FOUNDATIONS OF QUANTUM THEORY: NON-RELATIVISTIC APPROACH

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EPR Paradox and Bohr's Argument, Bohm's Argument and Mermin's Argument - Part 02

After EPR's argument, Bohr replied to their argument with his counter argument and he proposed the principle of complementarity. So, the Einstein's argument against the completeness of quantum mechanics, that is, it violates the element of physical reality, was based on the assumption that if we perform measurement of observable X on particle A, then we will know the state of particle B. And if we had performed the measurement instead of X, if we had performed the measurement on momentum P on particle A, then we would have known the position of or momentum of the particle P also. So, here it is a sort of contrafactual understanding or contrafactual measurements where we have performed measurement in X, so we know the position of particle B also. If we had performed measurement of momentum instead of position, we would have known the momentum of particle B also.

So, in that way this argument was very contrafactual. So, against that Bohr suggested, Bohr proposed the principle of complementarity and he said that the x and p are complementary measurement operators. They are complementary operators and we know that they do not commute and we know what is their commutation. Bohr argued that for the principle of complementarity, which suggests that certain pairs of physical properties such as position and momentum cannot be simultaneously observed or measured in any way. This means that different experimental setups can reveal different aspects of quantum systems, but no single experiment can reveal all this simultaneously.

So, it is known that we are given a pair of particles, particle A and B which are entangled. So, we can perform either X or P, we cannot say that we perform X and if we had to perform P, we would have gotten this result. That does not work. They are complementary observable. To have x measurement, you take an ensemble of particles perform x measurement, to have the outcome of p measurement we take another assemble of particle, we perform p measurement and the statistical answer is what we have finally. We cannot say that we if we had performed measurement in x we would have gotten something and if we had performed measurement in p we could have we would have gotten something else.

In that way, we can say how Bohr proposed. Position and momentum of operators are complementary making the choice to measure and measure one excludes the possibility of measuring the other. This is the principle of complementarity that position and momentum operators are complementary. Making the choice to measure one excludes the possibility of measuring the other. In that way, EPR argument may not be as sound as it sounded. So, that was kind of a loophole in ERP or very weak loophole in ERP which Bohr tried to emphasize. And about the non-local nature of quantum mechanics, Bohr's counter argument was that this is correct that if we perform measurement of XA observable on a particle A then we and we get x0 outcome then we know that the state of the system has collapsed to x0 plus L and if we had performed measurement PA and we would have gotten p0. Then we know that the state of the particle B is p0 which is sum over X exponential minus i p0 x.

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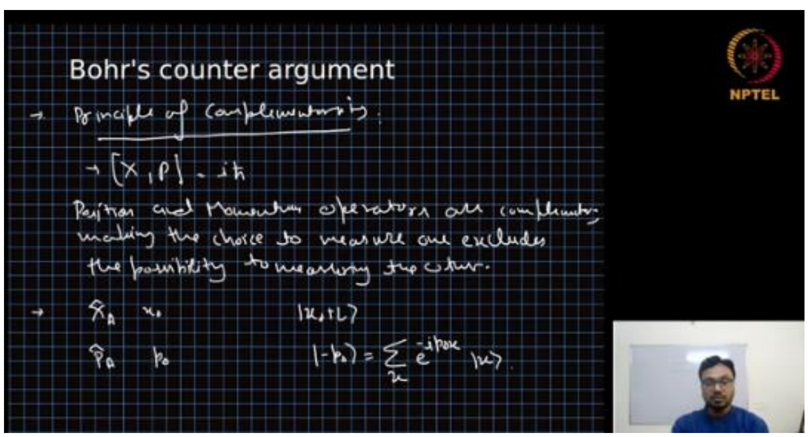

So, this is fine, this is understandable, but particle B or the Bob in this case will never know what is the state of his particle, whether it is x0 plus L or its superposition of all the x, unless he himself performs measurement of position on his particle. And when he performed the measurement of position, then it will be x0 or any other x but he doesn't know from this data alone from the output of the measurement of XB, he wouldn't know what was performed ,what was what measurement operator was used in the lab A. Let us say Bob measures XB observable and he gets x0 plus L in the first case or some x0 prime in the second case, some value of position in the X. For Bob, these are just two values of measurement. From this data alone, he cannot infer whether XA measurement was performed or PA measurement was performed on particle A. Because these are just position values.

