Introduction to Quantum Field Theory - II (Theory of Scalar Fields)
Prof. Anurag Tripathi
Department of Physics
Indian Institute of Technology - Hyderabad

Lecture - 31
Renormalization -Part 3

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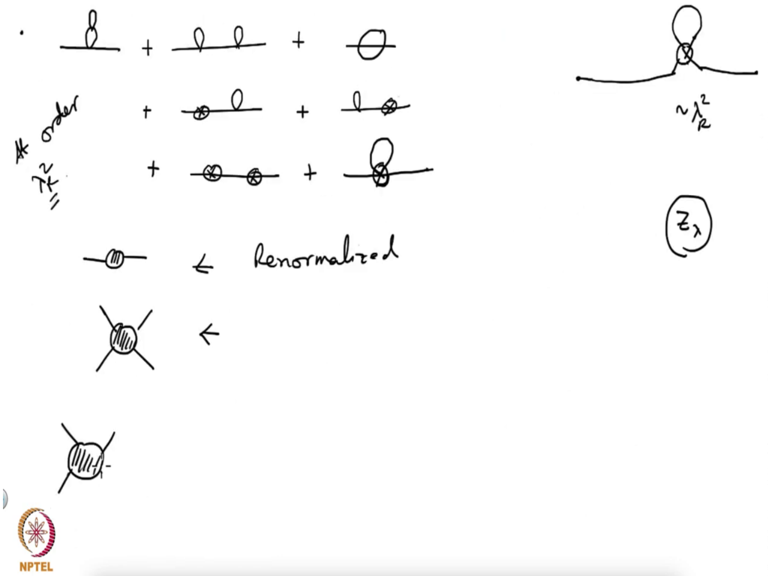

Let us review what we have done so far. So, we have succeeded in making 2 point function finite up to order Lambda r. So, we have made 2 point function finite up to order Lambda r remember Lambda r and M are they are finite parameters they have finite values and the Lambda and M which you had in the original Theory they become infinite as Epsilon goes to zero that is what we have seen.

So, more specifically at order Lambda r we had this contribution at one loop from the to the 2 point function. So, let me just write it. So, if you look at the 2 point function this is to the lowest order this just a propagator and then you have this diagram and of course the counter term diagram also and we made this finite up to order Lambda and of course there are other higher terms higher in order Lambda.

But we have not done anything for those we have whatever calculation we did last time was for order Lambda terms. So, these are those terms because here you have Lambda r and this vertex as you saw is also Lambda vertex it is a 2 point vertex but the order is of course

Lambda r and we have found that to make this 2 point function finite up to order Lambda r the constant the renormalization constants take these values.

So, Lambda Z of Phi was 1 + the order Lambda term order Lambda r term turned out to be zero. So, it was 0. So, it starts at then order Lambda r squared that term turned out to be zero and then Z M Square we found to be 1 + 1 over 16 Pi Square let us check it is correct spelling let me go back and see this is correct and also remind you that Z Phi what is that Phi. So, you had the field Phi in the original lagrangian that we wrote as Z PHI to the half fiery normalized and m.

We wrote Z M times M renormalized and Lambda to be Z Lambda times Lambda renormalized these are as I told you earlier these are called renormalized field and renormalized parameters and these ones are called bare parameters and bare fields I guess I said this before but I am not so sure. So, these are all bare quantities and these are all denormalized quantities. And as you see that Z M square is a singular one over Epsilon term is present here.

So, when you take Epsilon going to zero which is basically going to 4 dimensions then M becomes infinite. So, at the cost of making these where parameters for infinite singular we are able to renormalize the theory such that Phi or MRN Lambda r r finite objects. So, this diagram + this diagram so, right now I am not including the external propagators these propagators are not included you can put a cut you can put this little text to indicate that and you.

Remember this is what we had defined as minus i Sigma, Sigma is for the entire one particle irreducible diagrams. So, I will put a subscript one to say that we are at the lowest order and this is what we found last time to be I Lambda r over 32 Pi Square 1 + log of mu Square over m r squared this is good because log should have in its argument a dimensionless object that otherwise there is no meaning to the argument and this is correct because you have mass dimension 2 in the numerator and mass dimension 2 in the denominator.

So, log has an argument which is dimensionless and that is how it should appear and then log of 4 pi minus Euler gamma e minus sorry minus our gamma times m r Square and we know that how a 2 point function behaves. So, this is one particular reducible + these other and they re-sum to give you I times p Square minus m r Square minus Sigma 1 p square + Epsilon times the residue.

So, you see that I am this one p i includes all order diagrams but here I am restricting to only up to order Lambda Lambda r that is why I put the subscript one Sigma 1 and to this order the physical mass is given by I mean to any order the physical mass is given by the the zero of the denominator or the pool of the propagator pole of this 2 point function. So, I should look at where the denominator is zero.

