Introduction to Quantum Field Theory - II (Theory of Scalar Fields)

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Lecture - 32
Renormalization -Part 4

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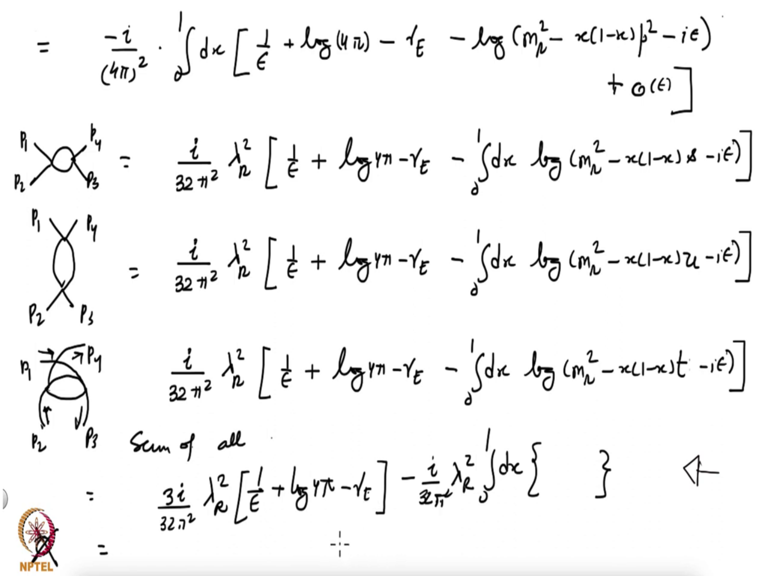

Let us continue our discussion on renormalization. In the last video we saw how we can evaluate some physical observables and that these observables depend on the choice of scheme and the result also depends on the choice of renormalization scale mu. I mean even though I have already told you that the result should not depend on the choice of mu here for example. But your predictions will depend on mu because we are truncating the perturbation series to some order in Lambda R.

If you could evaluate to all orders in perturbation theory then you would not have a a dependence on explicit dependence on mu sorry a dependence on mu. Also we saw that these schemes Ms and Ms bar and such schemes they are telling you what finite constants you remove together with the coupling const together with the infinite pieces. So, for example here the choice of Z lambda, Z M and Z PHI they specify in addition to this pole that you do not subtract any additional term you subtract only the singular parts.

So, that is M s scheme or minimal subtraction scheme and we have seen that these factors of log 4 pi and Gamma i they always appear in this combination at one Loop and if you subtract these also then the result is in M s bar scheme. And of course the physical result or the result for a physical observation observable that you get depends on the choice of scheme because here you see this is the sum of all these I should remove this thing on the next page.

Now here so, after we added this counter term and we removed this 3i over 32 Pi Square Lambda R Square 1 over Epsilon this term log 4 Pi - gamma e times this Factor together with this remaining term which contains the kinematic part is what gives you the four point function which enters the observables that you are calculating. Let us say you are calculating a cross section for two going to two processes in 543.

So, you scatter two particles and you observe two particles in the final State and in that case this these fundament diagrams will be used in that that calculation. So, the result that you get for the cross section will depend on what is what you input from here right what is this four point greens function but this result depends on the scheme choice whether you have kept log four Pi - gamma e or not or you have some other constant.

So, the result is going to depend on the choice of scheme M s bar scheme or modified I do not know whether I wrote it earlier let me write on the next page.

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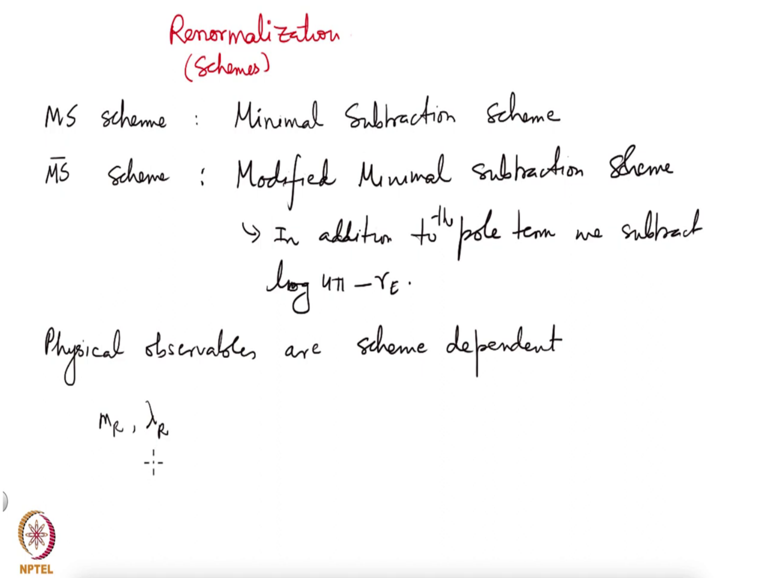