It is possible that he got X0A and he assumes that on the Alice side, on the side A, XA measurement was performed and the outcome was X0'. It is possible. Or on the other side, PA was performed and p0 came out as the outcome. So, in that way, from this data alone, it's not possible to infer what was measured in lab A. So, there is no message, no information being transferred from lab A to lab B by performing measurement on an entangled state. So, in that way, quantum mechanics does not violate the causality.

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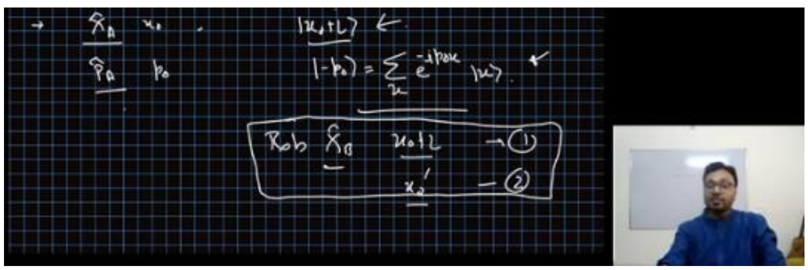

So, in that way, it's still local. The states are affected, but no information is transferred. That was the argument by Bohr's against EPR argument. An alternate way of presenting EPR argument was given by Bohm's. And his argument was using the spin systems where he proposed the following.

Again, we will call them A and B. And he considered the state of the joint state to be the singlet state. That is 01 minus 10 over root 2. So, this is the joint state of the two particles. Here, let us set the nomenclature here. So, sigma x acting on 0 will give us 1 and acting on 1 will give us 0 sigma y acting on 0 will give us i 1 and acting on 1 will give us minus i 0 sigma z acting on 0 will give us 0 and on 1 it will give us minus. So these are the, this is how we define 0 and 1 state.

Now, if we perform a measurement of Sx, Sigma X on the first particle, then whatever we get the outcome. The outcome of the measurement of sigma X on the second particle will give us the negative answer. If m1x is the outcome of the measurement of sigma X on particle A, then m2 of x is the measurement outcome of particle P, then it will be minus of m1. That can be verified very easily. Okay, similarly sigma y on first particle and sigma y on the second particle will result in the opposite outcome so m2y will be equal to minus of m1y similarly m1z and m2z they will have the opposite outcome.

From here, we can see two things. By performing measurement on the first particle, we know what will be the outcome of the second particle if the measurement is performed in the same basis. So, in that way, sigma x, sigma y, sigma z on the second particle is the element of physical reality. And m1x, m2x, m2y, m2z are the numerical values of those variables. We can reverse the argument and say that sigma X, sigma Y, sigma Z measurement on the first particle is the element of physical reality.

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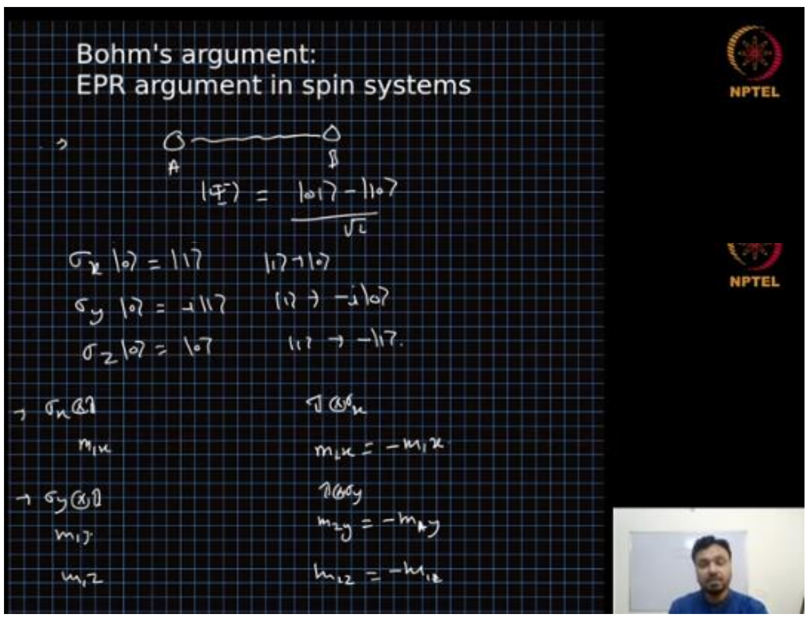

If we perform measurement on the second one, then we can predict the outcome of the first one. In that way, all these six observables are element of physical reality. Now, if we have two commuting observables, A and B, such that they commute, then we can measure them simultaneously. Let us say A measured on psi gives us outcome a. Observable B measured on psi gives us b. Then any function of a and b, we measure on the state psi will give us the outcome f of a, b. So, this is the recursive definition of element of reality.

