Introduction to Quantum Field Theory - II (Theory of Scalar Fields)
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Lecture - 30
Renormalization -Part 2

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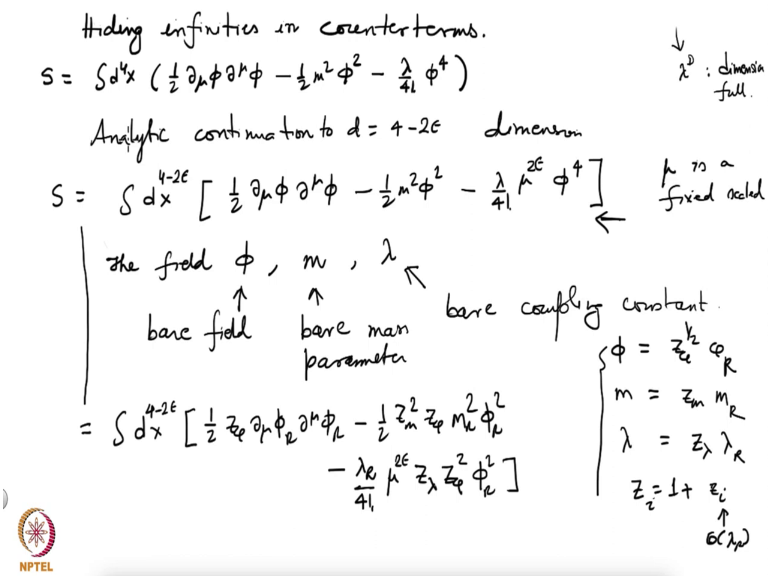

So, discussion on renormalization and today I am going to show you how to get rid of Infinities that you get in fundament diagrams. So, let us quickly summarize what we have been doing so far. So, we are interested in the 5 4 theory whose action is the following and this is in 4 dimensions. Let me reduce a bit brush palette and then as you know that because we have divergences sorry.

Because our integrals were not defined in 4 dimensions we decided to do analytic continuation 2D Dimensions which for which I mean which we are taking to be 4 - 2 Epsilon dimensions. So, the action with which we will work is this 4 - 2 Epsilon and then we have half Del mu Phi Del mu Phi - half M Square Phi Square - Lambda over 4 factorial mu to the 2 Epsilon 5 to the 4. And as you recall the reason why we have mu to the 2 Epsilon here is because the coupling constant becomes Dimension full as you go to D Dimensions.

So, in order to have the dimension dimensionless coupling I have pulled out a scale mu and average to the power to Epsilon. So, mu is a fixed scale if you decide on a mu and then you proceed otherwise you would have had a Lambda times mu Epsilon as some let us call

Lambda D some D dimensional I mean a coupling constant in D Dimensions which would be Dimension full.

So, instead of working with Lambda times mu Epsilon mu to the 2 Epsilon you could have worked directly with this but this would have been then Dimension full. So, also you should note that you do not have 3 parameters you have only 2 parameters M and Lambda mu you fix right that is not something. So, because see Lambda d is one parameter splitting the Lambda D into 2 factors does not make 2 parameters you cannot I mean if you could then you could have also split into ten other factors and said that you have ten parameters which makes no sense.

So, you really have 2 parameters M and Lambda and clearly your results should not depend on the choice of mu because you are going to vary only Lambda. Another way also when you go to Lambda going to sorry absolute am going to zero limit there is no mu because this Epsilon will drop out sorry this option goes to zero and this mu to Epsilon will drop out. So, in 4 dimensions when you go back again you see there is no mu.

But we will see that even though results should not depend on mu the physical observable should not depend on mu but when we calculate we are going to see dependence on mu we will make more remarks about that later but this is something that is going to happen and some notation now these files the fields that appear here the field Phi the parameter m and the parameter Lambda these are called bare parameters or where Fields.

So, this one is a bare field this is bare Mass parameter and this is bare coupling constant then we did the following we wrote the same action as. So, I am putting equal to. So, it is the same as it is not something different as 4 - 2 Epsilon d to the 4 - I forgot x here then we redefined Fields we said that the bare field is Z phi to the half and then fiery normalized. And similarly bare coupling or sorry bare Mass is Z M times M renormalized.

And Lambda the bare coupling constant is Z Lambda times Lambda renormalized where the Z started order one this is small z lowercase z and these z i's start at order Lambda r these fields Phi m r and Lambda r we will keep them finite that is something we demand and you are going to see that the fields Phi M and Lambda will become infinite. All that Infinity will be contained in the Z phi Z m’s and Z Lambda.

But let us anyway progress and see explicitly that this is what happens. So, with these definitions we wrote the same action not changing the theory it is the same theory. We wrote as half Z Phi Del mu Phi r del Mi Phi r - half Z M Square Z Phi M r Square Phi r square all I am doing is substituting these definitions in this section here and that is what gives you this thing and then - Lambda r over 4 factorial mu to the 2 Epsilon Z Lambda Z Phi Square Phi r square.

And then I added and subtracted the terms which you get in the action here but Phi and M and Lambda replaced by Phi r M r and Lambda because this is these are the ones which were originally there right. So, I just want to have Feynman rules which are which the Feynman rules contain the set of Feynman which I already had that is what I want to ensure. So, I just add and subtract this term with the fields and parameters replaced by renormalized fields and renormalized parameters.

