Fiber Optics
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Lecture – 11
Electromagnetic Analysis of Waveguides – 1

Now, in this lecture we will start the analysis of optical waveguides using electromagnetic theory. Let us first look at types of waveguides, this is the bulk medium.

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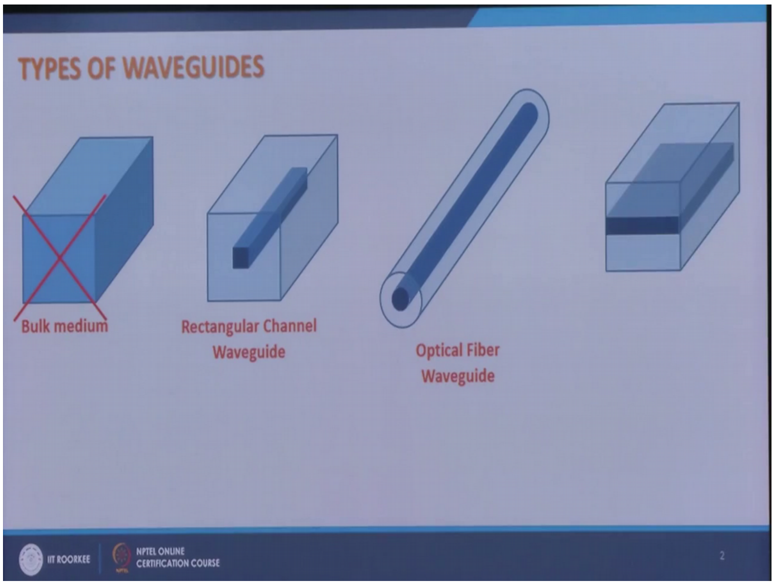

So, this is not a waveguide, when the dimensions are very large as compared to wavelength. We can have a waveguide like this where we have a channel or a material in this geometry and the dimensions are comparable to the wavelength and it is surrounded by a lower index medium like this. Then in this way I create a channel for light to flow along this. This is a rectangular channel waveguide. I can have this channel in the form of a cylinder high index refractive index cylinder which is surrounded by lower refractive index medium. Again all are comparable to wavelength the dimensions are comparable to wavelength and this is optical fiber waveguide. And then I can also have a configuration like this we are this width of the channel can be infinitely extended.

So, instead of having this kind of channel, I can have a slab thin slab of high refractive index. It is sandwiched between 2 other thick slaps of lower refractive indices. Then this is known as planner waveguide. In these 2, I have confinement in 2 dimension and propagation in this longitudinal direction. Here I have confinement only in one direction and propagation in this longitudinal direction. So, we are going to now do the electromagnetic wave analysis of such kind of media. We had seen that in infinitely extended medium where there were no such refractive index discontinuities, the solutions of the wave equations were plane waves.

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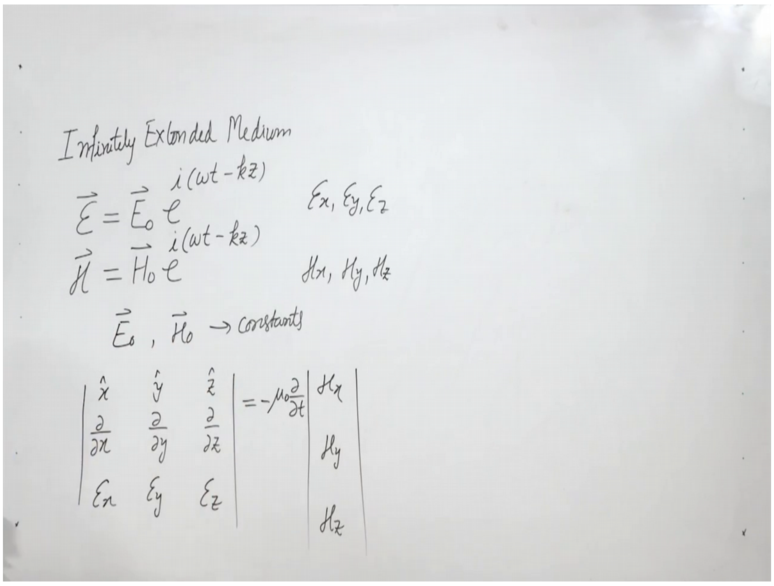

If you remember that the electric field associated with that wave in an infinitely extended medium if it is infinitely extended medium.