So, we have seen M s scheme modified sorry minimal subtraction scheme minimal in the sense that you just remove the singular pieces nothing more nothing less M s bar scheme is modified minimal subtraction scheme so, in addition here in addition to poles in addition to the pole. So, we are working at one Loop to the pole term we subtract log 4 Pi - Euler gamma physical observables as I have argued are scheme dependent.

So, you are calculating to some order in perturbation Theory and you see that your result is going to be dependent on the scheme when you calculate to a fixed order in perturbation theory. So, here I was arguing that some scheme might be better than other scheme even though they are all equivalent but because you are going to calculate the observables to fixed ordering perturbation theory a particular scheme choice might be more suitable.

Because the terms which are generated such as log 4 Pi - Euler gamma e and the such terms will also appear at higher orders and these are not very small terms. So, they keep appearing at higher orders. So, your convergence towards the correct answer will be better in one scheme compared to another scheme you can imagine that you instead of subtracting this you subtract a large number times Lambda R square and similarly at higher orders.

So, the prediction becomes very different now because you have some other large number here and you go to one order higher end perturbation theory a similar large number is getting subtracted there. So, your results are not going to converge towards the true result fast if you to make a bad choice of a renormalization scheme. So, here as you can see that M s bar scheme will be better than M s because such large terms which systematically appear at all orders in the perturbation theory are removed.

So, convergence will be better and that is why M s bar scheme is often used when one is working with Quantum Field Theory. So, this is one reason why people go to higher orders in perturbation theory because the dependence of the observables the calculated observables on the scales mu and the schemes becomes less and less as you go to as you include higher order terms.

Because I mean if you could if you could sum to all orders in perturbation theory the result should not depend on such choices. So, this dependence goes down as you include higher orders in perturbation Theory which means higher order terms in Lambda R. But these are not the only schemes that are possible I will tell you of another scheme that you can use. So, here when you're using such schemes subtraction schemes M s and M s bar.

Then you are in the lagrangian or yeah so, you are using M R Lambda R and such things as some fixed numbers that you use and then you determine the physical observables in terms of these parameters but sometimes you might want to also work directly with the physically physical observables that are measured in the calculation and that is also possible it is not impossible I will give you an example.

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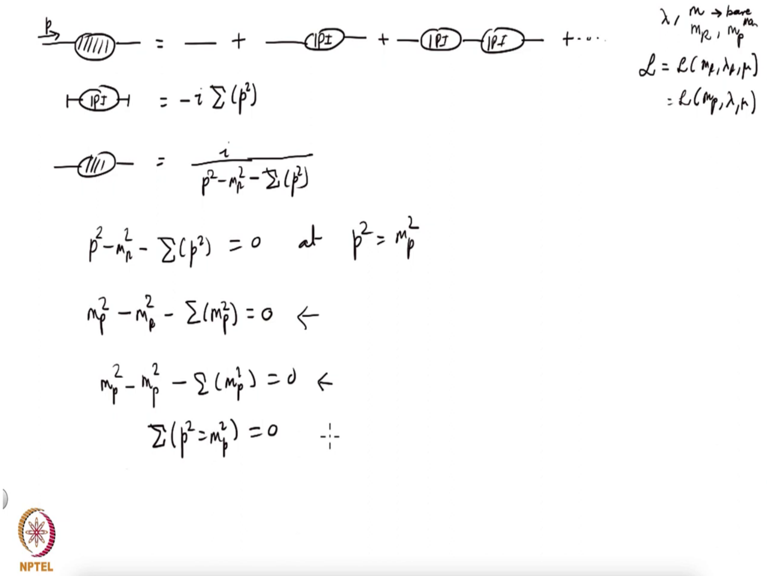

So, suppose instead of using M R you want to work with M physically. So, suppose you are given a theory and you have you have written the lagrangian in terms of M which is the bear Mass and Lambda which is bare coupling constant and then you split the lagrangian the way I had earlier told and use a fixed a finite number M R in the in the lagrangian. So, your lagrangian density is now a function of M R Lambda R and also mu and of course Epsilon also.

