Introduction to Quantum Field Theory - II (Theory of Scalar Fields)
Prof. Anurag Tripathi
Department of Physics
Indian Institute of Technology - Hyderabad

Lecture - 33
Renormalization -Part 5

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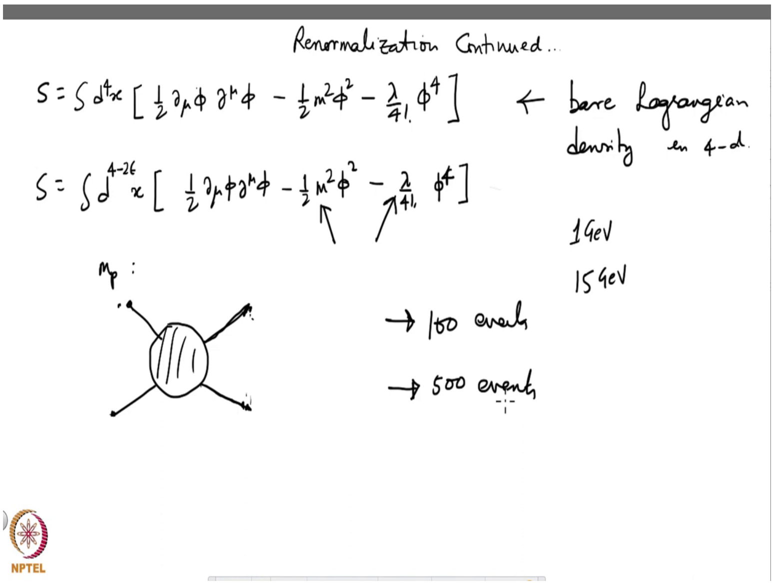

So let us continue from where we left last time. So, we have seen so, far that we can remove the divergences in Greens functions by renormalization and there are various renormalization schemes and I have mentioned a three of them M S, M S bar and also on shell renormalization. You can also have on half shell renormalization. So, and we have also seen that our results may depend on will depend on mu the grease functions that we write.

So, let us try to understand what things change and what things do not change when you do a renormalization. So, that is what I want to talk about in this in this lecture. So, let us first try to understand what we mean by a given theory. So, let us suppose that an action is given to you which is 5 4 action this is the where action or where lagrangian here in the square brackets or even more appropriately bear lagrangian density in four dimensions.

So, let us go to d dimensions where d is 4 minus 2 Epsilon. Let me not write it I will introduce that when I write the renormalized field here Lambda is dimension full as you know it has Dimensions mu to the 2 Epsilon or if you wish you can introduce some mu naught and put Lambda times mu naught power to Epsilon. So, let us keep Epsilon fixed I want to keep Epsilon fixed because then things are finite.

They become singular when Epsilon goes to zero but things are finite as Epsilon is fixed to some non-zero value. So, given this action and given these bear parameters. Now they are not. Now they are not singular they are singular only when Epsilon goes to zero. So, I fix Epsilon and the reason I am fixing Epsilon is. So, that I do not run into this trouble of Infinities everything is finite but large because Epsilon is not really zero.

I mean Epsilon is not Epsilon is somewhat close to four close to zero. So, things are large but not in finite because Epsilon is some fixed number. So, let us see what we mean by saying that this gives us a theory this is this specifies a theory choosing some values of M square and M and Lambda. So, suppose I take this action or take this theory and find out what the physical mass is.

So, I figure out what the physical mass is from this Theory what will be the inputs the input will be M and M Lambda right. You choose some value of Lambda some value of M and that will give you some value of M P physical Mass. Now if you were to change the values of M and Lambda these weird parameters if you were to change them then the value of physical Mass would change.

Now you have a different theory suppose some values of some choice of M and M Lambda gives you particles in this theory which have mass 1 Ge V. I am just randomly saying some number you can take other numbers also but let us say some choice of M and Lambda gives you particles in this Theory which have mass equal to 1 Ge V meaning M P is 1Ge V and if you were to change these values of M and Lambda to some other new values it could be that the new Mass becomes now let us say 15 Ge V.

Now this is a different Theory right because in the first Theory the particles had Mass 1 Ge V and in the second theory the mass is 15 Ge V the action looks the same but the parameters that appear take different values. So, changing these values will give you particles of different masses. So, you are in a different theory the same thing I could say in another way suppose you look at some cross section.

