FOUNDATIONS OF QUANTUM THEORY: NON-RELATIVISTIC APPROACH

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EPR Paradox and Bohr's Argument, Bohm's Argument and Mermin's Argument - Part 01

In the earliest period of quantum mechanics, when quantum mechanics was fully developed, many thinkers at that time doubted the quantum mechanics for many reasons. Many prominent scientists like Einstein and Mermin and Bohr, they discussed the problems and discrepancies and paradoxes in quantum mechanics theory. In the next couple of lectures, we will discuss those paradoxes, those inconsistencies and how they were removed and how they made quantum mechanics stronger. So, that will be the topic of the next few lectures. So, we start with the following.

Let us say we have a quantum system and it can have states psi 1, psi 2, psi 3. Then quantum mechanics say that any superposition of them, any normalized superposition of them is also a valid state of the quantum system. Similarly, if we have two quantum systems and first one can be in psi 1 and other one can be in phi 1, first one can be in psi 2, other one can be in phi 2 and so on, then their superposition is also a valid state. This superposition as we have discussed earlier contains states which we call entangled states which are highly correlated states between two or more particles of quantum system. We discussed two particle quantum systems but this idea can be extended to multi-parted entangled multi-parted quantum systems, so the consequence of these kind of states is that if we perform measurement on the first system some measurement of on the first system then the state of the second system also collapses.

If we perform, if let us say we have a state 0 0 plus 1 1 over root 2 and we perform measurement in 0 1 basis, that is the eigen basis of sigma z in subsystem A and we get outcome 0 then we are sure that the outcome of the B subsystem is also 0 if we perform measurement in sigma z. Similarly, if we get one here, we are sure that the outcome yeah on the B side or other system is also one. So, in that way it seems like we are able to affect the remote systems by performing measurement on our system, if the state is entangled and this entanglement is possible. Because of these properties of quantum mechanics, these axioms of quantum mechanics that if zero zero is a valid state of bipartite system and one one is a valid state of bipartite system, then their superposition is also valid state, this alone contributes to the concept of entanglement. The consequence of that entanglement is apparent uh faster than speed of light communication, okay. So these are these were the kind of things which puzzled people at that time. Now we will discuss the argument presented by EPR. EPR stands for Einstein, Podolsky and Rosen.

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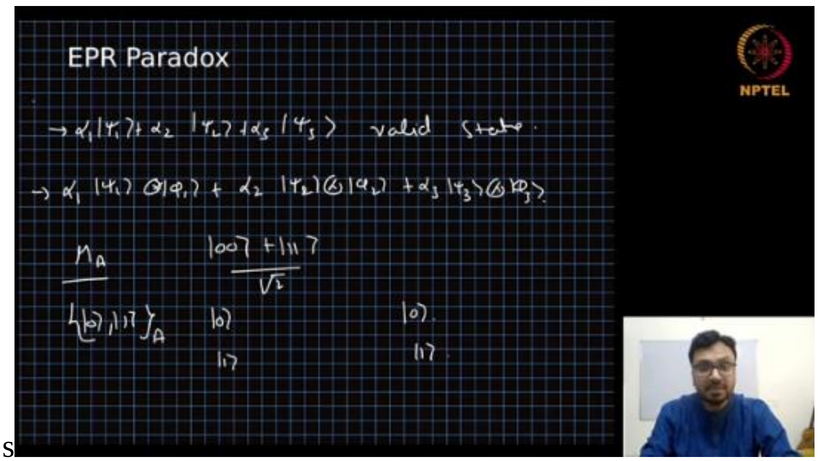

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So, EPR paradox we want to discuss and the statement in 1935 paper was that can quantum mechanical description of quantum mechanics of physical reality be considered complete? So, they're posing a question that can we consider quantum mechanical description of physical reality as a complete theory in the sense that, is it a valid theory to explain our reality, our world, our universe? Because the quantum mechanics is so bizarre, it has complex numbers, it has superposition, it has collapse postulate, which is counter intuitive for classical understanding from a classical understanding point of view, many of the concepts of quantum mechanics looks bizarre looks fairytale type. But this was the question posed by Einstein, Podolsky and Rosen that can we consider, can we think of quantum mechanics as the theory to explain the physical reality of our universe? Is the quantum mechanical theory complete?

Does it have some inconsistency? Can we find some inconsistency in this? So, this question, although it is a small question, very straightforward question, it has some very heavy, very prominent words or phrases which need to be understood. So, what do we mean by physical reality? So, for that, EPR, Einstein and fellows, they define the element of physical reality.

