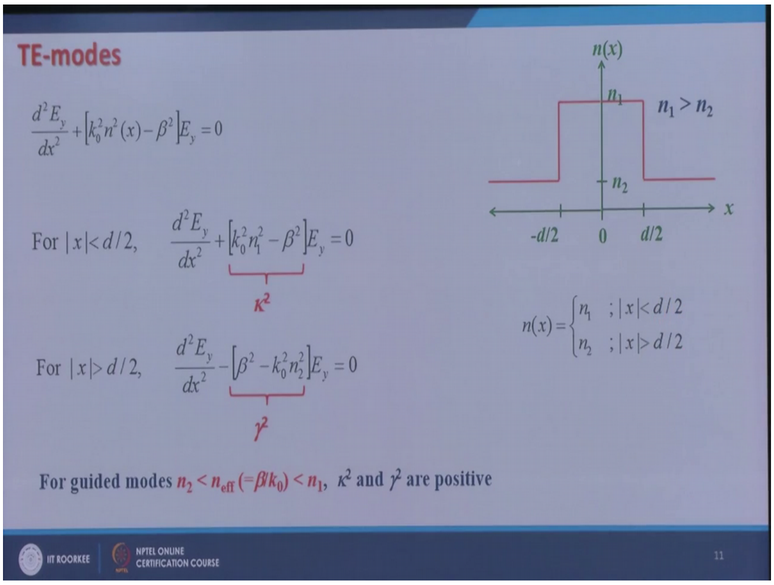

So, this is the wave equation for TE-modes and this is the n x. So, how do I solve it? Well I find that in this region and in this region and in this region in all the 3 regions the refractive index is constant although when I go from here to here and here to here I encounter index discontinuity, but on this side and on that side the refractive index remains constant.

So, I make use of this and write down the wave equation in this region and in this region that is in this region and in these 2 regions. So, for mod x less than d by 2 I write it down as d 2 E y over d x square plus k naught square n 1 square because n x is equal to n 1 in this region minus beta square times E y is equal to 0. And I also keep in mind that my effective refractive index which is given by beta over k naught. Effective refractive index n effective which is given by beta over k naught this should lie between n 2 and n 1 for guided modes for guided modes n effective would lie between n 2 and n 1 which means that beta would lie between k naught n 2 and k naught n 1. So, beta would be greater than k naught n 2 while it would be smaller than k naught n 1.

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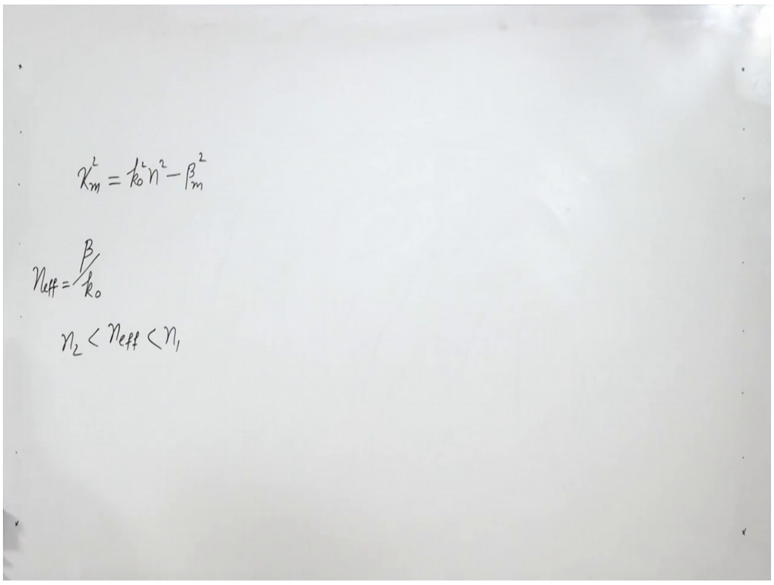

So, when I write this equation now in these 2 regions which are represented by mod x greater than d by 2 I write it as d 2 E y over d x square minus beta square minus k naught square n 2 square times E y because n x is equal n 2 here. And why I have written it in this fashion. So, that this is positive here and this is also positive here because I am solving it going to solve it for this condition which is the condition for guided modes for which the energy would be confined into this region. So, I represent this as kappa square and I represent this as gamma square and this kappa square and gamma square are positive for guided modes as I have explained. So, let me write these equations down again here.