So, you are doing a scattering in some Collider and you Collide let us say 2 particles to five particles these scalar particles and produce 2 final State particles. And you want to find out the the cross section for this production that you can calculate in putting your chosen values of M and Lambda you will get a certain prediction and using that cross section you can try to find out that if you were to collide let us say a billion of billion such pairs.

So, billion of this and these particles input that into your calculation. So, use cross section and this this number of collisions and multiplying them will give you the number of events in which you are going to see these final state particles. Whatever you have kinematical configurations you have specified for these particles according to that you can find out the number of events that you are going to see when you Collide let us say 1 billion of such pairs.

Let us say you get a hundred let us say you are going to see 100 events in which these final states are produced when you collide one billion such pairs of particles in the instant in the incoming State and that result is for some choice of M and Lambda. Now you change the change the values of these bear parameters M and where parameter Lambda to some other value keeping Epsilon fixed.

I do not change Epsilon otherwise Things become infinite it becomes difficult to talk. So, let us keep Epsilon fixed let us pretend our world is not four dimensional but 4 -2 Epsilon dimension. Now changing these values of M and Lambda would change the cross section cross section has this mod M squares amplitude squares and which is what contains the Dynamics the remaining parts are just kinematical.

So, these numbers will change and maybe. Now you get instead of 100 let us say 500 events depending on what M and Lambda you have chosen. So, you get 500 events this time. So, clearly that choice of M and Lambda which gave you 100 events is describing a different theory then this one in which you get 500 events. So, changing these values of M and Lambda will give you a different theory in which the predictions for physical masses for the for the number of events that are produced in a particle in particular collisions they all will change.

Even though the form of the action is the same but the parameters are different and that results in different in different physical observables different values of the physical observables. So, now what I will do is we will stick to one theory and by sticking to one theory means having a particular value of M and Lambda. So, that all the outcomes are fixed there is no ambiguity in uh what you are going to get in a particular scattering or what you are going to get for the physical Mark they are all fixed now.

So, M in Lambda are fixed keeping Epsilon fixed. So, that I do not run into the trouble of infinities. So, now I want to see the effect of doing renormalization what things change when I do a renormalizations and renormalization and what quantities do not change when I do a renormalization. And we will also come back to this discussion here in a in a while you will see why I was talking about this.

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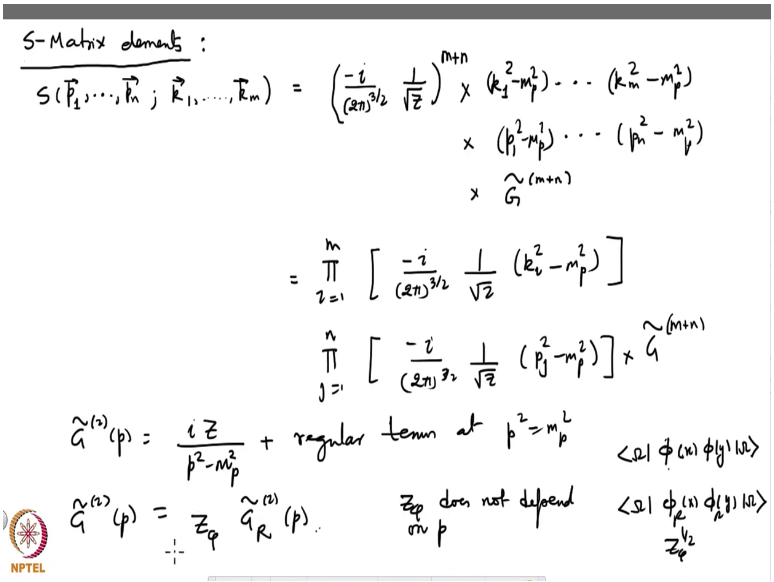

So, let us first look at the S Matrix elements see we have seen already that the Greens functions they change. So, greens functions change when you do a renormalization because each field when it gets renormalized picks up a factor of Z Phi or square root of Z Phi. So, let me. Now Look at the asymmetrics elements what happens to them and of course I am going to keep the theory fixed.