So, what is the value of p Square for which this is 0 and that is that value of p Square is the physical mass square because that is the structure right you know it it has to go as I over times some some residue p Square minus M physical Square. So, I am just saying that when p square is equal to MP Square the denominator vanishes. So, that is why I am going to take this denominator and put it equal to 0. and the value of p Square for which that happens is the physical Mass.

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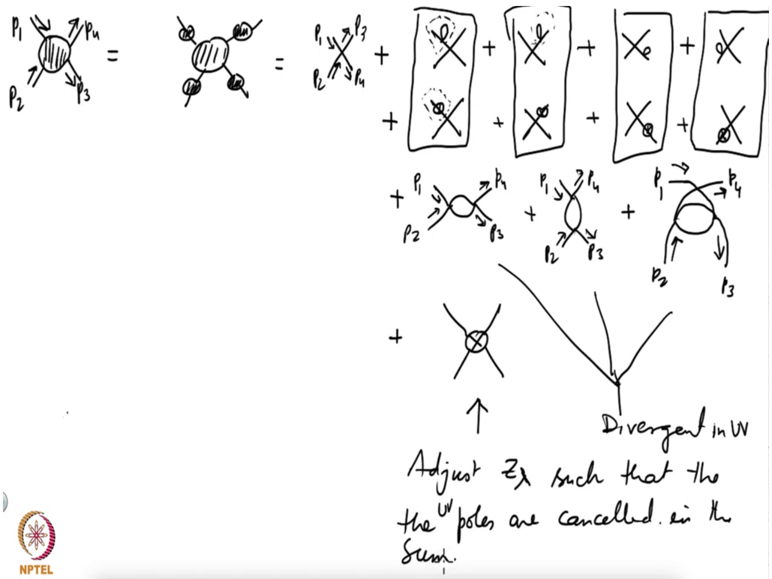

So, what is the physical mass in this Theory with these choice of parameters is piece given by the condition p Square minus M Square minus Sigma 1 of p square is equal to 0. So, let us solve that and it is easy because it is easy anyway but also note that Sigma 1 of p Square is independent of p square up to order Lambda there was no p Square dependence and you know why because this diagram does not involve an integral which carries the momentum p.

So, what do you get from here you get that p Square minus m r Square minus Lambda r over 32 Pi square and the other factors that is the condition we have and when we solve it put you get that this is 0 for p equal to the this object which means the physical Mass MP square is M r square + Lambda r over 32 Pi square m r Square + Lambda r over 32 pi squared log of mu Square over m r Square + log of 4 Pi minus gamma e m r Square that is the physical mass.

So, you see that to the lowest order in Lambda or Lambda r physical mass is same as the renormalized mass they do not differ up to our at the zeroth order at the lowest order then there is a correction. So, Physical mass is different from the renormalized mass square physical mass square is different from the normalized mass square by this order Lambda correction term and of course these are also the terms which are present.

So, this is the total correction and right now this result is in a particular C scheme it is called M S scheme right because I have in writing this only subtracted a pole term right Z M had only a pole term only the singular piece nothing no finite terms. But as I told you last time you could also subtract finite terms meaning you could also subtract this one which is proportional to Lambda r or any other term which is which you want to remove proportional to Lambda r.

So, if you remove log 4 Pi minus gamma e then that result is called a result in M S bar scheme. So, you see that the physical mass depends on the scheme which is basically which is equivalent to doing a finite field and mass parameter and this mass parameter renormalization and field renormalization by a finite amount. But anyway let us just say that this is a particular scheme in which we are giving the physical mass.

Good and also the result depends on mu which is the renormalization scale and also should again remind you I think I told here. So, find good now a couple of points. So, this diagram did not contribute a term which was proportional to p Square but as I told it was very specific to this particular diagram but in general let us say yeah in general you will have to be square dependence but let us look at a special class of diagrams that appear at 2 Loop.

So, at 2 Loop we will have these kind of diagrams. So, this is a 2 Loop diagram. So, you see I am drawing all the diagrams which are which are Lambda r square terms. So, this has 2 factors of coupling. So, it is Lambda r square diagram and then you have this also. So, what I am doing is I am drawing all the diagrams that contribute to the 2 point function at order Lambda r square.

So, this diagram definitely contributes this also contributes then you have this one that also contributes because you have a Lambda here and Lambda there this is called Sunset diagram in the literature and these ones are called Cactus diagram they look like cactus. So, these ones of course again do not depend on p Square these integrals do not carry the moment of p Square.

So, the p Square dependence is solely in the propagators but these integrals do not contain this one is the same situation whatever p enters in here never enters into the loop and it just goes out without entering the loops but here it will enter. So, if I draw this diagram maybe a little bigger. So, let us say you put l 1 here l 2 here p you have entering here. So, this will be p l 1 + l 2 + p that is what we'll enter here right.