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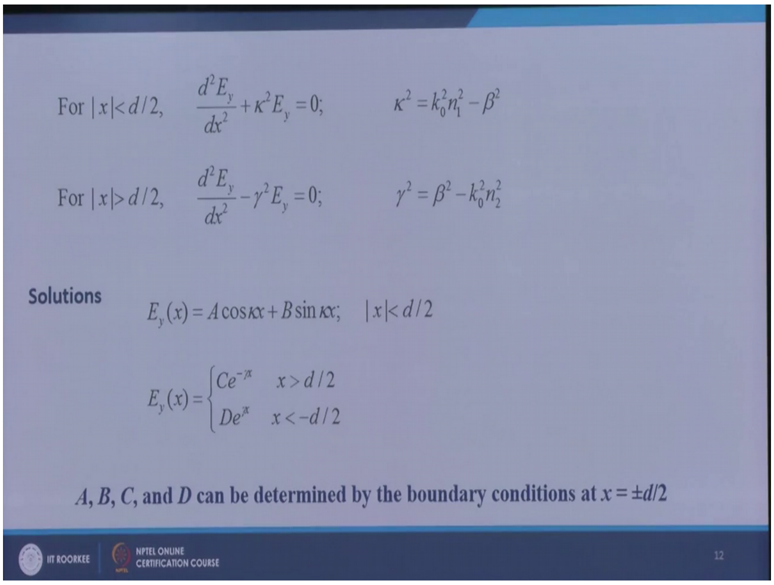

So, for mod x less than d by 2 I have a got a got an equation d 2 E by over d x square plus kappa square E y is equal to 0 and for mod x greater than d by 2 it is d 2 E y over d x square minus gamma square E y is equal to 0 where kappa square as this and gamma square is this.

What are the solutions? I know the solutions of these differential equations very well, this equation gives me oscillatory solutions e to the power I kappa x or e to the power minus i kappa x or sin kappa x cosine kappa x while this equation gives me exponentially amplifying and decaying solutions. So, if I write the solutions in the regions mod x less than d by 2 and mod x greater than d by 2 I find that in this region it is oscillatory solution A cosine kappa x plus B sin kappa x and in the region mod x greater than d by 2 I have exponential and decaying solutions, but out of these 2 solutions I retain only exponentially decaying solution while exponentially decaying solution and not exponentially amplifying solution because I want a solution which gives me guided modes and for guided modes the energy should be confined into the high index region and it should decay down as you go away from the high index region.

As you go towards infinity x is equal to plus minus infinity the energy should decay down. So, I cannot take exponentially employee find solutions that is y for x greater than d by 2 I have chosen the form of solutions Ce to the power minus gamma x and for x less than minus d by 2 I have chosen the form De to the power gamma x this A B C D are some constants which can be determined by the boundary conditions. What are the boundaries? Boundaries are x is equal to plus d by 2 and x is equal to minus d by 2, but here I can make some more simplification what simplification I can make is to utilize the symmetry of the problem how can I utilize the symmetry of the problem let us see.

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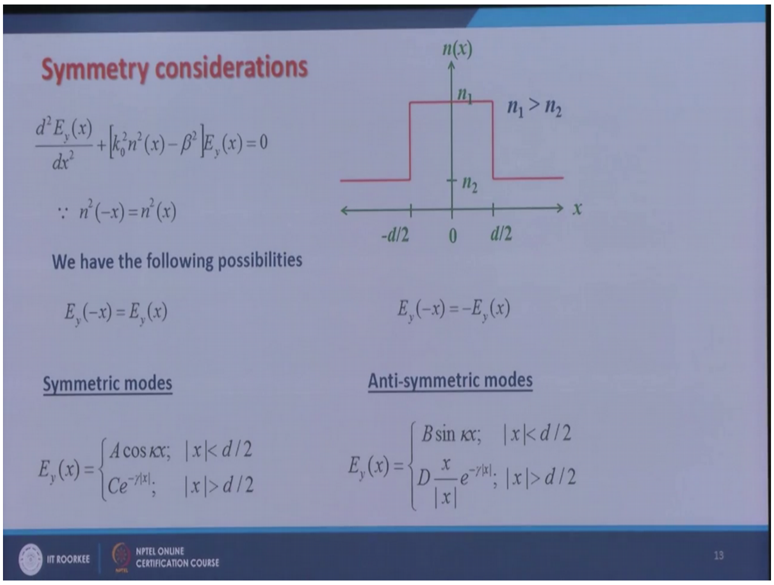

I have the wave equation for TE-modes like this and this is the n of x. I see that this is symmetric about x is equal to 0 which means that n square of minus x is equal to n square of x.