I am not going to change the bare parameters M and M Lambda they will not change. So, S Matrix element you know they are giving you the the probability amplitude of a state with labels K 1 to K M transforming into a state with labels P 1 to P N and you would expect that this quantity would be a fixed I mean once it is given what theory you have these probabilities would get fixed or these amplitudes will get fixed right.

So, that is the expectation. So, I expect that s should not change if I keep the bare parameters in the theory fixed and even if I change other things like renormalized parameters and renormalized Fields even if I change them but my bare parameters are fixed the theory should not change and these asymmetrics elements should not change and let us verify whether that is indeed the case.

So, you recall how it is given you can go back in the notes and you will find that the expression is this I'm writing it slightly differently compared to what I wrote earlier I mean it is not different it is just minor reshuffle of the factors here and there it was this remember if we had one factor of 1 over root Z for each external leg and that is why you have one over root Z M + n and you have these factors M P is a physical Mass times the greens function M + n Point greens function where G tilde is the Fourier transform.

The tilde always denotes the Fourier transform that is what we had seen and you also know that G tilde contains the poles coming from the propagators right and what is left after pulling out those contributions from the external leg corrections is the amputated greens function. So, clearly the asymmetrics element is the residue of these greens functions if you take the residue of this.

So, you know this as a pole. So, take the residue of the pole that will give you the S Matrix. So, I should have written it slightly differently anyway I can do it now. So, let us write it like this minor modification I am just writing the incoming and outgoing part separately I am just distributing these factors over these products J equal to 1 2 n times. Now recall what is the structure of 2 point function and that is relevant because the external legs they have the structure of 2 point function.

So, let us recall what is a 2 point function and again I am writing everything in the bear Theory this there is nothing I am not using the renormalized grease function. So, this is G tilde are calculated with the bare fields. So, G tilde 2 was we found that it is i z over P Square minus M P Square where Z I had introduced earlier this Z is not the wave function renormalization constant that you have seen here that I denote by Z Phi plus regular terms.

Let me see if I can find out easily where that expression was it is not in this file anyway you can go back and see this expression there is a structure we already know that G tilde R 2 that is the 2 point function has a pole at physical mass. So, that is what is this denominator telling you and the residue is I times Z and then you have other regular terms at P Square equal to M P Square and also when we use the renormalized fields then G tilde 2.

So, here this is the correlator made out of this thing can you take the Fourier transform of this but the fields are bare fields that is the 2 point function but in this for this one it is constructed out of fiery normalized and you take the Fourier transform and that is what gives you digital R 2 and because Phi is z till I don't think I'm putting tilde Z Phi to the half. So, you see you get a factor of Z Phi here.

So, that is how the 2 point function with bare Fields defined with bear Fields is related to the 2 point function defined with the renormalized fields. So, R is the subscript I use for indicating that I am using renormalized Fields here. So, that is the relation also note that Z is a constant this renormalization constant is really a constant it does not depend on P. So, Z of Phi does not depend on P or P Square.

So, the pole structure here see these the left hand side is a function of P the right hand side is a function of P and this factor is a constant does not carry any P dependence and I know that the left hand side has a pole at physical Mass. So, it it has a form of 1 over P Square minus MP Square. So, it has a pole and then it implies that on the right hand side also since this is the only factor which has P dependence this should have a pole at physical Mass right.

So, that the uh. So, that it is the same function otherwise it cannot be that the left hand side has a pole at P equal to M C square and right side does not have because it has to be because left hand side is same as the right hand side. So, we should have poles on both the sides. So, it is clear that G tilde 2 defined with renormalized fields also has a pole at the physical Mass let us look at the residue now.

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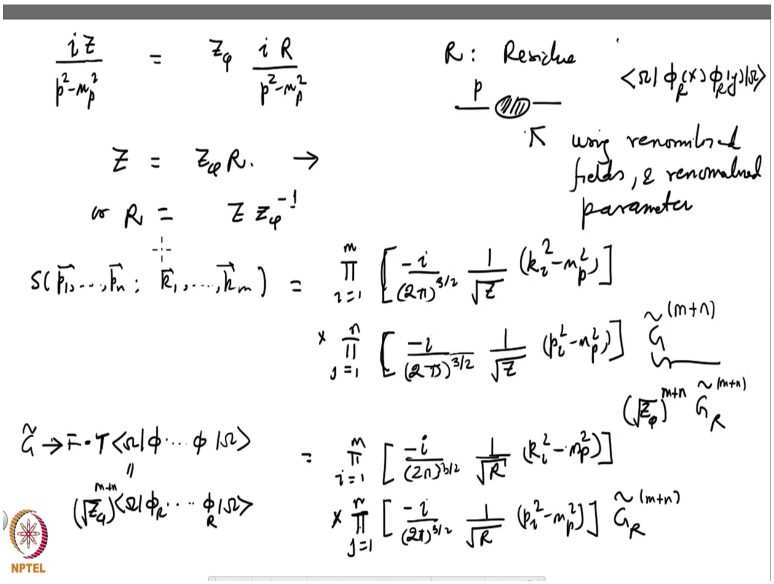