I had seen that E was given by E 0 e to the power i omega t minus k z and H was H 0 e to the power i omega t minus k z and I had these vectors.

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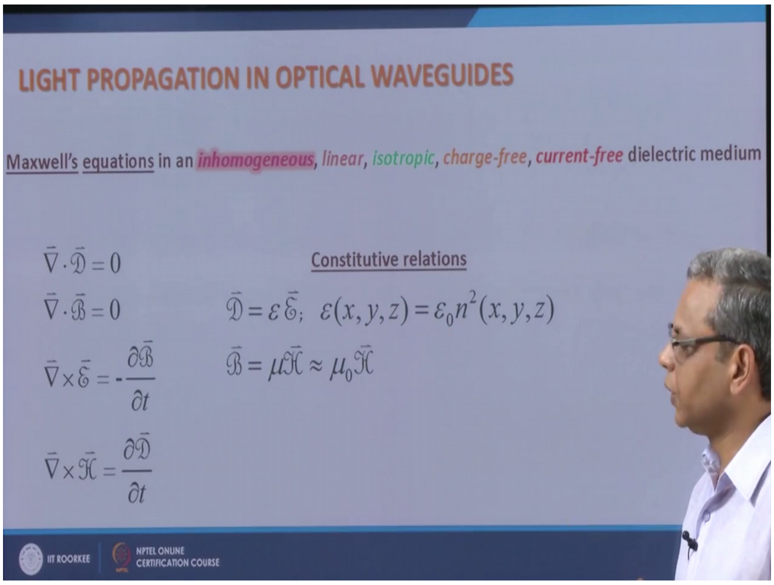

So, they have components E x, E y, E z, H x, H y, H z, this is H z. And the point to be notice here was that, these E 0 and H 0 they were constants. So, E 0, H 0 purely constants; what we could see that in case of infinitely extended medium our direction of propagation was z. So, I had z part coming out like this and then time part coming out like this. Here also what I will do? I will take this direction which is propagation direction is z and z part would be like this itself. So, in order to now find out the propagation of light waves in such kind of medium where we have refractive index discontinuity refractive index is not uniform, when I consider Maxwell's equations in homogeneous, linear isotropic charge free current free dielectric medium.

In the previous case it was homogeneous medium now it is in homogeneous medium. And again I write down all the Maxwell's equations Maxwell's equations go like this. But now if I look at constitutive relations, when D is equal to epsilon E this epsilon is now in general a function of x y and z, which is nothing but epsilon naught times n square of x y and z. So, this is not a constant scalar now, which was the case for infinitely extended medium. And again I am talking about dielectric. So, B is equal to mu H which is closely equal to mu naught times H because it is non magnetic medium. So, what should I do? Know again I should form the wave equation. So, that I can find out what are the electric and magnetic field solutions. So, the usual procedure to find out the wave equation is I take the curl of this equation and use other Maxwell's equations into it.

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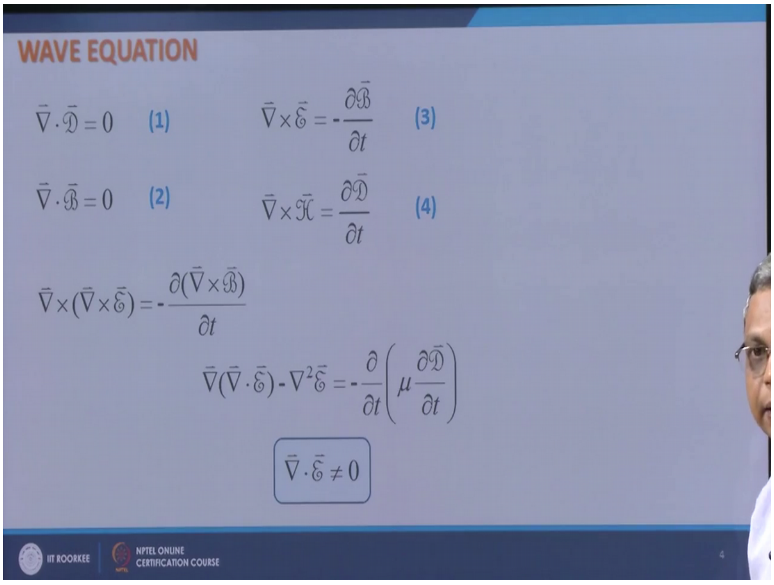