If A and B are the element of reality, that is we measure them and we get the values A and B, then any function of them should also give us the same function in terms of the outcomes A and B. Now we see that sigma i tensor identity where i can be x y z, any of those operators commute with identity tensor sigma j but j can also be x y and z, so sigma x tensor identity coming with identity tensor sigma x identity tensor sigma y and identity tensor sigma z, it commutes with everything. Similarly, we can choose any operator here and there they will come out observable on two different systems commute it means Sigma X tensor identity commute with identity tensor Sigma Y. Sigma Y tensor identity commute with identity tensor Sigma X. So, it means we can measure these two simultaneously. We can measure these two simultaneously and we get Sigma X tensor Sigma Y. And we have Sigma Y tensor Sigma X. And these two operators also commute.

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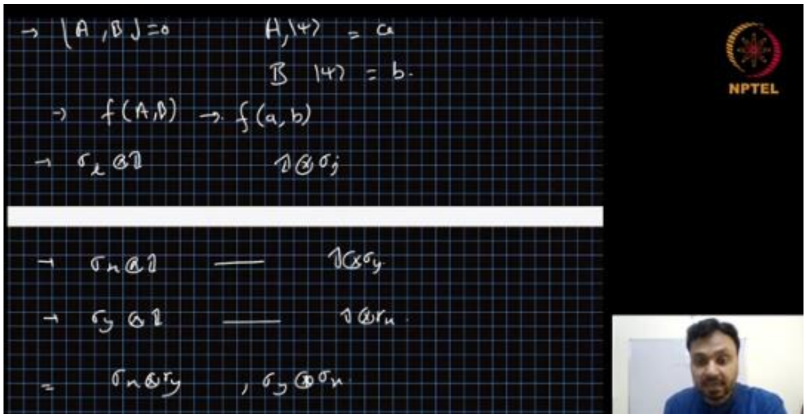

So, let us say the outcome of this Sigma X was M1 X, Sigma Y was M2 Y. Then the outcome from this measurement will be M1 X, M2 Y. And from here, it will be M1 Y, M2 X. And we have established earlier that M1 X is equal to minus of M2 Y, M2 X. And M1y is minus of M2y. That is, sigma x measurement on first will give us exactly opposite results for sigma x measurement on the second particle. Similarly, sigma y on first will give you exactly opposite results for sigma y on the other. So, the results will be negatively coordinated.

This says that M1x M2y is same as M1y M2y. So, outcome of these two measurements should be same. But if we see sigma x tensor sigma y plus sigma y tensor sigma x on psi minus, this was the singlet state. We get sigma x tensor sigma y acting on 0 1 minus 1 0 over root 2 plus sigma y tensor sigma x acting on 0 1 minus 1 0 over root 2 and from here we can see sigma y acting on the first one will flip the bit. The 0 will become 1 sigma y acting on 1 will flip it 0 but with a phase that is minus i. Similarly, the second one, 1 will go to 0, 0 will go to 1, but with the phase i over root 2 plus sigma y acting on 0 will make it 1, sigma x acting on 1 will make it 0, but 0 goes to 1 with sigma y with the phase i minus 1 will go to 0 with phase minus i, so plus i here and 0 will go to 1.

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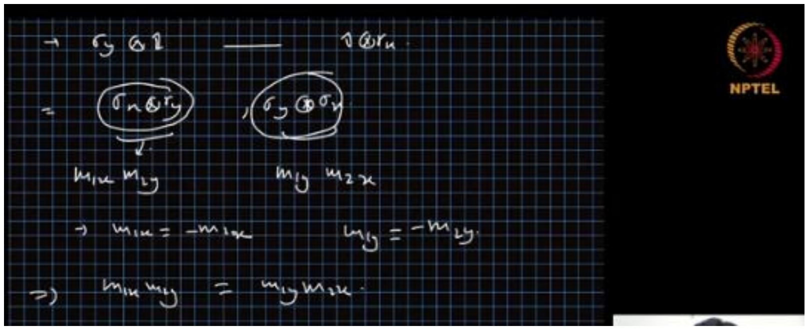

And this is 0. So, when we perform measurement of sigma x tensor sigma y, the sigma y tensor sigma x on psi minus, the outcome is 0. So, what does it mean? This means m1 x, m2 y is not same but negative of each other, m1 y, m2 x. So, this is the contradiction shown by Bohm.