So, now you see that when you write down the integral for this one it will be what it will be ddl 1 ddl 2 up these factors also and then you will have I over L1 Square I over l 2 square and I over l 1 + l 2 + p Square I forgot to write M r square + Epsilon and now you see that when you evaluate this integral the result will depend on p Square and M r square. So, this is going to be a function of p Square.

So, you get in dependence on p Square which was absent in this diagram and at one Loop which means that the Z PHI will be non-vanishing when you go to 2 Loops this is a 2 Loop diagram sorry not Z PHI will be not Vanishing Z PHI will be will be having a contribution at order Lambda r square. So, there will be some Auto Lambda r square term coming from these kinds of diagrams.

So, that Z PHI will have non-vanishing Auto Lambda r square contribution let me go back yeah you see Z PHI minus 1 this term is proportional to p Square right because it is coming from the Del mu Phi Del mu Fighter the counter term had a Delphi Del Mi Phi let me fire let me fire. So, there is a p-scale dependence here and you are going to generate such a term from here as well good.

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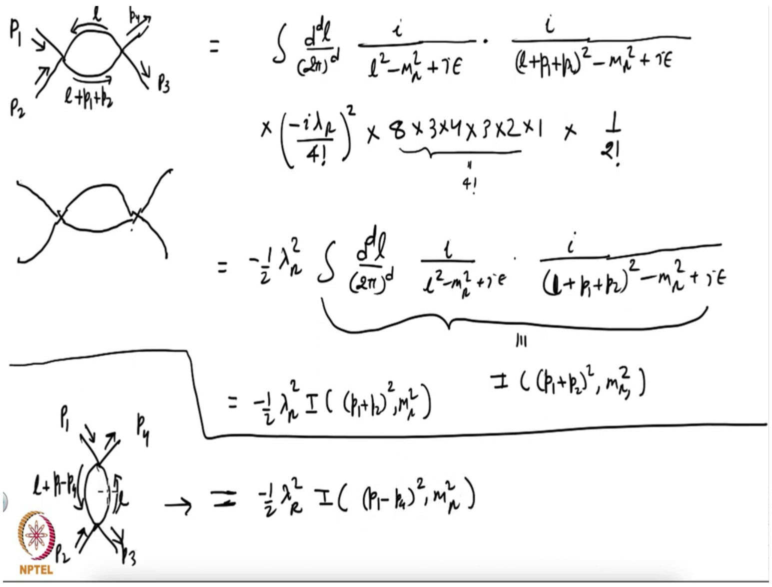

So, before I proceed with new things let me just also point out that if you were to calculate a 2 Loop you will have to include many more diagrams not just the ones which I have drawn. So, let me draw them. So, of course you have the characters and then you will have this diagram. So, all you have to do is take all the vertices that are present and connect them in all possible ways with these 2 external points which will give you the external propagators.

So, you can also draw this is another diagram that is possible and then you have this vertices present. So, you can draw one such vertex connect them and take this regular vertex then you can also have then this this is also possible and because these are both this vertex is order Lambda r. So, 2 of these makes m dot Square and then also you have the following.

So, suppose you take this 4 point vertex that is you remember this is order Lambda r square. Let me show you if you remember how come it is not here you see this we had this counter term vertex which had an explicit factor of Lambda r and because Z Lambda and Z Phi are one + order Lambda terms Lambda terms this thing contributes at order Lambda. So, Lambda r times Lambda it is a Lambda r square term.

So, this counter vertex is order Lambda r square and that is what I am using because I am drawing diagrams which are correct up to order Lambda r square. So, this vertex if I take and these are 2 of my external lines then I can connect it like this. So, this is another order Lambda r squared term that contributes this will bring in Z of Lambda with it 2 loops good. So, let us now try to make things finite completely at at one Loop for the 5 4 theory.

So, we have already renormalized the 2 point function but the goal is that we should renormalize Everything All Greens functions up to one Loop. So, if there is any function given a green function given to me I would like to have a finite prediction up to order Lambda Lambda r or let us say up to order lambda r's up to up to one Loop. So, I have already done 2 point function already renormalized by choice of Z M and Z Lambda.

Now let us go to a 4 point function this is Divergent now this object is Divergent but we still have Z Lambda at our disposal that we have not yet fixed. So, we can hope that a choice of Z Lambda can fix or can kill the singularities that are present in the diagrams in here. So, let us see what kind of 4 point functions we have we have already learned that the general structure of a 4 point function is this that you can write a general 4 point function.