So, I get del of del dot E minus del square E is equal to minus del t of mu times del D over del t, thus in the same way as I had done in the previous case. But now here I should check that del dot E is not 0 I should be careful. In the previous case del dot E was 0, but now I have del dot D and D is equal to epsilon E epsilon depends upon x y and z.

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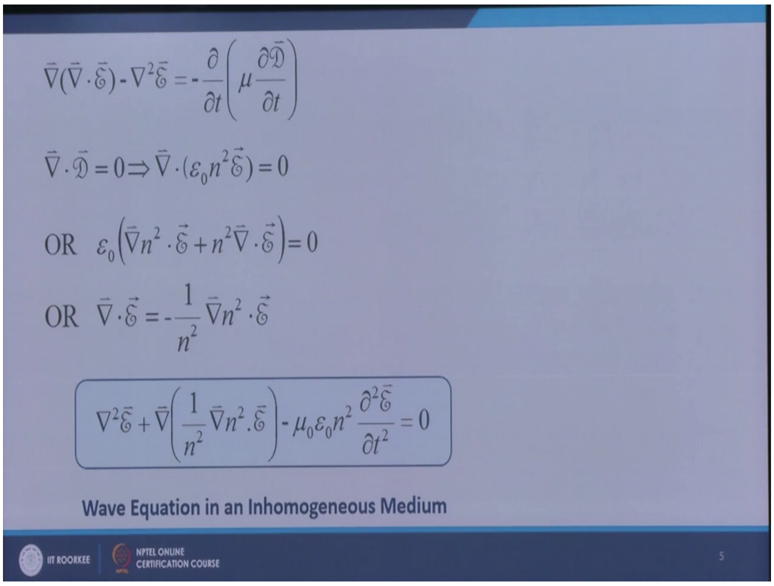

So, I cannot take that epsilon out of this del operator. So, del dot E would not be 0 if del dot E is not 0 then how this wave equation is going to change? So, let me know find out what is del dot E for that I take del dot D is equal to 0 and put D is equal to epsilon not n square E n square is a function of x y and z. So, I write it down as epsilon naught del of n square dot E plus n square del dot E and that should be equal to 0. This gives me del dot E is equal to minus gradient of n square over n square dot E. I put this back into this equation and rearrange the terms to get a wave equation in this particular form.

What it is del square E plus del of 1 over n square del n square dot E minus mu naught epsilon naught n square del 2 E over del t square. So now, you can notice an extra term here, this is an extra term that we have got it as compared to the case of infinitely extended medium.

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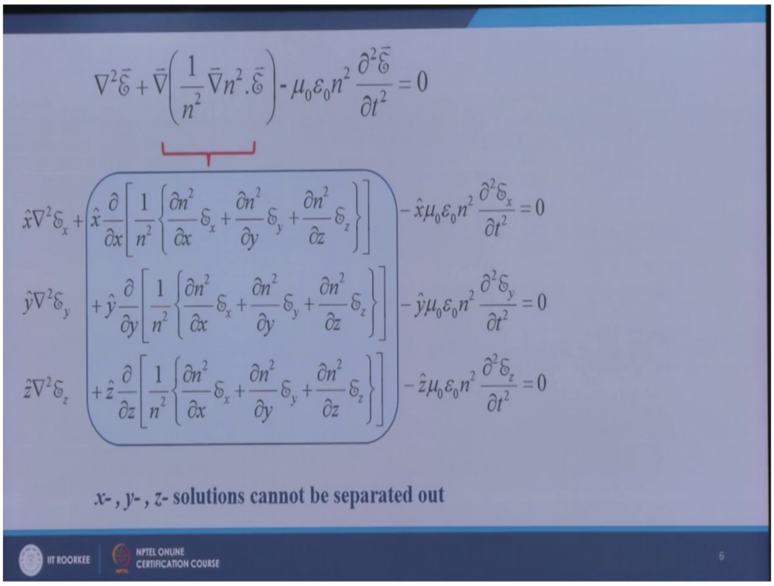