This argument was sort of based on the EPR argument but in the spin language, in half particle language. So, from here the Bohm argued that the recursive definition of the element of reality which appears obvious is incompatible with quantum mechanics. This is what we are trying to show from the Einstein's argument and from this argument and from subsequent couple of arguments that whatever our understanding, our common sense dictates about quantum mechanics or about the realism and the reality of the universe that is incompatible with the quantum mechanics. Also, this may also suggest that the quantum systems may not possess a well defined state of the system before we perform measurement. So, the superposition of principle is now revealing itself and it might be that all these contradictions are because of superposition principle that the particle is not in 0 or 1 state before we perform measurement, but it is in a superposition state and only upon the act of measurement we make it collapse to 0 or 1 state.

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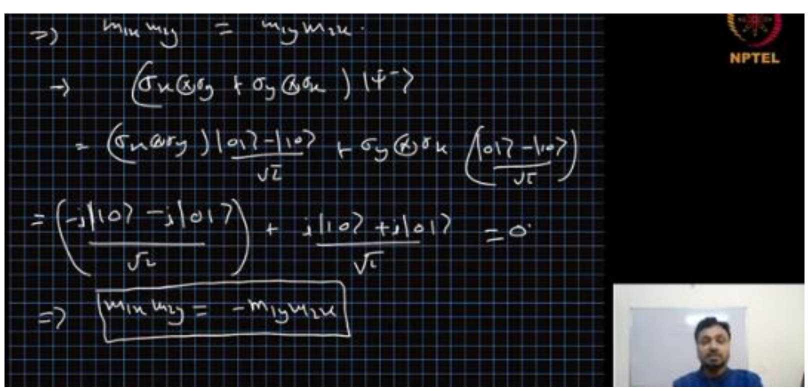

So, it's like saying that the balls are not red or blue before we perform measurement, but they were both red and blue at a given time, at the same time before we perform measurement. And when we perform measurement, because of some internal thing, it collapses to either red or blue. So, in that sense, maybe there is some kind of hidden parameter. Hidden variable inside each of the quantum system which will make it collapse to either zero or one state in another way either to red or blue color if we are talking about colors. The Bohm's argument for spin half, two spin half particles was extended to three particle model by Mermin and in this he considered the state psi which is the three particle state 0, 0, 0 minus 1, 1, 1 over root 2.

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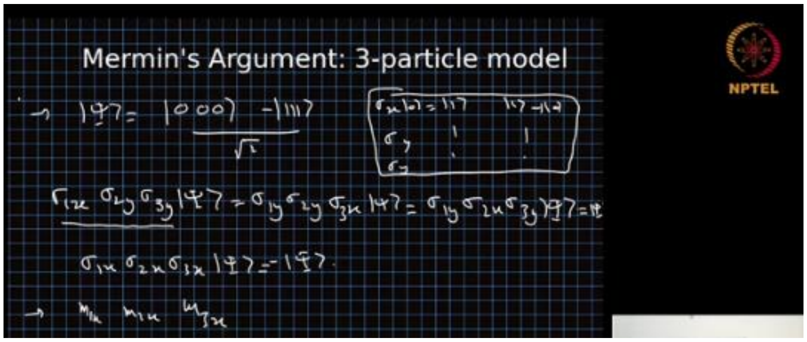

So, he is considering three particle which are in this kind of entangled state. We have not discussed about three particle entanglement, but you can take my words on it that this is an entangled state. Now, consider the following observable. The x observable on the first one, y observable on the second one and y observable on the third. When we apply it on the state, that is same as if we had applied y on the first one, y on the second one and x on the third.

And this is same as y on the first one, x on the second one and y on the third. So, it doesn't matter if we have two y's and one x does not matter, their relative positions, they will acting on this state psi, they will give us the same result and that is the same as the state psi. But if we apply sigma x sigma 1 x sigma 2 x sigma 3 x on psi we get minus psi. So, applying x operation on the three will give us the negative of the state. These things can be checked very easily, we have already established the relation that sigma x acting on 0 will give us 1 and acting on 1 it will give us 0 and all the other relations about sigma y and sigma z. Using those relations, we can verify this claim that sigma 1x, sigma 2y, sigma 3y acting on psi is same as sigma 1y, sigma 2y, sigma 3x, sigma 1y, sigma 2x, sigma 3y.