So, for that I will do I will just write the expression close to the P Square equal to M P Square. So, the left hand side is 1 over P Square minus M P Square I times Z I am just looking at the pole terms and what do you have on the right hand side you have a Z Phi this factor and then I have argued that this has a pole this is the same pole. So, it should have I times some residue.

Let us call it what I am calling it R the residues are divided by P Square minus M physical Square where R is the residue for is the residue which you get here. So, this is the 2 point function and here the fields are renormalized fields. So, as I wrote earlier this is just the Fourier transform of this and normalized parameters this is this object. So, when you calculate this you know that that is going to give you a pole at the physical Mass.

And it will have some residue and whatever residue that you are going to get I am denoting by R or rather i times R. So, from here you can see that Z is Z phi times R or the residue that you are going to get in the renormalized theory is Z over Z Phi and of course if you choose a renamerization scheme such that this R is one that you can choose such a renormalization scheme so, that you get we have already done that before.

Let us see here. So, with these choices so, we had demanded that we want to put a we want to use a scheme such that the propagator comes out to be i over P Square minus MP Square. So, that the residue is one and then you can find out what conditions you have to impose. So, this is what we have done here. So, you could do you could ask for that and if you ask for such a condition to be satisfied that R should come out to be one.

Then Z is same as Z phi or the wave function renormalization constant Z Phi is same as the Z which we had introduced earlier but in general if you are not using such a scheme you will have a residue R which will be different from one and it will be related to Z Phi and Z by this relation or this relation. So, now let us see what happens to the asymmetrics element given that we know how the residue changes when you change when you do a renormalization.

So, we see that s Matrix element and this I am writing using the bear quantities. So, my understanding is that x asymmetics elements are physical observables these are physical quantities because you are you are talking about what is the probability amplitude of a state with with these labels given to K 1 to K m appearing as another state with labels this P 1 to P M and it should not change and let us see what it gives us.

So, I have product I equal to 1 to m minus i over 2 pi 3 halves 1 over square root of Z k i Square minus M P Square yeah sorry. So, let us go here. So, here let me let me write it it will take little while to write but it will be it will be better to do this let me write it again I am just writing the same thing again now what I will do is I will put this result that Z is Z Phi times r. So, here I will put Z Phi times R.

So, you will get a root R here one over root R here another one over root Z Phi and remember I am not I am still writing the same as Matrix element this is still in the bear theory I am just replacing this n this is just like what you have here with 2 Fields this is an object constructed out of M plus n Fields and each field when you turn to renormalize field will give you a factor of root Z root Z of Phi.

So, what I am saying here is that G tilde is the this has stopped working for some strange reason for some strange reason it is Fourier transform of just a second let me see why it is not working. So, it is a Fourier transform of such M plus n Fields and when I write this as correlation function with Phi R here I will have Z Phi M + n. So, root Z Phi M + n. So, here if I replace this factor this will have root Z of Phi M plus n g tilde M plus n but the renormalized one.

And you have seen here already that you get 1 over square root of r and then you have yeah root Z of Phi in the denominator. So, roots are defined the denominator and root are in the denominator and there are M such factors here and N such factors here. So, together they will give you root Z Phi M plus n in the denominator from here and you have root Z Phi

to the M plus n in the numerator coming from this part. So, the same expression I can write as the following 1 over r square root of r where square root of where R is the residue when you which you get when you used again not writing when you used renormalized fields and renormalized parameters this is getting hopeless. So, n - i over 2 pi 3 half 1 over square root of R minus M P Square times G tilde R this is with the renormalized field.