So, this is a wave equation in an in homogeneous medium. What is the implication of this term now? What complicates it can introduce in our analysis? Well if I expand this then I write down this x y and z components and if expand this what I see that now it is not possible for me to separate out x y and z. T can be separated out, that is not a problem. But I cannot separate out x y and z. X y and z solutions cannot be separated or which I could do in case of infinitely extended medium. So, what do I do now? Well similarly if I do it for the magnetic field I will obtain an equation in H something like this, again there would be a middle terms which makes it impossible to separate out x y and z solutions.

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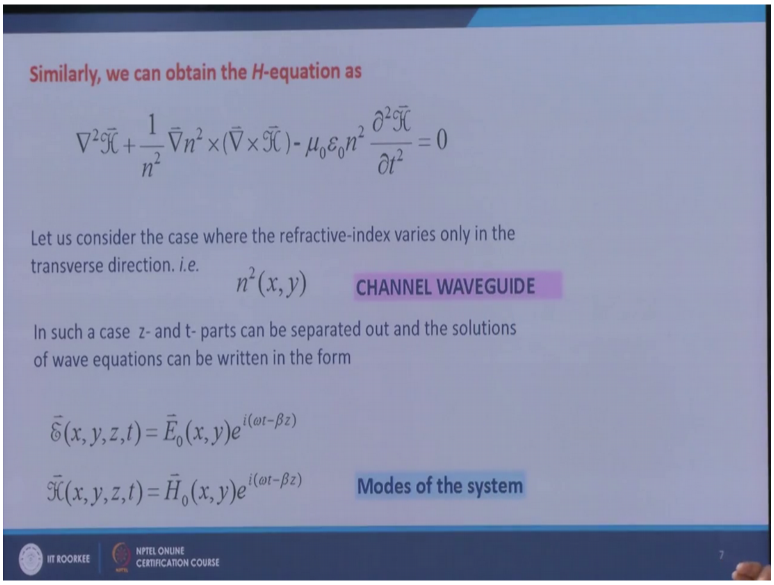

So now let me consider a case where refractive index where is only transfers direction which is the case of optical waveguides and optical fibers. So, so I take in general a case where n square is a function of x y which can be a channel waveguide or optical fiber. So, n square does not vary with z, in this case I can separate out z and t parts. And if I can separate out z and t parts then the solution z and t solution can be written in the same way as this. So, I write down z and t solution like this. And x y solution is still remains. So, I put it with E 0. So, the associated electric field I can write as E x y t is equal to E 0 of x y times e to the power i omega t minus beta z Similarly H.

Now what I have got? I have got that the solutions here have this form. So, this is a function of x and y and this is the propagation in z direction. So, as if some function of x and y is propagating in z direction with some propagation constant beta, similarly for H. So, these are the modes of the system. And as I will find out that there can be only certain such functions possible which sustain their shape and propagate with certain propagation constant beta these are the modes of the system. So now, my problem reduces to find out these functions, E 0 of x y and H 0 of x y and their corresponding propagation constants beta.