But suppose you wish to write this lagrangian density not in terms of M R and Lambda R for something that you can measure physically. For example you may want to directly work with physical mass in the lagrangian density and some other Lambda that is measured I will tell you what that Lambda could be in the in the in the theory. So, let me describe one such scheme.

So, let us look at first two point function and we have seen that in the perturbation theory if I sum all the terms this is one particularly irreducible and so forth where one particularly reducible. So, here I will just put these two vertical lines to you to tell that these are amputated that these amputated veins limbs are cut. So, these legs are cut they are not part of part of the diagram meaning you do not have to include a factor of I over P Square or I sorry I over P Square - M R square for this.

And for that that you don't have to include and this is what we had defined as - i Sigma and with this the structure of two point function becomes and you already know that the structure of two point function is that it has a pole at the physical mass and the zero of the denominator which is where the pole lies gives you the physical Mass right. So, this gives you the condition that we where the physical gives you the condition the zero of this gives you the condition to determine the physical Mass.

So, let us see P Square - M R Square - Sigma p square that is 0 at p square is equal to M P Square. So, if I put p square is equal to M P Square I get M P Square - M R Square - Sigma of P equal to M P Square. So, I have put P square is equal to M P Square here and that is the condition you get now suppose I want to choose a value of M R no suppose I have measured in the experiment and I find that the particles have mass M P.

Now instead of choosing some Mr in the theory which is finite and determining M P in terms of M R I want to choose physical mass as the parameter that appears in the lagrangian itself in the pre-normalized LaGrange. So, I do not want to use some M R but I want to use M P and that is possible because that would require that I choose M R to be M P and the condition this condition now turns into this condition.

Where I put M R to be M P that is what I want to have that is a choice I want to make and this says that the sigma at P square is equal to M P Square should vanish. So, that is the condition if I can satisfy this condition then the then I can I can use the physical mass for M R and one can then determine what is the what is what does this imply for the renormalization constants because that is something you still have to fix.

So, you see that making this condition will make the physical mass appear in your in sorry making this choice will make M R equal to the physical mass.

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$$p \; \text{—}\!\!\bigcirc\!\!\text{—} = \frac{i}{p^2 - m_p^2 + i\epsilon} + \text{regular term at } p^2 = m_p^2$$

← The residue $= i$.

← becomes the propagator of a free particle of mass m_p.

Denominator —◯—

$$p^2 - m_p^2 - \Sigma(p^2)$$

Taylor expand $\Sigma(p^2)$ around $p^2 = m_p^2$

$$\Sigma(p^2) = \underset{\substack{\parallel \\ 0}}{\Sigma(m_p^2)} + (p^2 - m_p^2)\left.\frac{\partial \Sigma}{\partial p^2}\right|_{p^2 = m_p^2} + \frac{1}{2!}(p^2 - m_p^2)^2 \left.\frac{\partial^2 \Sigma(p^2)}{\partial (p^2)^2}\right|_{p^2 = m_p^2} + \cdots$$

$$\Rightarrow = p^2 - m_p^2 - (p^2 - m_p^2)\left.\frac{\partial \Sigma}{\partial p^2}\right|_{m_p^2} - \frac{1}{2}(p^2 - m_p^2)^2 \left.\frac{\partial^2 \Sigma}{\partial (p^2)^2}\right|_{m_p^2} + \cdots$$

$$(p^2 - m_p^2)\left[1 - \left.\frac{\partial \Sigma(p^2)}{\partial p^2}\right|_{m_p^2} - \frac{1}{2}(p^2 - m_p^2)\left.\frac{\partial^2 \Sigma}{\partial (p^2)^2}\right|_{m_p^2} + \text{higher order terms in } (p^2 - m_p^2)\right]$$

Now I also want to make another condition requirement that I wish to have the two point function to have the following structure. So, I have already written here that the; I mean sorry that is that is the structure. So, there is nothing special here also you have other regular terms at P Square equal to M P Square the other thing that I have done is you see you we always had a residue here it was not 1.

So, the residue was one when we had free theory but the residue was not one when we had a interacting theory. So, but let us say I wish to have the structure of the propagator in the renormalized theory such that the residue is one here. So, that the propagator is that of a free particle of mass M P and that we can arrange and let us see how. So, the condition is that the residue is equal to it will be more appropriate to say I rather than one because you have an i here.

So, that is what I want. So, that this looks like becomes the propagator of a free particle that is what I wanted to be like and then I should determine what should the renormalization constants be if I require this to happen. So, let us look at that um. So, what I am asking is that. So, now let us go back here this is how it looks like i over P Square - M R Square - Sigma P Square.