So, you see whether you use the renormalized perturbation theory renormalized fields and parameters and then calculate G tilde with the renormalized fields and put the residue that you get in the renormalized theory that one or you work with the under normalized fields. And parameters you get the same result which is good because that is something we should expect because s is a physical quantity right.

It is telling you the these probabilities of transitioning from here to there and that should not that should be fixed once you fix the theory and the theory gets fixed when you fix the bare couplings and a bare bear Mass parameters.

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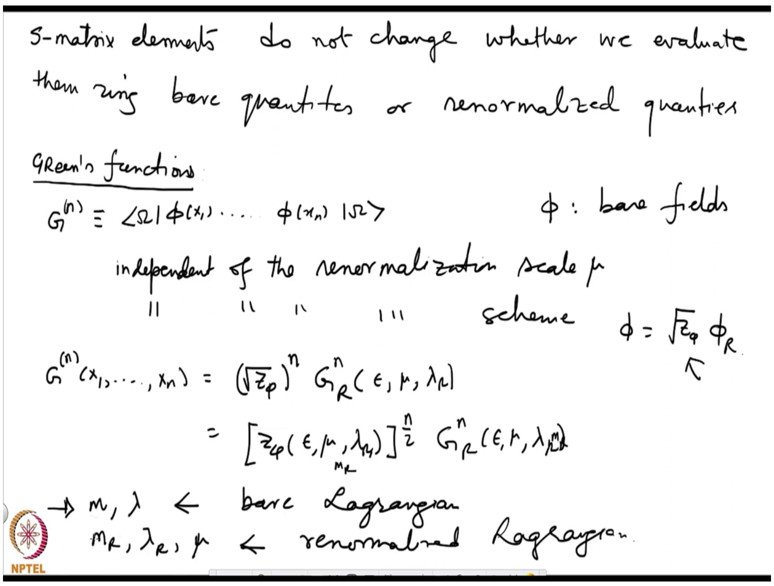

So, this is good let me write this in words S Matrix elements do not change whether we evaluate lots of difficulties today whether we evaluate those using bare quantities or renormalized quantities my quantities I mean fields and parameters. So, that is about asymmetics elements let us look at greens functions. So, let us hope this works. Now yeah I was looking at Greens functions.

So, let me write down a green bare greens function which I will write as G n where these files are bare fields. Now this object is independent of the renormalization scale mu right because bear quantities do not depend on you there is no mu in the in the action this mu comes when you write the fields and the couplings in terms of renormalized fields and renormalized parameters.

So, when you have Lambda R then only you will get mu but here because this is in terms of where quantities there is no mu. So, it does not depend on mu and of course it does not depend on any scheme also independent of the renormalization schemes we have not subtracted any infinities. So, there is no reason why scheme should affect this. Now let us write this greens function in terms of renormalized greens function.

So, G n is equal to this object which you have here. Now each field Phi I will write as Z Phi times Phi R and remember these are constants they do not contain any momentum dependence or any X dependence. So, this is root Z Phi to the n times G n but now with renormalized fields because we are denoting by G R the Green greens greatest function that you construct with G R with normalized fields.

And of course this is a function of Epsilon mu and Lambda R let me write this Z also explicitly with this its arguments. So, Z of Phi it depends on Epsilon mu n Lambda r see since the left hand side is independent of all it is Mu and scheme and all these things but G depends on it. So, that G depend that dependence in G has to be cancelled by the dependence in Z. So, Z also has to depend on all these variables Epsilon mu and Lambda R and M R also I should have written M R also.

These are the parameters in the theory but now you see that the original action or the lagrangian had only M and Lambda these were the only parameters in the bear action or bear lagrangian but when you do renormalization when you write everything in terms of renormalized fields you are writing in terms of M R Lambda R and mu right. So, you see that you have 2 parameters here. So, your theory is really controlled by these 2 parameters.

So, once you specify what M and Lambda is I am still thinking of Epsilon as being finite. So, once you specify for a given Epsilon what M and Lambda are then you have fixed everything in the theory meaning what is the physical mass of particles that is fixed what is the probability of producing a certain event that is fixed those things are fixed but here you see you have for the same action for the same theory instead of 2 three parameters.