Let us say we have a system, a particle, an atom, or a ball or anything. If without affecting in any way, this system, if we can predict with certainty the value of a physical quantity, then there exists an element of physical reality corresponding to that. Let me repeat this statement. If without affecting in any way a quantum system or a system, if we can predict with certainty the value of a physical quantity, then there exists an element of physical reality corresponding to this physical reality. This can be understood as follows. Consider a ball, a baseball ball.

So, without us measuring the ball, the speed of the ball, the position of the ball, the weight, the temperature, the size of the ball, without even measuring that, this ball possesses all this quantity. The ball is present at a definite position. It has a definite momentum. It has a definite shape, size, weight, mass, temperature, everything at a given time of the day at a given location and if the parameters are kept constant. So, we don't need to measure any of these things to say that it has some definite mass, it has some definite temperature and definite position and momentum and everything.

So, all these quantities are the element of physical reality for the ball. And their values will be the numerical value of those physical quantities or the element of physical reality. So, let us say we are talking about the position or location of the ball. It is present at one place whether we measure it or not. So, the position of the ball is the element of physical reality.

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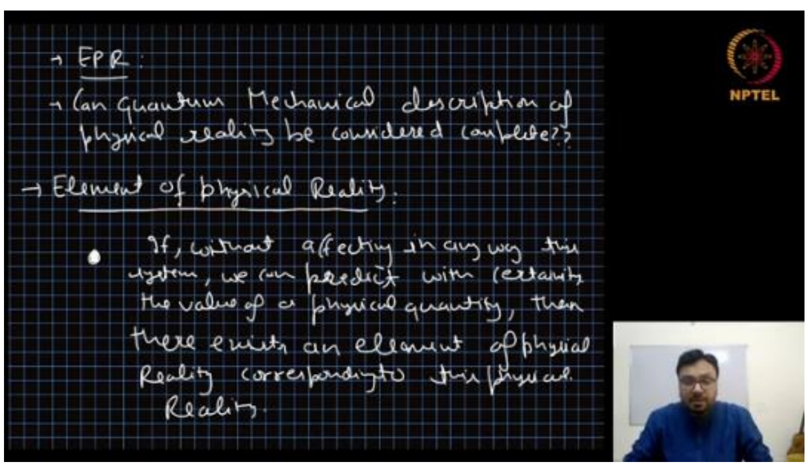

And where it is present at a given time that becomes the numerical value of that element of physical reality. So, this is the argument which resulted in many of the statements. One of the profound statements people have discussed often and people have read it on social media is, does the moon exist when we don't look at it? So, the existence of moon is it an element of physical reality or not? So, of course, it exists when we don't look at it because it is there.

That is, what is the essence of this statement, that if without affecting in any way, if we can predict with certainty the value of a physical quantity, then there exists an element of physical reality, that reality is the existence of that thing. So, these are a little bit more philosophical issues, but they reveal some profound properties of quantum mechanics and entanglement being one of those. After defining the element of physical reality, we need to define two more things. One is the completeness of a theory. A theory is considered complete if it contains the counterpart to all the element of physical reality.

So, if we have element of physical reality, then there should be an observable corresponding to that. Observable corresponding to all the element of physical reality. So, element of physical reality, I will say element of physical reality. I didn't realize that element of physical reality also becomes EPR. So, it means whatever is measurable or whatever is supposed to exist in a given particle, all of those quantities should be, there should be a corresponding observable for all of those quantities.

And other is the correctness of a theory. A theory is considered correct if it agrees with experimental results. So, when Einstein, the fellow scientist, wrote the paper, 1935 paper, in which he raised all these doubts about the quantum mechanics, at that time Einstein did not have problem with the correctness of the quantum mechanics because it was predicting everything which was measured in lab which, was the experimental result. at that time einstein did not have problem with the correctness of the quantum mechanics because it was predicting everything which was measured in lab which, was the experimental result. He was not very happy with the quantum mechanics and he gave a very interesting argument which showed doubts, which showed some problems in the theory of quantum mechanics.

And that argument, we call it Einstein argument. It goes like this. So, now let us consider two particles, particle A and particle B, and their state is prepared in such a way that it is a delta function in the position and it is a delta function in momentum. The state of these two particle systems can be written as sum over X, state X of particle A and state X plus L of particle B. So, it's in the x representation, if we write it in the p representation, we can see, what is the relation between x and p states.