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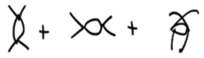

$$= -\frac{1}{2}\lambda_R^2 I(s, m_R^2) - \frac{1}{2}\lambda_R^2 I(t, m_R^2) - \frac{1}{2}\lambda_R^2 I(u, m_R^2)$$

$$I(p^2, m_R^2) = \int \frac{d^d l}{(2\pi)^d} \frac{i}{l^2 - m_R^2 + i\epsilon} \cdot \frac{i}{(l+p)^2 - m_R^2 + i\epsilon}$$

$(p_1+p_2)^2 = s$

$(p_1-p_3)^2 = t$

$(p_1-p_4)^2 = u$

$$= \int_0^1 dx_1 \int_0^1 dx_2\, \delta(x_1+x_2-1) \int \frac{d^d l}{(2\pi)^d} \frac{i^2}{\{x_1(l^2 - m_R^2 + i\epsilon) + x_2[(l+p)^2 - m_R^2 + i\epsilon]\}^2}$$

Denomenator:

$$[l^2 + (1-x_1)p^2 + (1-x_1)2l\cdot p - m_R^2 + i\epsilon]^2$$

$x = 1 - x_1$

$$I(p^2, m_R^2) = \int_0^1 dx \int \frac{d^d l}{(2\pi)^d} \frac{i^2}{[l^2 + xp^2 + 2x\, l\cdot p - m_R^2 + i\epsilon]^2}$$

Denomenator $[(l+xp)^2 + x(1-x)p^2 - m_R^2 + i\epsilon]^2$: $k^\mu = l^\mu + xp^\mu$

$k^2 + x(1-x)p^2 - m$

I should go to next page I think let us go to the next page. So, 4 point function or any endpoint function the general structure is this you have Corrections on the leg and the Big Blob in the center is what is called amputated greens function right that is what you have seen earlier. So, if you look at this then in terms of fundament diagrams you get such such diagrams I am looking only at the connected diagrams. So, you can have a correction here you can have a correction here or on here or here.

These are all possible diagrams then remember we should use all vertices not just this vertex but also the counter vertices. So, you can also get this diagram and how do I remove this let us drive this way this can be misleading. So, I will remove yes and you can have this diagram this is also present and then let us call P 1 P 2 P 3 P 4 let us assign momenta this way so here also and similarly on all these other diagrams and then we have P4 and you have P 1 P 2 P 3 P 4 and then you have.

So, this is P 1 and this is P 3. So, meaning P 1 and P 3 attach at the same vertex see here P 1 and P 2 attach at the same vertex P 1 and P 4 attach at the same vertex this is the diagram in which P1 and p 3 attach at the same vertex this is not a Vertex here before and p 2. This is not a vertex. So, these are the diagrams is any other diagram missing. So, these are all order Lambda r square diagrams there is one more possible which is this and because this itself is auto Lambda r square.

So, now you see that because we have renormalized a 2 point function and this this part of this leg right this is a 2 point function this is another 2 point function but the divergences in this are canceled by divergences in this diagram so these 2 together cancel the all the similarities these 2 cancel their singularities. So, they pairwise the singularities are cancelled in this. So, as far as these diagrams are concerned they are finite.

Because of whatever we did earlier but these are one Loop diagrams that contain divergences and we have a counter term available to us with the parameter that is with a constant Z Lambda which is still not fixed and we will try to adjust that Z Lambda says that the divergence is contained in these 3 diagrams the sum of it is removed. So, our goal is um. So, here let us right these are divergent in UV and our goal is to adjust Z of Lambda such that the pole the UV poles are cancelled in the sum. So, let us evaluate one of these I mean all of these diagrams.

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$$I(p^2, m_R^2) = \int_0^1 dx \int \frac{d^d k}{(2\pi)^d} \frac{i^2}{\left[k^2 + x(1-x)p^2 - m_R^2 + i\epsilon\right]^2}$$

use

$$\int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 - \Delta + i\epsilon)^N} = \frac{i(-1)^N}{(4\pi)^{d/2}} \frac{\Gamma(N - \frac{d}{2})}{\Gamma(N)} \left(\frac{1}{\Delta - i\epsilon}\right)^{N - \frac{d}{2}}$$

$$N = 2; \quad \Delta = m_R^2 - x(1-x)p^2$$

$$I(p^2, m_R^2) = i^2 \frac{i(-1)^2}{(4\pi)^{d/2}} \cdot \frac{\Gamma(2 - \frac{d}{2})}{\Gamma(2)} \int_0^1 dx \left[\frac{1}{m_R^2 - x(1-x)p^2 - i\epsilon}\right]^{2 - \frac{d}{2}}$$

$$(4\pi)^{\frac{4-2\epsilon}{2}} = (4\pi)^2 (4\pi)^{-\epsilon}$$

$$\Gamma(2 - \tfrac{d}{2}) = \Gamma(\epsilon)$$

$$= \frac{-i}{(4\pi)^2} \cdot \left[\frac{1}{\epsilon} - \gamma_E + O(\epsilon)\right]\left[1 + \epsilon \log 4\pi + O(\epsilon)\right]$$

$$\times \int_0^1 dx \left[1 - \epsilon \log (m_R^2 - x(1-x)p^2 - i\epsilon\right.$$