So, let us look at the denominator of this which is as I showed you on the previous page P Square - M R square but m R I have chosen to be M P already - Sigma of P Square. So, that is what it should it will look like when I put it in here where M R I have chosen to be M P.

So, now let us write it in the following manner. So, I will taylor expand Sigma of P Square. So, Sigma of P Square around p square is equal to M P Square where the pole is.

So, Sigma of P Square is Sigma of M P Square + P Square – M P Square evaluated at p square is equal to MP Square + other terms + higher order terms higher order terms in P Square – M P Square you are expanding close to M P Square. So, these are terms of lower order compared to P Square – M P Square whole Square now this first term the constant term Sigma M P squared that is 0 from here Sigma of M P square is zero that is the condition of having the pole at this the condition of having M R equal to M P.

So, that is 0 now I have this term and other terms. So, what becomes of this denominator it is P Square - M P Square - this term P Square - MP Square I will just write in short here I'll put MP Square and then you have this term with the - sign - half P Square – M P Square whole Square Delta 2 Sigma over Delta P Square whole Square value to that M P square and then other higher order terms.

So, let us now this is equal to this now let us factor out P Square – M P Square and that gives you 1 - Delta Sigma over Delta P Square evaluated at M P Square - half P Square – M P Square. So, one factor of P Square – M P square is pulled out it is here times Delta 2 Sigma over Delta P Square whole square at M Square + higher order terms in P Square – M P Square. So, the next term will be P Square – M P Square whole Square.

So, this is what you get in the denominator. So, what you have at this stage is here. So, at this stage let us call it denominator this is um.

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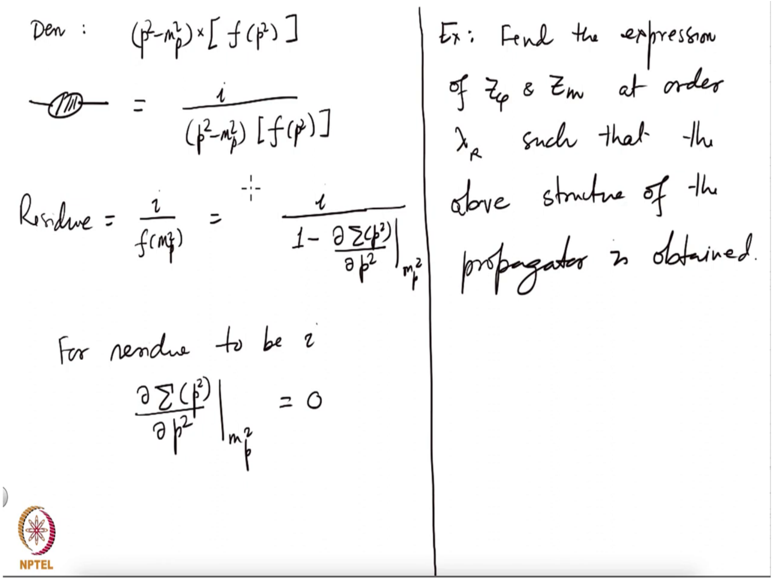

So, denominator now is P Square – M P Square times let us call it f of P Square whatever that f is. So, your propagated the two point function sorry is i over P Square – M P Square times f P Square and what is f P Square f of P Square is determined in the square brackets. So, now you can read off the residue. So, the residue is i over f of M P Square right you have to put in i over f P Square P Square equal to M P square and that gives you the residue of the pole.

So, the residue is this and how much is that that is i over let us go back. So, when you put P square is equal to M P Square this term goes away all other all other higher order terms also drop out because they all contain factors of P Square – M P Square only these two terms survive. So, you get 1 - Del Sigma over Del P Square. So, you get residue is 1 - Delta Sigma of P Square over Delta of P Square evaluated at M P Square and we want the residue to be i.

Because we wanted everything to look like a propagator of a free particle. So, for that to be true this derivative should vanish when you put P square is equal to M P Square. So, we get the condition that is another condition. So, I will leave it as an exercise it is easy we have done already equivalent of this in the case of M S and M S bar. So, this you will be able to do. So, find the expression of the Z's and Z phi’s if these conditions have to be satisfied.

Find the expression of Z Phi and Z M such at one loop at order Alpha is so, not Alpha sorry Lambda R such that the above conditions are satisfied. So, you will get with this you will get the residue to be Iota and the renormalized mass will be equal to the physical Mass. So, find out what is that Phi and Z M you should choose right because infinity the infinite term the singular term.