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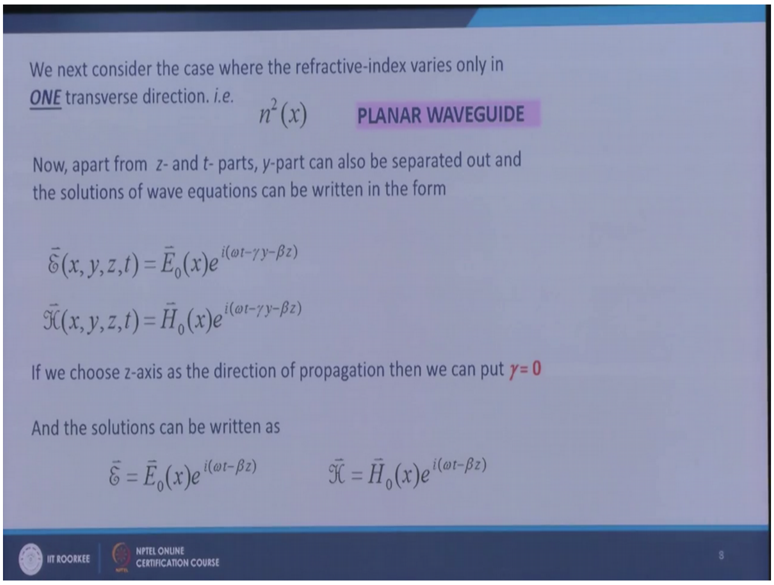

Let me the start with doing a very simple problem where I remove any index discontinuity even in y direction. So, I take the simplest case we are the variation of refractive index is only in x direction.

So, I have n square as a function of x only and this is the case of say planner waveguide something like this, where you have this is x this is y and this is z. So, where y is infinitely extended, z is infinitely extended and you have index cn discontinuity only in x direction. So, here you have different refractive index here different and here different. So, n square is the function of x only. Now if it is a function of x only then I can separate out y part also. And the solution I can write as e to the power i omega t minus some gamma y minus beta z. And x part will be now associated with E 0. So, E 0 is not a constant is it is a function of x some function of x. Similarly H is equal to H 0 x e to the power i omega t minus gamma y minus beta z.

So, these are the form of solutions now. What I can do? I can always choose my direction of propagation if the light is propagating in this direction I can label it as z or I can label it as y it is up to me to choose my access. So, what I do? I choose z axis as the direction of propagation then without loss of any generality I put gamma is equal to 0. So, you can see that if this is infinitely extended, this is infinitely extended index discontinuity is only in x. So, you can launch light into this in this direction and let the light propagate along y or you can let the light propagate along z you can launch it from here. So, I choose to take the direction of propagation as z. So, I put gamma is equal to 0.

Then I can write the solutions as E 0 of x e to the power i omega t minus beta z and x 0 of x e to the power i omega t minus beta z. So, I have got similar form of solution, but it is not the same. Here E 0 and H 0 are constants; here E 0 and H 0 are the functions of x and these functions now I want to find out.

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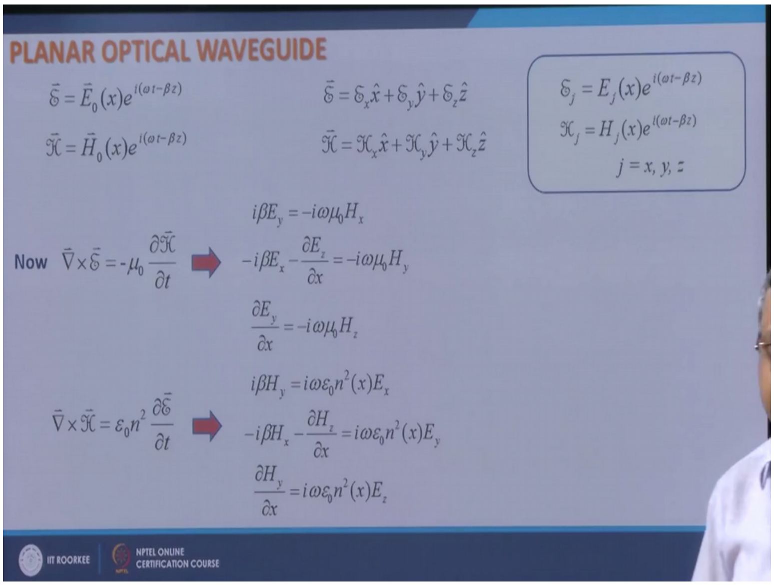

I need to find out how E varies with x and how H varies with x, that is what I need to know and they will give me the modes. So, I have the solutions here and do not forget that I have vector sins here which means this E is nothing but E x x cap plus E y y cap plus E z z cap and similarly H. So, basically I have 6 such equations 3 in E and 3 in H. So, if I write down the components of this then I can write them as E j is equal to E j x e to the power i omega t minus beta z and similarly H z and similarly H j where j can be x y or z.