So, let us evaluate this I am not going to write the external propagators because they are common in all these diagrams. So, see if I mean I could write the external propagator here I over piece p one square minus M r square + Epsilon and similarly for these 4 but that is the same here. So, you can take factor that out and whatever remains is coming from the integral in this Loop and the Z vectors and everything else in here and that is what I should arrange.

So, let us look at this one p one p 2. So, these are the momenta that enter then you have Loop momentum l and this is l + P 1 + P 2 flowing in this direction P 3 P 4 and the expression is let me first write down the propagators. So, d 4 l over 2 pi sorry not d 4 ddl over 2 pi to the d i over l Square minus M r square + Epsilon l + P 1 + P2 whole Square minus m r squared + Epsilon we have 2 propagators carrying the loop momenta and this is here then I should write down the vertices minus i Lambda r over 4 factorial there are 2 of them.

So, square then the combinatoric factor. So, let us check what that will be. So, you have these 2 vertices and I have these 4 external points. So, P 1 can connect to any of this and this right. So, that is eight possibilities then this guy should connect only on this vertex it cannot go to that vertex because that will be a different kind of a diagram. So, there are 3 and this one can go to any of the 4.

This one can again can go to any of 3 should have any of the 3 then this can go to any of the 2 this and that and that one can go to only this 1 8 into 3 into 4 into 3 into 2 into one which is a which is basically 4 factorial this is 4 factorial and the reason you have one over five 4 factorial here is precisely because you get a 4 factorial. So, that cancels times 1 over 2 factorial remember when we are at order Lambda r square we have to include 1 over 2 factorial if we were at order Lambda r to the n then we would include 1 over n factorial ok.

So, that is this factor and this is now that i square gives you minus 1 this is a half here this 4 factorial consists that is 4 factorial. So, you get minus half Lambda r square ddl over 2 pi to the d ah. So, I will define this integral to be I of P 1 + P 2 whole Square and of course it will depend on M r square as well set that is the definition you see this integral is going to depend on apart from the Epsilon or d these values right because when look at this integral this is Lorenz invariant integral.

So, the result will depend on whatever Lawrence objects you can construct out of it and P 1 and P 2 are present but they are present in the sum they appear together P 1 + P 2. So, I mean very nicely you could say it will it will depend on p one square P 2 square and 2 P 1 dot P 2 but this will not it will not depend separately on this but only as P 1 + P 2 whole square right because that is what appears here as a sum.

So, the square of this and of course M r Square n if you want Epsilon also but let me drop Epsilon it is understood. So, good let us see this diagram. So, this is minus half Lambda r square i P 1 + P 2 whole square M r Square. Now let us see this diagram P 1 P 2 P 3 P 4 let us assign Loop momentum like this it does not matter which way you assign the loop momentum l + p 1 minus P4 l enter set this vertex P 1 also enters here and P 4 exits.

So, l + P 1 minus P 4 that is what you have. So, if you write this integral the expression for this you will get exactly all the factors will be same all the propagators will be the same except for the difference coming here. So, instead of l + P 1 + P 2 you will have l + P 1 – P 4 square right this one will still remain l Square minus M r squared. So, you get exactly the same thing and the expression would be minus half Lambda r square the same integral but the argument will be this time P 1 minus P 4 square M r Square that P 1 – P 4 is coming because of this one this propagator l + P 1 – P 4.

So, the same argument which I gave earlier tells you that the result will depend on P 1 P 1 - P 4 whole Square. So, let me write it down here neatly all the 3.

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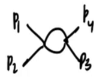
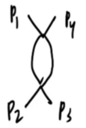
$$= \frac{-i}{(4\pi)^2} \cdot \int_0^1 dx \left[\frac{1}{\epsilon} + \log(4\pi) - \gamma_E - \log\left(m_R^2 - x(1-x)p^2 - i\epsilon\right) + O(\epsilon) \right]$$



$$= \frac{i}{32\pi^2} \lambda_R^2 \left[\frac{1}{\epsilon} + \log 4\pi - \gamma_E - \int_0^1 dx \, \log\left(m_R^2 - x(1-x)s - i\epsilon\right) \right]$$



$$= \frac{i}{32\pi^2} \lambda_R^2 \left[\frac{1}{\epsilon} + \log 4\pi - \gamma_E - \int_0^1 dx \, \log\left(m_R^2 - x(1-x)u - i\epsilon\right) \right]$$

$$\frac{i}{32\pi^2} \lambda_R^2 \left[\frac{1}{\epsilon} + \log 4\pi - \gamma_E - \int_0^1 dx \, \log\left(m_R^2 - x(1-x)t - i\epsilon\right) \right]$$