So, clearly all the three are not independent because here you have 2 here you have three. So, it is possible to choose different values of mu or let me put it this way. So, let us look at the theory from this point of view where you are using only M and Lambda the bear parameters. So, you have concluded all your conclusions you have found out what is the physical Mass when you have chosen a particular value of M and particular value of Lambda.

So, given that fixed choice of M and Lambda bear parameters you have found out what the physical mass is what is the probability of producing up particular final State let us say 2 body final State when you are colliding 2 particles a three body final State when your produce Pro colliding whatever number of particles you are fixing the initial State you are fixing the final State you get numbers depending on the initial and final momenta you have all these big table of tables of numbers.

Now to choose some value of mu given that value of mu that you have chosen find out Lambda r and m r these numbers what their value should be such that you reproduce those entries in the table meaning you reproduce exactly the same physical observables having the same physical values given the choice of mu you have made and the Lambda R and M R. So, once you do that you are still talking about the same theory because all physical observables are same whether you choose M Lambda or M R Lambda and mu.

Now what you do is because you have three parameters here you choose a different value of mu. Now choosing a different value of mu will still be because you will be able to change Lambda R and M R because not all three are independent. So, accordingly you change values of Lambda R and M are such that all the physical observables are reproduced exactly the way they were before.

Meaning your new choice of mu and accordingly adjusted value of Lambda R and M R they should give you exactly the same physical mass as you had earlier and the same values for other observables. So, that you can claim that you are still talking about the same Theory and not different Theory and this will happen because simply because you have 2 parameters here and you have more than 2 parameters here.

So, there is there is a redundancy in choosing in these parameters. So, you have 2 here three. So, you can choose one of them and Vary it and accordingly you should vary these Lambda R and M R such that M and Lambda do not change the effect is what you want to arrange is that changing mu should not change M and Lambda and that you achieve by correspondingly adjusting the values of Lambda R and M R.

Which means that once a theory is given to you and by theory I mean these observables are values of these observables are fixed then these parameters M and Lambda are really dependent on the choice of mu you make.

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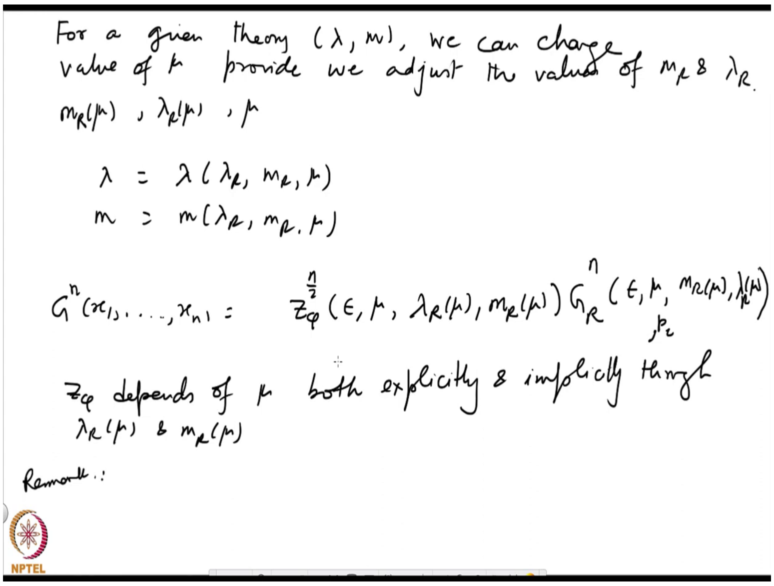

So, instead of writing just M R and Lambda R I should write m r of mu Lambda R of mu these things. So, for a given Theory meaning given choices of Lambda and M we can change values of value of mu provided we adjust or we change the values of M R and Lambda R such that Lambda and M do not change right because Lambda is a function of Lambda r m r and mu and M is a function of Lambda R, M R and mu.

And if I could arrange that Lambda and M do not change even when I change mu then clearly all the predictions for the observables they will not change. So, I will I should write then G n x 1 x 2 up to x n is equal to Z Phi n over 2 where Z depends on these variables. Now I am making this notation explicit by putting mu as an argument of Lambda R and of course it also depends on external momenta that are also parameter.