Now, let me put these solutions into curl equations. Why I am doing this? Ultimately I want to find out how E varies with x and how H varies with x. So, I need to form a differential equation in E with respect to x. In order to do that and I know from here I will get del E over del x del H over del x terms. So, that is why I put these into these equations now. When I do this then this will give me 3 equations, one corresponding to H x then H y and H z. And this will also give me 3 equations, E x E y an E z x y and z components here x y and z components here, let me do it. The x component from here we will come out to be if you expand this, because you know that del cross E del cross E you can write as x cap y cap z cap del del x del del y del del z. And then you have E x E y E z is equal to So, this is del cross E minus mu naught del del t and then you have 3 components here H x H y H z.

So, from here you can find out 6 equations, the first one would be i beta E y is equal to minus i omega mu naught H x. The x equation from here would be i beta H y is equal to i omega epsilon naught n square of x times E x. The second one from here would be minus i beta E x minus de E z over del x is equal to minus i omega mu naught H y. And here it would be minus i beta H x minus del H z over del x minus i omega epsilon naught n square of x E y.

Third one would be del E by over del x minus i omega mu naught H z and here it would be del H y over del x. i omega epsilon naught n square of x E z. So, I have got 6 equations which relate the electric and magnetic field components E x E y E z and H x H y H z. What do I do with these equations? Well, what I notice one thing that I can simplify the situation to certain extent. And how can I simplify the situation? Well if I have if I have a waveguide and I launch light into this when launching light into this I have some control on light and that is I can launch this polarization or this polarization.

This is x axis this is y axis. So, if I decide to launch this polarization that is E y is non 0 and E x is 0 then let me see which equations do I invoke. Do invoke all the 6 equations or I invoke only a few of them?

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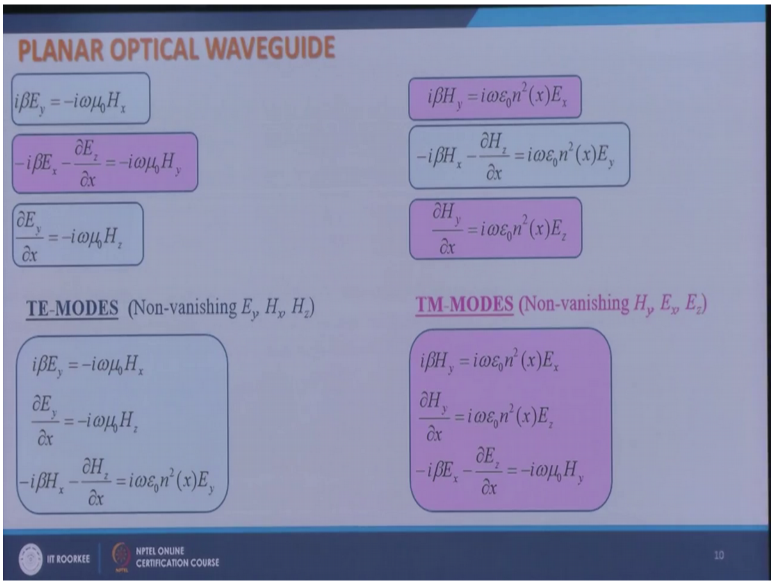

And what I find that the equations which have E y non 0 and E x 0 are this one has E y non 0 E x is equal to 0. This one has E y non 0 and this one has E y non 0. So, so if I if I launch y polarized wave then I invoke these 3 equations and when I launch exploitation I invoke these 3 equation. So, 3 equations can be involved at a time. So, this gives me this gives me a room to simplify the problem, because I need to now consider only 3 equations at a time. These 3 equations the blue ones are these and what I see there is they have only 3 non vanishing components of E and H and they are E y H x and H z.