All of them acting on the state psi will give us the state psi. But sigma 1x, sigma 2x, sigma 3x acting on psi will give us minus. Since sigma x, sigma y, sigma z are the element of reality as was proven from the Bohm's argument. And let us say when you perform measurement of sigma 1x, we get m1x, m2x, m3x for measurement of sigma x on second and third particle. Similarly, m1y m2y m3y and m1z m2z m3z measurement outcome, we perform those measurement corresponding measurement. So, when we perform sigma 1x sigma 2y sigma 3y, then outcome is m1x m2y m3y and since we are getting the same state back, so the outcome, the product of the three is one.

Similarly, m1y, m2x, m3y is 1, m1y, m2y, m3x is 1, but m1x, m2x, m3x is minus. So, from these four equations, we got these four equalities. Let me repeat. If m1x is the outcome we get when we perform measurement of sigma x on the state psi, and m2x and m3x are the corresponding measurement outcome on second and third particle. Similarly, sigma m1y, m2y, m3y, m1z, m2z, m3z.

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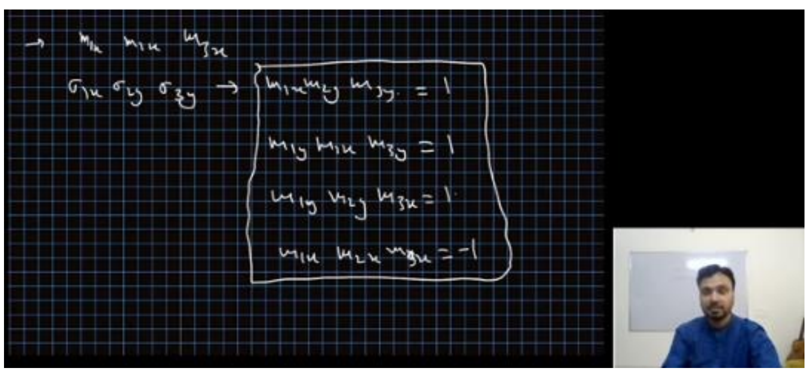

Then from this sigma 1x, sigma 2y, sigma 3y, sigma psi equals psi, we get m1x, m2y, m3y equals 1. Similarly, the other three equations. Now from here, we can see, if we take the product of all four, then we get m1X appears twice, m2X appears twice, m3X appears twice, similarly m2Y, m3Y and m1Y. So, when we take the product, we get m1 x squared, m2 x squared, m3 x squared, m1 y squared, m2 y squared, m3 y squared. And when we take the product of the right hand side, we get minus 1.

So, we have product of numbers, this is m1 x, m2 x, m3 x, m1 y, m2 y, m3 y, all are real numbers. So, they can be either plus or minus 1. And when we take the squares, it will always be 1. So, the product of 1s is giving us minus 1. This is a contradiction.

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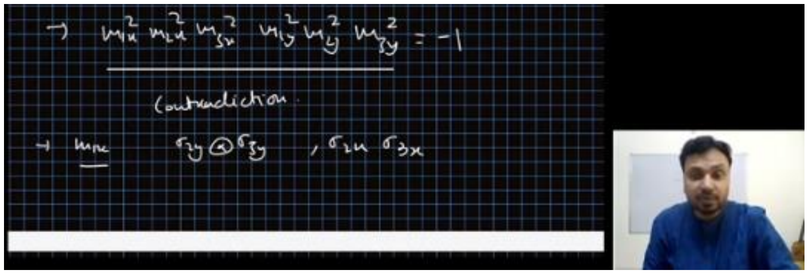

So, this seems to point, this is obvious contradiction, this seem to point towards the following things the measurement outcome m1x is different depending on whether we perform measurement of sigma 2y or sigma 3y or sigma 2x sigma 3x on the other two systems, m1x is the measurement outcome of measuring sigma x on the first. And the outcome, it seems like it depends on whether we perform measurement of sigma 2y sigma 3y or sigma 2x sigma 3x on the other two systems. This seems like the contextual nature of quantum mechanics, that is, depending on the measurement being performed at the other system, the outcome of the desired system might change. So, the outcome of a measurement is highly context dependent, whether we are performing measurement of sigma 2y sigma 3y or sigma 2x sigma 3x, that will tell us what was the outcome of the measurement on first particle. Another example along these lines of contextual nature of quantum mechanics, we can consider two particles, usual, we have A and B.