Sum of all

$$= \frac{3i}{32\pi^2} \lambda_R^2 \left[\frac{1}{\epsilon} + \log 4\pi - \gamma_E \right] - \frac{i}{32\pi^2} \lambda_R^2 \int_0^1 dx \left\{ \quad \right\}$$

$$=$$

So, you will have the third diagram as well which is which is this one and it is obvious that instead of P 1 - P 4 you will get P 1 - P 3 you can assign the momenta and check for yourself then that is what you are going to get. So, let me write it down this will be half i of. So, also I want to Define that we have already seen that P 1 + P 2 whole square is what is called s these are the Mendel stem invariance P 3 Square this is called t and P 1 - P 4 square that is called u.

So, this is a function of S and M r square and Epsilon also the order has changed now I should have drawn this diagram first because that is what corresponds to this one but anyway expression is still correct t M r Square I of u m r Square. So, we have these 3 terms and we should evaluate these. So, let me evaluate i of P Square M r Square instead of choosing St or u I will just write p Square where later I will put p Square equal to s and t and u n then add the add these things.

So, this is ddl over 2 pi d I am just writing p now introduce 5 1 parameters. So, that we can combine the combine these two integrands into one so, this I can write as integral 0 to 1 dx1 integral 0 to 1 dx 2 Delta of x 1 + x 2 minus 1 integral ddl over 2 pi d and then you have i Square i into i makes i Square times x 1 of l Square minus M r square + Epsilon + x 2 l + p whole Square + x 2 l + p whole Square minus M r square + Epsilon.

And you have to square it and the other factor coming from the gamma is one in this case because you have only 2 denominators. So, gamma n - 1 that gives you a factor of 1, now you should integrate over x 2 because there is a delta function that integral is easy and and that will just leave behind only x 1 which you can then use to write the denominator as the following.

So, check that the denominator will be this l square + 1 -s x 1 p square + 1 – x 1 2 l dot p minus M r square + Epsilon Square that is what you'll get now you can redefine x to be 1 – x 1 and then do the integral I mean change the variables and you'll get the following. So, after a little bit of algebra you will find the following now we should do what we should complete the square.

So, let us complete the square the denominator you can write as this denominator you can write as l + x p whole Square you should check that this is something you can get b square minus m r squared + Epsilon now we should shift the momentum and we are allowed to shift the momentum because we are taking d to be sufficiently small that the integrals are integrals are finite.

So, I will then use that and define k equal to l + x p r or we can write it more explicitly like this with this the denominator becomes k square + x 1 - x p Square minus m r squared + Epsilon whole Square this is what you get after the shift.

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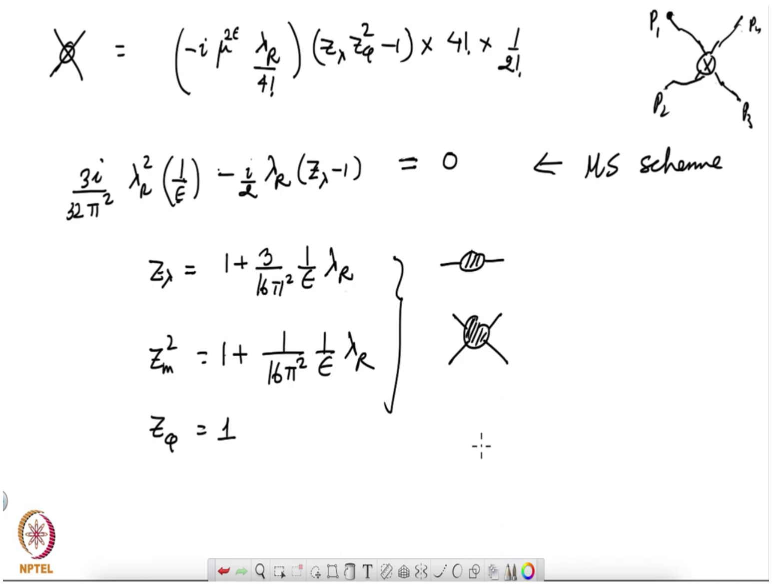

So, what is the integral i now so, i is integral 0 to 1 dx ddk over 2 pi to the d and then you have i Square over k square + x 1 minus x p Square - M r square + Epsilon now we should use the formula that I had written earlier so use integral d d k over 2 pi d 1 over K Square minus Delta + Epsilon 2 the capital N is this is the result which I had obtained earlier. So, in that I should put capital N equal to 2 and and what and Delta to be Mr Square minus x times 1 - x p Square because here Delta has a negative sign in front of it.