So, you see that z Phi depends on mu both explicitly and implicitly both explicitly and implicitly through Lambda R and mu R and M R sorry. So, there is a mute dependence that comes through the arguments of m r and Lambda that is the implicit dependence and then you also have an explicit dependence on mu here I let me go back and show you somewhere here I do not think we have it here.

But you see what finite parts you subtract is up to you and any dependence on mu that you get when you are renormalizing that also you can subtract because mu is some finite you have chosen it to be some number that also you can subtract some new dependence you can subtract. So, z this renormalization constant Z will in general depend explicitly on.

So, on mu because you are subtracting some finite parts. Now here is a remark that when you are doing M S scheme or M S bar scheme in these schemes in these renormalization schemes you only subtract the the pole parts and also in the M S scheme and in M S bar you also subtract things like log four pi and Euler gamma. So, at one Loop of course this is how it appears and clear it is clear that at one loop Z Phi will not depend explicitly on mu.

Because what you are subtracting will only have the pole and no mu dependence let me go back and show you I hope I will find it this time here. So, this Z Lambda Z M they depend only on Lambda R here there is no explicit mu dependence. So, here now I have already argued that Lambda R should really be written as Lambda R of mu. So, the mu dependence in Z comes through the dependence on Lambda r that is a statement of course true at one Loop because that is something you have seen explicitly.

But it is true that this statement is even true to all orders no matter at what order in in Lambda R and R the dependence on mu will be only through Lambda and also M R will not depend on will not appear in Z phi.

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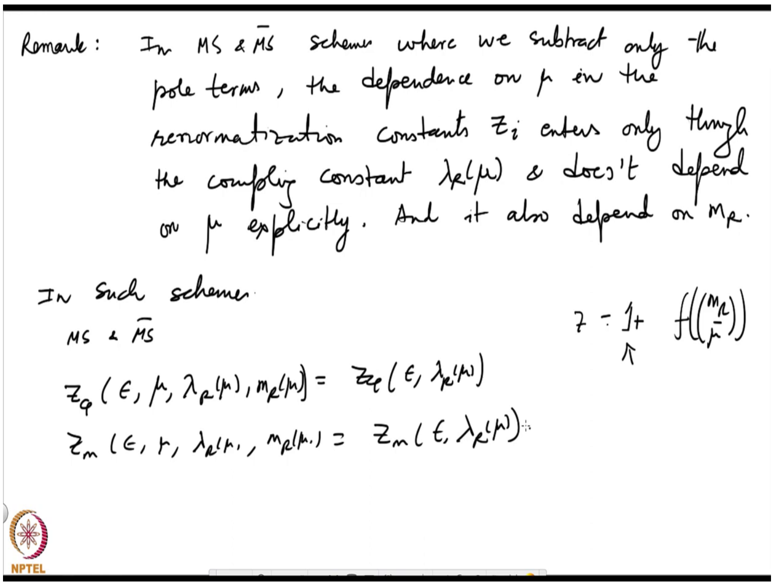

So, let me this is an all order statement I am not giving you the argument for it but I am just stating the result so the remark is the following in M S and M S bar schemes where we subtract only the pole terms the dependence on mu in the renormalization constants Z i where i is Phi or M enters only through the coupling constant and it does not depend on mu explicitly and it also does not depend on M R it cannot depend on M R.

Because you see Z is one plus something now one is dimensionless and if mu is not allowed it cannot appear explicitly then a marker cannot also appear because if M R appeared then the object will be Dimension full the term that you will be writing will be dimensional the only way M R could appear would have been if mu also appeared because then you can make such ratios M R over mu which is dimensionless.

So, it would make sense to add such a dimensional dimensionless function to a dimensionless quantity here but since mu cannot appear M R can also not appear. So, it is not. So, evident from one Loop calculation that mu will not appear but it is is something you can argue that whatever mu dependence might appear gets cancelled when you are looking at M S or M S bar scheme. So, in such schemes which is M S and M S bar our renormalization constants as we should write as.

So, in general these are dependent on all these objects but in these schemes they only depend on Epsilon and Lambda R. So, this is good something we know about the dependence of these renormalization constants on mu and that is something we are interested in because if you look at the bare greens functions this does not depend on mu right hand side does and G n depends on mu.

In a certain way square root of Z depends on mu in certain way such that the dependent gets cancelled because on the left hand side you have mu Independence. So, we will analyze further these objects in the next video.