In these 3 I get that there is only one component of E and that is transverse. Then these modes are also known as TE modes or transverse electric modes or transverse electric polarization. While the other 3 have non vanishing components of E and H as H y E x and E z and I see that there is only one component of magnetic field and that is transverse component then they are known as transverse magnetic modes. Or TM polarization and this will correspond to these 3 equations. So now, let me do the analysis of TE modes first. So, what I want to do? Again do not forget I want to find out how E and H vary with x and I need to find out a differential equation in E or H with respect to x. So, for TE modes I write down these 3 equations.

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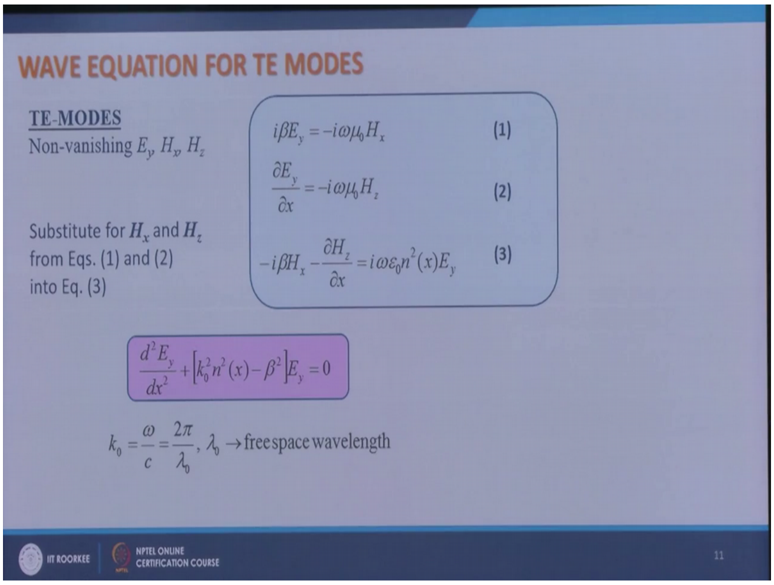

And what I can do now since these 3 equations relate E y H x and H z then if I know one of them then I can find out the others. So, for example, if I know E y I can find out H x from here and H z from here ok.

So, what I do let me find out E y. So, I substitute for H x and H z from these 2 equations into the third equation. And when I do this I form a differential equation in E y. And this comes out to be D 2 E y over D x square plus k naught square n square of x minus beta square times E y is equal to 0. Where I have use the effect that k naught is omega be c, which is also 2 pi over lambda naught where lambda naught is free space wavelength. So now, I have got now I have got a differential equation in E y for a given n square of x. So, if I know my planner waveguide that is I know n square of x then I can solve this equation for the given n square of x and obtain E y. And that will give me the modes, that will tell me how electromagnetic wave propagates in that medium of n square x refractive index variation. So, let me apply it to a very simple waveguide, which I call planner mirror waveguide. What is a planner mirror waveguide?

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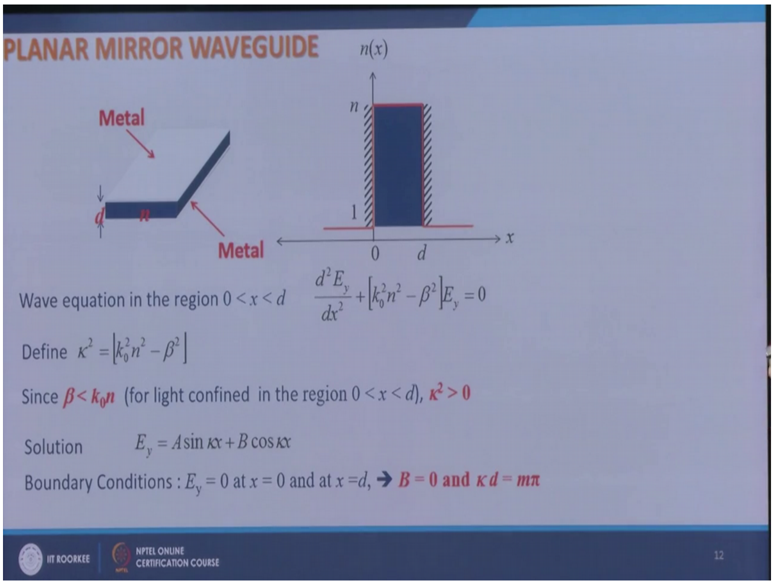

You take a very thin slab of refractive index and let us say glass. It has got a width D and refractive index n. And I polish and I sorry not polish and I deposit metal here and here. When I deposit metal on top and bottom and if I launch any light then that light will be reflected back and forth from this mirror and from this mirror and should be guided.