So, using this I get I to be this Vector Phi Square then you have I coming from here then minus 1 to the 2 which is 1 and you have 4 Pi d over 2 then you have gamma of 2 - d over 2 capital N is 2. So, this thing over gamma 2 and then you have integral 0 to 1 dx and 1 over M r Square minus x 1 - x p Square minus Epsilon this entire thing to the 2 minus d over 2. So, that is what we get and you see that you get a simple Pole from here.

So, let us extract that pole we are almost there. So, 4 pi 4 pi to the 4 minus 2 Epsilon over 2 is 4 Pi Square times 4 pi power minus Epsilon and gamma of 2 - d over 2 is gamma of Epsilon and Gamma of 2 is 1. So, with that I get i is equal to - i over 4 Pi Square times let us expand the; what has happened. So, at this I Square this minus 1 square is 1 this i square is minus 1. So, - i so, that is why you have a -i and this 4 pi Square from here and this 4 pi to the minus Epsilon I am going to take care of yeah and this in this is in the denominator.

So, this will go into the numerator it will become 4 pi to the Epsilon. So, here expanding the gamma around Epsilon equal to 0 you will get 1 over Epsilon minus solid gamma + order Epsilon term then you have coming from 4 pi to the Epsilon 1 + Epsilon log 4 pi + again or Epsilon term s 0 to 1 DX 1 minus Epsilon log of m r Square minus x 1 - x p Square minus i Epsilon + our Epsilon Square terms this Epsilon is different now.

All we have to do is multiply all these terms and look at the singular finite yeah singular and finite terms and drop the order Epsilon terms.

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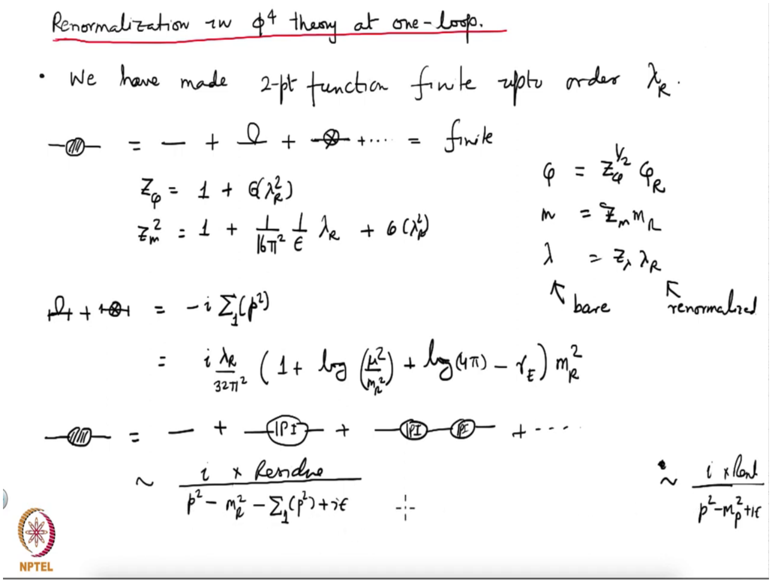


So, this is equal to minus i over 4 Pi Square to integral 0 to 1 dx you should check that this is what you get. So, you get a 1 over Epsilon how do you get when you multiply this one over Epsilon with one here and with one there. So, that produces the most singular piece then the finite terms. So, look at minus gamma e times 1 times 1 that is minus gamma e then you can take the order Epsilon term and multiply with the singular term here and with the one here.

So, that gives you log 4 pi and then you can similarly multiply this order Epsilon term with the pole term here together with this one that will give you log of this object. So, that is the result I am writing. You will always find this log 4 Pi minus Euler gamma at one Loop + order Epsilon terms good. So, now we see how the singularity looks like for that diagram. So, the contribution of this is now we have to multiply with the factor of minus half Lambda Square I of s m r Square that is what this diagram is.