We perform the sigma x measurement on the first particle is an element of physical reality and sigma x measurement on the second particle is also an element of physical reality. Let us say the outcome is m1 x and this is m2 x. So, sigma x tensor sigma x is an element of physical reality, that will be m1 x m2 x the outcome. Similarly, Identity tensor sigma z, that is measurement of the second particle, is an element of physical reality and the outcome is m2z. Sigma z tensor identity, element of physical reality, that is outcome is m1z. And the product of them, sigma z tensor sigma z, is also an element of physical reality and we get m1z, m2z. So, till now we were going the row wise that we have one element of physical reality, another element of physical reality.

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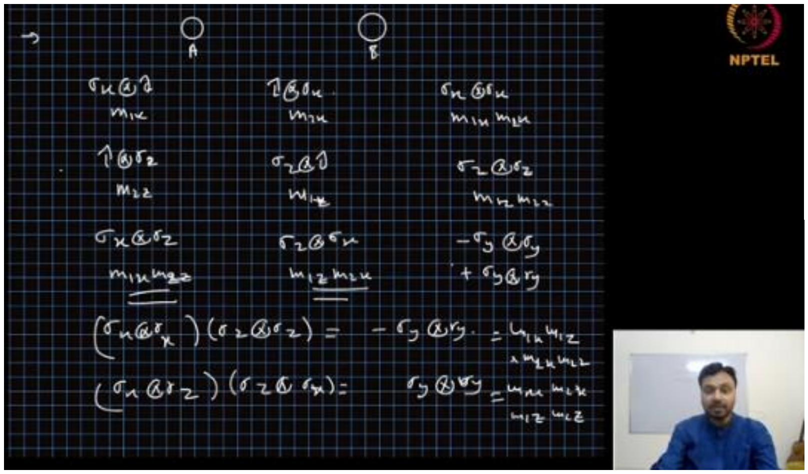

They commute and so their product is also an element of physical reality and we get the product of their outcome as the outcome of the product. Now we can go column wise. Sigma x tensor identity and identity tensor sigma z they commute. So, the element of physical reality sigma x tensor sigma z will have outcome m1x m2z. Similarly, here sigma z tensor sigma x and the outcome is m1z m2x. Now the question is, what element of physical reality we should put in this box here. There are two ways, one is that sigma x tensor sigma x commute with sigma z tensor sigma z, this can be verified. So, the product of these two operators will be another element of physical reality and that will be sigma y tensor sigma y but with minus. So, the outcome is minus m1 y m2 y which is a product of m1 x m2 x m1 z m2 z.

And if we take the product of these two operators, they also commute, sigma x tensor sigma z, commute with sigma z tensor sigma x and the product will be sigma y tensor sigma y, this plus sign. So, the product of the operator that is sigma x tensor sigma x with sigma z tensor sigma z is minus sigma y tensor sigma y. And sigma x tensor sigma z times sigma z tensor sigma x, they also commute and the product is sigma y tensor sigma y, but their values from the outcomes of the individual element of reality is m1 x m1 z times m1 m2 x m2 z and same here m1 x m2 x m1 z m2 z, so the outcome the product of the outcome is the same in both the cases. But the product of the operators is different, it is opposite, it's negative of each other. This is another mathematical contradiction we see which seem to challenge our understanding of the reality using the quantum mechanics. This simply indicate that whether we perform sigma x tensor sigma z or and sigma z tensor sigma x to calculate sigma y tensor sigma y or we do sigma x tensor sigma x, sigma z tensor sigma z, to calculate sigma y tensor sigma y, they will give us opposite results.

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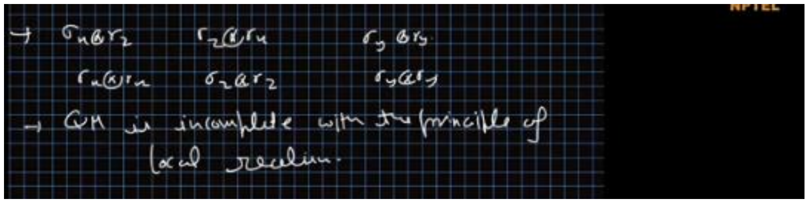

One will give us the positive result and one will give us the negative result. So, with all these arguments, the EPR argument, the Bohm's argument, Mermin's argument, this argument, we can argue, we can conclude that quantum mechanics is incomplete with the principle of local realism. There seem to be, there might be a hidden variable attached with each of the quantum system which can help rectify this problem. Perhaps the quantum mechanical description can be improved by including some hidden variable which might explain all this contradiction.