So, this is the simplest waveguide I can think of let me do that. So, I deposit metal on top and bottom if I look at the refractive index profile. Then I find that in this region between 0 and D I have refractive index n and here at the boundaries I have metal. When it boundaries I have metal then the electric field at the metal boundary should be 0 that is what I know. So, what I do know I write down the wave equation the equation which I obtained in the previous slide. And I put n square of x as n square and I write it down in the region between 0 and D.

So, this would be the equation. So, I from here I can find out how E y varies in this layer. And I know that I know that the fields has to be 0 here. Now let me defined this k naught square n square minus beta square as some kappa square since I know that beta which is the propagation constant of the wave in this region has to be less than k naught n it cannot be greater than k naught n, because propagation constant cannot be greater than the propagation constant of the medium itself k of infinitely extended medium.

So, beta is less than k naught n. So, kappa square is always greater than 0 which means that the solution of this equation would be E y is equal to A sin kappa x plus B cosine kappa x. Now my feel has to be 0 here and here. So, I apply these boundary conditions E y is equal to 0 at x is equal to 0 and at x is equal to D and this gives me B is equal to 0 and kappa D is equal to m pi. So, since B is equal to 0. So, this term goes off and kappa is equal to m pi over D.

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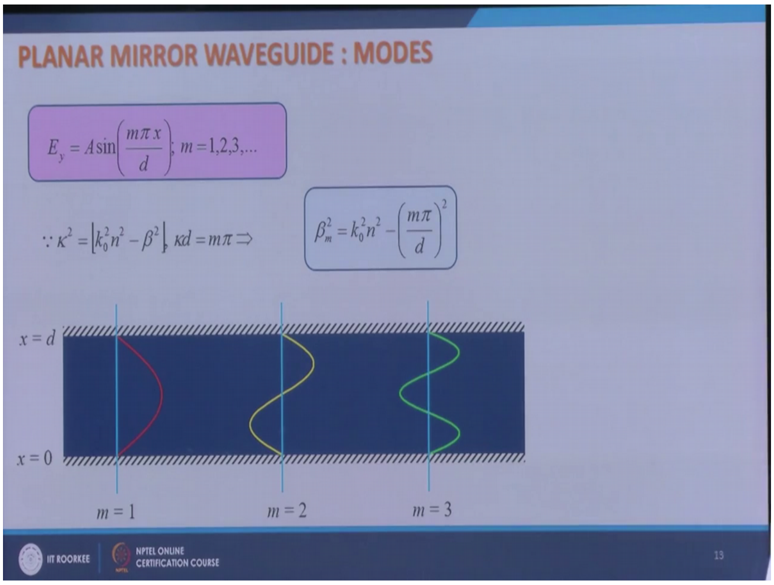

So, I put kappa is equal to m pi over D. So, my solution now becomes E y is equal to a sig m pi x over D where m can take integer values now what are. So, I have got E y of x what is left corresponding beta from where beta are coming from here because I know kappa D is equal to m pi and kappa square is equal to k naught square n square minus beta square. So, this gives me that there would be only certain discrete values of beta defined by beta m and given by beta m square is equal to k naught square n square minus m pi over D whole square.

So, I have got for a planer mirror waveguide only certain functions only certain functions which has certain propagation constants they will be sustained. If I plot them then for m is equal to one it will look like this for m is equal to 2 it would look like this m is equal to 3 like this. They are nothing but if you if you look carefully they look like as the modes of vibrations of a stretched string modes of vibrations of a stretched string. So, it is similar to that. So, in the next lecture I am going to understand what do they exactly represent. I know that in a waveguide in a waveguide if I launch if I launch ray like this then it will we reflected back and forth or if I launcher wave then wave would be reflected back and forth, but how do these represent the guidance in an a waveguide? So, let us understand it in the next lecture.

Thank you.