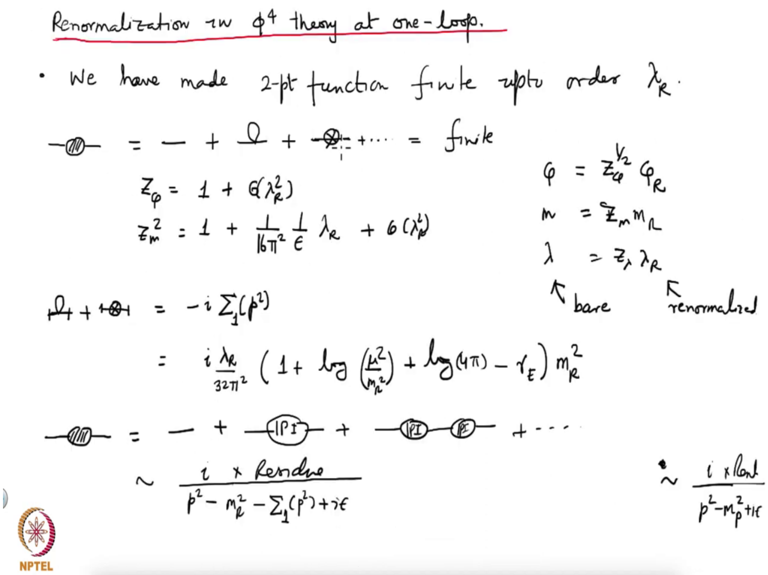
So, minus half Lambda r square times minus i over 4 Pi Square. So, I will combine it it gives you I over 32 Pi Square Lambda r square times 1 over Epsilon mine + log 4 pi minus gamma e and this term minus integral 0 to 1 dx log of m r Square - x 1 minus x as minus Epsilon this is P 1 P 2 P 2 and then you have this diagram let us go back this one involved P 1 - P 4 that is u. So, let us write this.

So, this is again the same thing let me see if I can just copy paste let us see I can do it again yes I can good. So, here I should have u and here I should have t. So, let me do that also this will be u and this will be t. What is not it right and this is let me draw the diagram differently let us draw it the same way as I drew earlier Jaws P 1 P 3 and P 2 P 4 it looks ugly but that is what it is.

So, I have to add up all these uh. So, let us look at the singular part or the sum of the 3 sum of all these is equal to see these terms are the same in all the 3. So, it becomes 3 times of that. So, 3 I over 32 Pi Square Lambda r square 1 over Epsilon + log 4 pi minus solid gamma + or minus I over 32 Pi Square Lambda r squared times this integral 0 to 1 dx and then you have all these terms that you can fix.

So, good now we; are almost there except for the fact that I have forgotten one term which is this also contributes right this I should include. So, let us write down the contribution of this actually I should not have deleted it does not undo shift come on that is it. So, let me go to the next page or maybe here itself let us see whether it works.

(Refer Slide Time: 59:07)



So, this where is that. So, for this one we have minus i I think it is a bad idea to write here let us go to the next page. So, this is equal to minus I mu to the 2 Epsilon Lambda r over 4 factorial and then you have Z Lambda Z Phi Square minus one you have a Z PHI Square because you have 4 lines coming out because it is a Vertex coming out of 4 fields that that is the vertex that we had earlier.

Let me show you here you see minus i Lambda r over 4 factorial mu to the 2 Epsilon and this space and remember this is Lambda r square vertex good. But then you have to have combinatoric factor which is 4 factorial because if you look at this vertex and you want to connect this P 1 P 2 P 3 and P 4. This one can go to any of the 4 then the second one can go to any of the 3 4 and the other 3 and any of the 2 then one. So, that is 4 factorial and then you also have one over 2 factorial because we are at order Lambda r square in the perturbation theory.

So, now to cancel the infinities I require that the sum of this singular piece let us work in M s scheme minimal subtraction I will remove only the one over Epsilon pole. So, this together with the counter term should vanish and because I am interested only in the poll this Epsilon I can put to zero. So, you can think of expanding it in powers of Epsilon and you pick up the first term which is one and you are not interested in epsilon log Nu Square term.

So, then you get 3i this is what you have on the previous page 3i over 32 Pi Square Lambda r Square 1 over Epsilon. Let us see 3i over 32 Pi Square Lambda r Square 1 over Epsilon that is what I have written here + this term which is minus I over 2 that is coming from here 4 factorials have cancelled. Then you have Lambda r and then here Z Lambda is 1 to order Lambda. So, I have only Z phi Square minus sorry Z phi is 1 to order Lambda it is 1 + 0 + Lambda r + some number times Lambda r square.

So, Z Phi I am going to put 1 and here you have Z PHI Z Lambda minus 1. and that should be finite or because I am working in minimal subtraction scheme I choose that finite to be zero that is in Ms scheme and from here I can find that um. So, that I goes away and I get Z Lambda is equal to 1 + 3 over 16 Pi Square Lambda r times 1 over Epsilon you see that with these choices of Z M Z Lambda and Z Phi I can make both the 2 point function and the 4 point function finite.

So, let me collect the result here Z M Square was one of Sixteen bar Square on our Epsilon Lambda r and this one is one + 2 sixteen pastel Lambda r perfect. So, with these choices I can make the 2 point function finite up to one Loop and also the 4 point function finite of 2 one Loop and do I need how about whether I need more parameters or more constant to fix let us say a six point function or a eight point function.

Do I need those I will leave it to you think about it. Actually we have already talked about this earlier and then I will continue the discussion in the next video.