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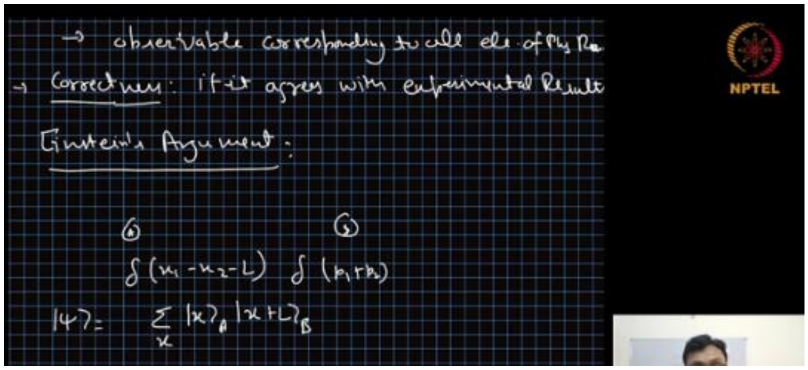

It is without the normalization, I'm not not writing the normalizations here, so I'm just saying proportional, so x is proportional to exponential of i p x p. So, we can write psi proportional to sum over x sum over p, this is sum over p, sum over p prime, exponential of i, x, p, exponential of i, x, plus i, p prime, p, p prime for A and B. And this can be written as sum over p, p prime, sum over x, exponential of i, x, p plus p prime and exponential of i l p prime p p prime. Now sum over x exponential of i x p p prime p plus p prime with the delta function is proportional to a delta function p plus p prime and then we take some over p prime then every p prime can be replaced with a minus p so we get some over p exponential of i l p minus p p and minus p so this is the state in the p representation. So, it does not matter whether we use the P representation or X representation, the state is the same. So, let us consider the X representation again. Psi is proportional to X, X plus L sum over X. So, there is a normalization constant and our treatment is independent of that normalization constant.

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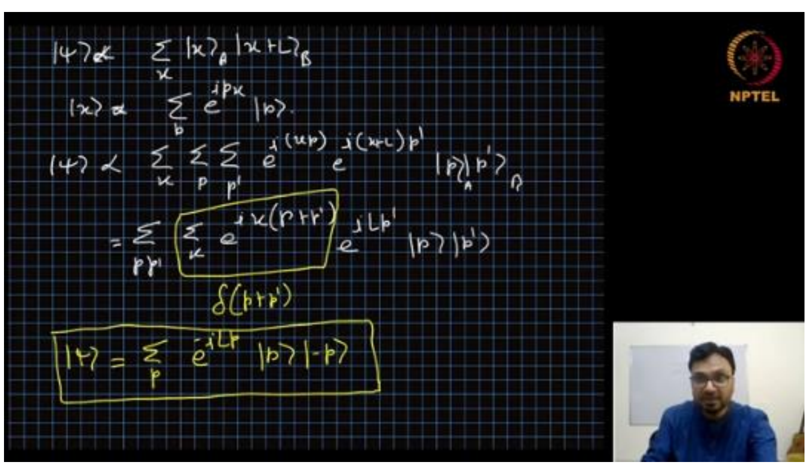

Now, we see that X is an orthonormal basis in HA, in the first step. And if we displace x, this is for all the x in the in the space in the real space all the positions so x plus L is also the same is just shifted this is also an orthonormal basis in HB. So, when we have one orthonormal basis and another orthonormal basis and only one sum, this looks like a Schmidt decomposition. So, this is an entangled state. Not just that, we see that the coefficient of each of these terms are the same. It's just the normalization constant.

So, when the Schmidt coefficients are same, all of them, then it is maximally entangled state. If we perform measurement of X operator on particle A, then we will get some value X0. And this value can be anything between minus infinity and plus infinity. So, we cannot control the outcome X0. It can be anything when we perform measurement on particle A, positive measurement on particle A.