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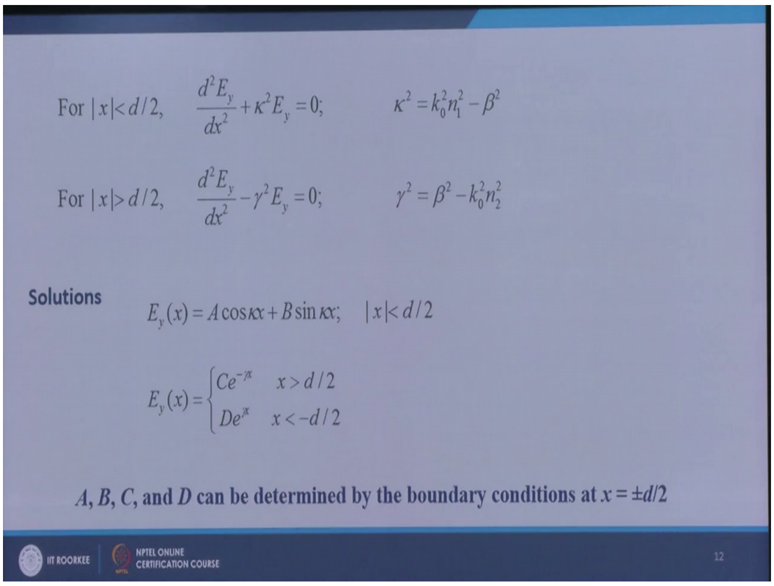

Now, in this equation if you replace x by minus x if you replace x by minus x, then you get d 2 E y of minus x divided by d of minus x square that is x square plus k naught square n square of minus x minus beta square E y of minus x is equal to 0 and this is nothing, but n square of x because of the symmetry of the profile which means that which means that I now have 2 possibilities which are E y of minus x is equal to E y of x if I put E y of minus x is equal to E y of x I get back the same equation or if I put E y of minus x is equal to minus E y of x then also I get back the same equation.

So, there are now 2 possibilities one is this another is this and they are known as symmetric modes because this is the property of a symmetric function and this is the property of anti symmetric function. So, these modes are known as symmetric modes and these modes are known as anti symmetric modes. The simplification it introduces is that out of those 2 functions cosine and sin I can pick one here and one here since cosine is the symmetric function and sin is the anti symmetric function.

So, symmetric modes would now be represented by E y of x is equal to a cosine kappa x for mod x less than d by 2 and Ce to the power minus gamma mod x for mod x greater than d by 2 for anti symmetric modes I will have E y of x is equal to b sin kappa x for x less than for mod x less than d by 2 and then I have De to the power minus gamma mod x for mod x less than d by 2, but I shall have to take care of sin when I go from left side or right side. What do I mean to say is this if this is x and this is d by 2 this is minus d by 2 this is x is equal to 0.

Now, if you plot a sin function in the region mod x less than d by 2. So, it goes something like this something like this and now in the n 2 region for positive x values you will for x greater than d by 2 you will approach from here and then it decays down while for x less than minus d by 2 it you will approach from here from negative side. So, here you start from positive side and decay down here you start from negative side and then decay down. So, to take care of that sin I will have to put x over mod x here.

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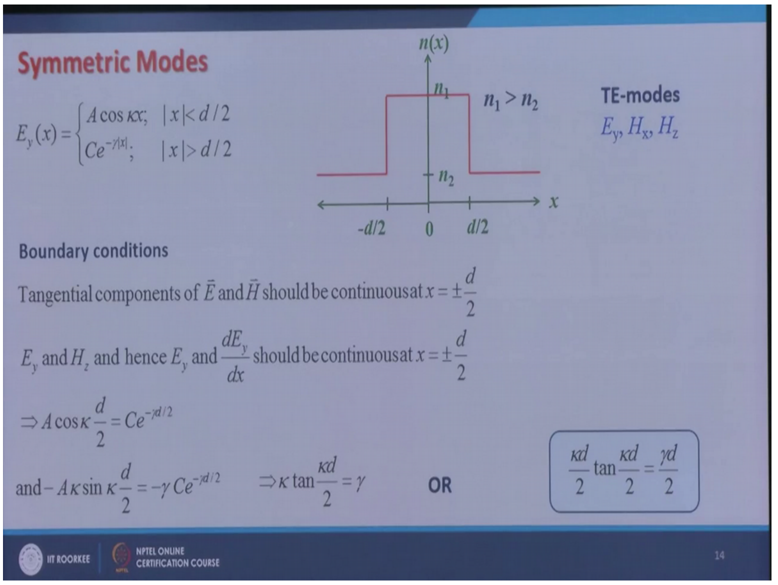