You have to subtract anyway and these differences in the skin can only be accommodated by changing the finite constants that you subtract. So, that is going to change only the expressions of finite parts of Z M, Z Phi and whatever else you have so because as you have already experienced that looking at the propagator you only fix Z Phi and Z M not the Z Lambda. So, that is why this in this exercise I have asked you to find out what Z Phi and Z Lam Z M R.

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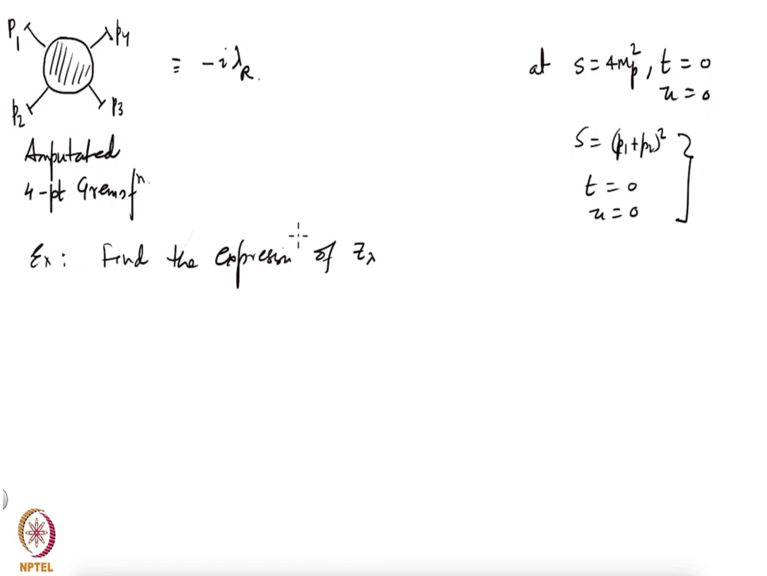

Now let us look at the four point function how come I have got this I should add more pages here. So, let us look at four point function now and again these are amputated. So, I am looking at an amputated four point Green function and that amputation I will denote by putting these small ticks so, amputated for one function. So, suppose you measure this which is which is something you can do by measuring some cross section of 2 going to two scattering because this will be an input.

So, it gets measured that way and suppose I want to work in a scheme a renormalization scheme in which I say that this object at s equal to 4 M P Square and t equal to 0 and U equal to 0 has a value which you want and you call it - i Lambda R. So, in this case you see you are not choosing some arbitrary value of Lambda R and then finding what this fourth Point function would be but rather you are saying that I want to use that Lambda R in my perturbation theory which equals the value of this amputated four point Green function at a certain scale.

Remember you have to choose a scale because then only you will get a constant for the terms that differ in these where did that go what is it just a second here see this Z Lambda is at M Z Phi I could have a constant here a constant here and another constant here at R L Lambda. So, these terms which you have here they are constants these constant differ when you go from one scheme to another scheme.

And that is why to have a constant you choose a scale because otherwise the result will depend on the kinematic factors also. So, choose a fixed scale and on that scale you say that the value of this four point greens function is Lambda R and. So, you say that when s value which is you remember that s is P 1 + P 2 whole Square. So, you are saying when this is 4 M P Square and T is equal to 0 and U equal to 0 you should figure out the physical meaning of this that would be a nice exercise to do.

That at these values the Green function is - i Lambda R when you demand this this will give you this will fix the Z Lambda that you should use. So, this is another exercise I leave it for you to do find out the find the expression of Z Lambda and the calculations that we have done earlier is what you are going to use to determine what Z Lambda should be to order Lambda R.

So, I hope that this will help you clarify that clarify the nature of these renormalization schemes and those renormalization constants how to fix them you can choose to write the the perturbation Theory using physical parameters like physical mass and Lambda R of this kind which I have written here in front of you I call it physical because this is something the left hand side you measure in experiment maybe directly maybe indirectly.

But you do measure it and you say that that is whatever value I get at some scale such as s is equal to 4 M P square and t equal to 0 and u equal to zero that is a physical number that you have measured right. So, that you can use in the perturbation Theory or you can use schemes like M S or M S bar where you choose some finite Mr and Lambda R and determine the physical observables in terms of those.

So, either way it is all they are all good whichever is suitable for your for your purpose typically in the field here is like QCD you use M s or M s bar schemes they are more suitable in that context. We will stop here and we will continue our discussion in the next video.