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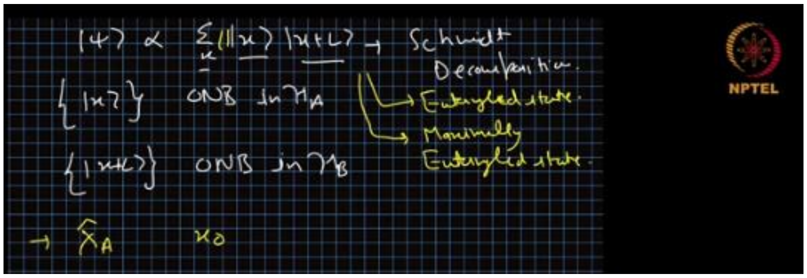

What we are sure is if we get X0 for particle A and we perform measurement XB on particle B, then we will get X0 plus L. So, the outcome of the first measurement is not known but outcome of the second measurement is we know it without even performing measurement. We know if particle a is in x0 and then particle b will be in x0 plus L. So, the state of the particle is particle through particle system is such that if particle a is at x0 then particle b will be exactly at l distance away from it x0 plus L. This is how they are correlated. Their positions are correlated. Similarly, if you perform measurement of momentum on particle A, we get some value P0.

And then if you perform measurement on particle P, we get minus P0. This is the meaning of this state here. If particle A is in state P0, then the particle must be in the opposite state, minus P0. This also we cannot control this, but this one we know for certain. So let me repeat we perform measurement on particle a, we get some value x0 and we know what is the outcome of the particle B, that is X0 plus L, even without performing measurement on the particle B. Similarly, if we perform measurement of momentum on particle A, we get P0 and then we know for certainty what is the momentum of particle B. That will be minus P0.

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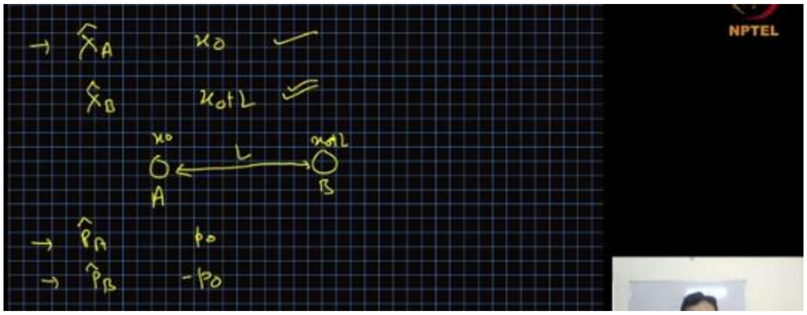

If we consider that these two particles are far apart, that is L is much much greater than 1. Far apart is not a technical term. We make it technical. Let us say they are space-like separated. It means they are so far apart that we cannot signal between these two particles, within the time scale we are interested in, the time it takes to perform measurement ,to assemble the data and to do whatever we want to do.

In that much time scale, we cannot communicate any information from particle a to particle b or from particle b to particle a, so those are those events have the space like separated event you they there is no communication possible within a given time frame. Now if that is the case and we can find the value of XB and PB without performing measurement, without affecting them in any way, then these two should be the element of physical reality and the corresponding values are x0 plus L and minus p0. Let me repeat. The definition of element of physical reality is that if without affecting a system in any way, if we can predict with certainty the outcome of the measurement, then there exists the element of physical reality corresponding to that measurement. It means whether we measure it or not this point that particle had that property. To understand standard in simple terms, let us say we have a box with two particles, one red and one blue and we cover it, so that no one can see, like, we divide the box into two parts and we cover this these boxes, so that no one knows which part it has the red ball and which part has the blue ball. One of the part is sent to lab A and other part is sent to lab B. Then we perform measurement of color here and we find that we had red ball in lab A. Then we know that the lab B must have gotten the blue ball.

Without performing the measurement on the color of the ball in the particle, color of the particle in lab B, we know that the particle must be blue in color. So, the color of that particle is the element of physical reality. Another example is if we have a bombshell, which is at rest in the beginning and it explodes into two parts. One part goes in this direction and one part goes in other in other direction part one part goes to lab A, other part goes to lab B and in the event of explosion, they start rotating, these two, they acquire certain angular momentum, the two part. Then, whatever since in the beginning the total angular momentum was zero, then whatever angular momentum we get here, we will get the opposite of that angular momentum on the part B. So, once we perform measurement here, we know the value of the angular momentum of the face of the bombshell in lab B. Hence, without affecting the, without performing measurement on the system, we can predict it.

So, it must be an element of physical reality. These quantities are attached to the particles to the systems, whether we perform measurement or not . The moon exists, whether we look at it or not. So, that is the element of physical reality. Along the similar line, if we can perform measurement on A system and we get the outcome of, with certainty we know the outcome of the B system for X and P, then X and P must be the element of physical reality for the B subsystem. Similarly, we can argue that XA and PA are also the element of physical reality for subsystem A, if we had done the same experiment by performing measurement on the B subsystem and predicting the state of A. In that way, the values of X and P observables are always present with the particle and we should be able to perform sharp measurements and get those values without worrying much.