So, these are the symmetric modes and remember that I am doing the analysis of TE-modes have non vanishing components of E and H has E y, H x and H z. What is left now? I need to find out the relationship between A and C I need to find out how these solutions are connected at the boundaries for that I will use boundary conditions. And I have learnt that the boundary conditions are when you encounter an interface between 2 dielectric media the boundary conditions are tangential components of E and H should be continuous.

Tangential components of E and H should be continuous what are the tangential components to the boundaries. This is the waveguide this is x this is x is equal to plus d by 2 this is x is equal to minus d by 2, this is y this is z. So, to this interface which is x is equal to plus d by 2 or x is equal to minus d by 2 here the tangential components are y and z y and z, x is a normal component and here the non vanishing components are E y, Hx and H z. So, the tangential components are E y and H z. So, these E y and H z should be continuous and if you look back to the 3 equations which relate these 3 components of E and H then I find that H z is nothing, but some constant times d E y over d x. So, E y and d E y over d x should be continuous at x is equal to plus minus d by 2. So, let me apply these boundary conditions, let me do it for x is equal to plus d by 2 the same would be obtained for x is equal to minus d by 2.

So, I put E y should be continuous at x is equal to plus d by 2 which means a cosine kappa d by 2 should be equal to Ce to the power minus gamma d by 2 and then for derivative d E y over d x. So, I will get minus a kappa sin kappa d by 2 is equal to minus gamma Ce to the power minus gamma d by 2. If I divide this by this I get kappa tan kappa d by 2 is equal to gamma and remember that these kappa have dimensions of 1 over meter. So, I can make them dimensionless, so I can multiply both sides by d by 2 and get an equation like this which is kappa d by 2 tan kappa d by 2 is equal to gamma d by 2.

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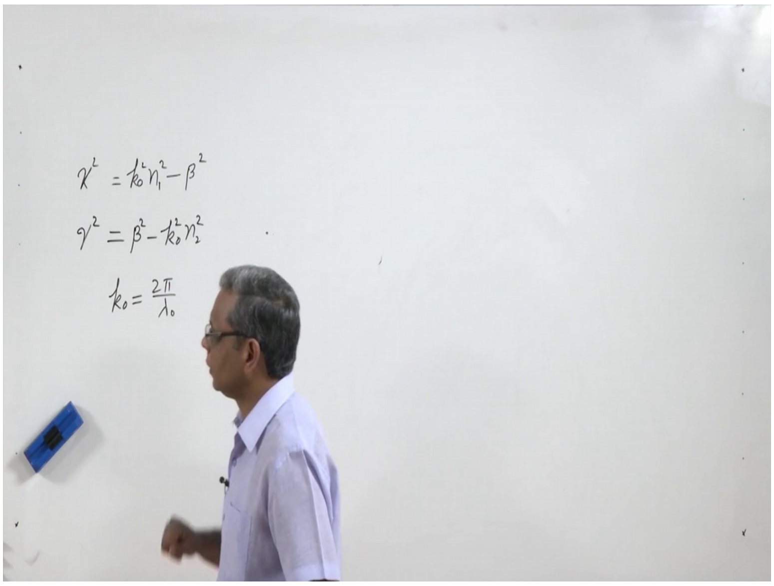

What is kappa and what is gamma? So, you remember that kappa a square is equal to k naught square m 1 square minus beta square and gamma square is beta square minus k naught square n 2 square k naught is equal to 2 pi over lambda naught. So, for a given waveguide and wavelength for a given waveguide and wavelength the only unknown in this in these kappa and gamma is beta. So, this is nothing, but a transcendental equation in beta. So, beta satisfy this equation. So, there are only certain values of beta which are possible and those values of theta satisfied this equation only those values of beta are possible. So, this makes this makes the modes discreet.

So, from here I find out what are the possible values of beta and for those beta I can find out the fields. So, that is how I can get the modes of a symmetric planar waveguides and these are symmetric modes.

In the next lecture I will continue with this to solve to find the solutions for anti symmetric modes.

Thank you.