But quantum mechanics also says that X and P are incompatible observables. It means they do not commute. This implies that we cannot, with certainty we cannot measure X and P simultaneously. So, Einstein's argument says that since x and p are the element of physical reality, we should be able to measure them with certainty and get the outcomes. And the Heisenberg uncertainty principle says that since x and p are non-commuting observables, that is the commutator of x and p is i h bar, they cannot be measured simultaneously with certainty. If we perform measurement on one with high certainty then the certainty and the other one goes low or uncertainty with the product of the uncertainty is conserved or low bounded.

Delta x square times delta p square is always greater than or equal to h bar over 4, h bar square over 4. So, that is the Heisenberg Uncertainty Relation. So, this is what Einstein has shown. This hypothetical scenario gives you a contradiction between the laws of quantum mechanics and the predictions of quantum mechanics and what we should observe physically. So, this was the argument given by Einstein which raised some doubts about the theory of quantum mechanics.

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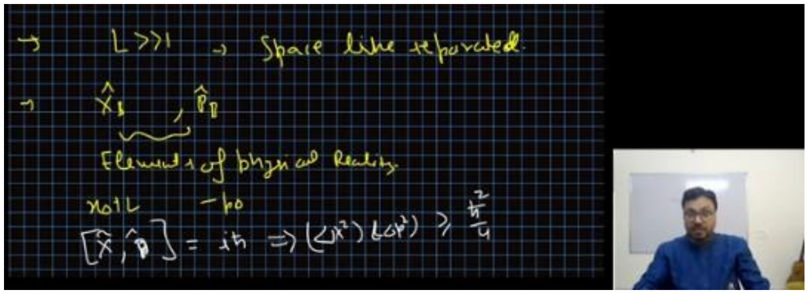

From this obvious contradiction between the physical reality and the predictions of quantum mechanics, EPR concluded that the quantum mechanical description of physical reality is not complete. Further, there is doubts about the non-local nature of quantum mechanics. That is, the choice of the measurement done on the first particle affects the results of the second particle. So, it seems like we are varying the superluminal communication or we are violating the causality. It means, this can be understood like this.

If we perform measurement of XA on particle A, then we get outcome X0 and the state of the particle B is X0 plus L. But if we perform measurement of PA on the on the first particle and we go to p0, then the state of the second particle is minus p0. And in terms of the x representation, it will be sum over minus i xp0 x, sum over x. So, in that way, in the two cases, the state of the second particle is very different. In the first case, it is an eigenstate of the position of the position operator x, that is x0 plus L. In the second case, it is the eigenstate of p operator, so it is equal superposition or only the phase difference but equal superposition of all the positions of the all the eigenstates of the position operator x. So, in that case, in the first case, the particle is localized in the X-space and in the second case, it is completely delocalized in the X-space. So, depending on the measurement on the first particle, the state of the second particle can be drastically different.

And this is the non-local nature of quantum mechanics. We are doing something on one side and we are affecting the state of the second system completely. So, then Einstein remarked or concluded that quantum mechanics is violating the realism and it is violating the locality. It is possible that these two things are related. So, Einstein said that quantum mechanics is either violating the realism or it is violating the locality.

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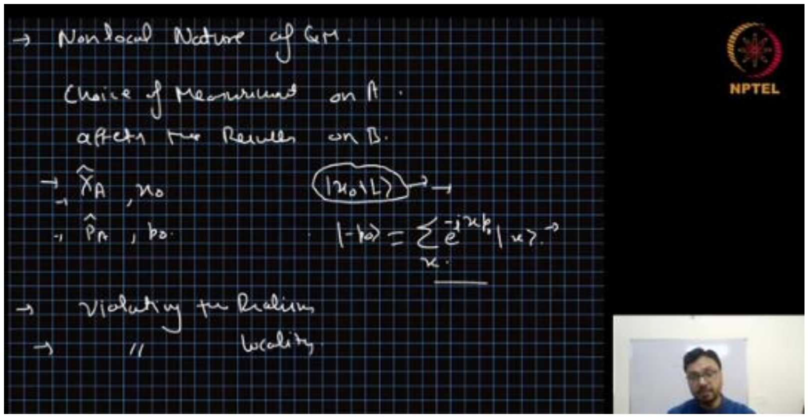

That was the conclusion of the EPR paper. The final conclusion.