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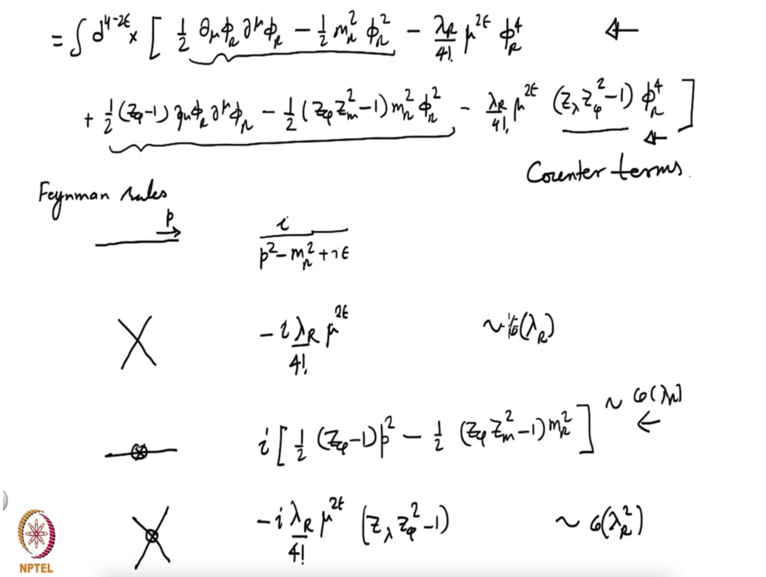

So, the same action I rewrote as d 4 - 2 Epsilon x and half Del mu Phi r del mu Phi r - half M r Square remember M r has the mass dimension 1. So, that is why there is no need of any any additional mass scale and then we had Lambda r over 4 factorial mu to the 2 Epsilon Phi r to the 4. So, I added this and then I subtracted and I get this + let me start from here + half Z phi - 1 Del mu Phi r del mu Phi r - half Z Phi Z M Square - 1 M r Square Phi r square - Lambda r over 4 factorial mu to the 2 Epsilon Z Lambda Z Phi Square - 1 phi r to the 4.

Good is the again the same action as before but written slightly differently. These the first line that is what we had in the original action with the replacement of the fields and parameters the second line is new and these are called counter terms. So, all the terms in here are called counter terms. Remember I have not put any additional terms as you as you know ok but still I am calling these as counter terms and this has the original thing because it looks like what you had originally even though the fields have been rescaled and mass parameter and coupling constant has been rescaled.

So, that is just a way of saying that what we have here is counter term and you will see what it is going to counter okay. So, and then we had the Feynman rules momentum space Feynman rules. So, the propagator is i over p Square - M r Square + i Epsilon right because this is what is going to provide you with the propagator and that is the M r here. So, that is M r square + i Epsilon then as earlier you have this vertex which gives you a factor of - i Lambda over 4 factorial okay.

And then you have a 2 point vertex and remember I called this is a vertex because Z - 1 is of order Lambda r. So, this term comes with the order Lambda r and this has 2 Fields Phi. So, it will have 2 external 2 2 legs. So, this vertex just like this vertex has 4 legs because you have 4 Fields here. If you go back and see how we derive fundament rules why this has 4 legs you will see that for the same reason this has 2 legs just a second and this you should see that in the in the Fourier space this will give you the following the moment of space Feynman rule is the following.

And we also had the vertex coming from this one this has 4 legs and this also starts at order Lambda r right because Z Lambda is of order one sorry Z Lambda is 1 + order Lambda Z phi square is one + order Lambda. So, when you subtract one it will give you a term which is of order Lambda the same is true for this one and this one all these terms. So, when I wrote this, this one I have included both of these here.

And now I am writing this one again to remind that this is coming from a counter term I have put this circle here and the Feynman rule is the following just like you had - I Lambda of 4 factorial here you have the same thing here I have missed mu to the 2 Epsilon mu to the 2 Epsilon Z Lambda is at Phi squared - 1. So, since this factor is of order Lambda r you already have an explicit order explicit factor of Lambda r this vertex or this term is of order Lambda r square and that is why this vertex is of order Lambda r Square.

So, it is one order higher compared to this vertex and this vertex these are all of order Lambda r and remember when we do Perturbation Theory we have to do things order by order in the parameter Lambda r right because Lambda r is our expansion parameter if we are organizing the perturbation Theory using the parameter Lambda r. So, a term which is of order Lambda r square is a higher order term compared to an order Lambda r term.

So, if you want to get a result which is correct only up to order Lambda r you do not have to worry about terms that appear at order Lambda r square. So, as of now I have not really done anything I have just renamed things beyond that I have not done any anything. Now let us go and try to calculate some physical thing in the theory and in the same theory there is nothing it is not a new theory it is the same theory.

And let us try to figure out the physical mass that is one thing one would like to know what are the physical masses of the particles and that is something I want to calculate. So, what you should notice that I will choose some value of M r sum value of Lambda r some value of mu okay these will be fixed. So, they are inputs or choices and then I ask if I have made these choices then what is the physical mass that I get and of course that physical mass will depend on these choices right M r and Lambda r. Let us see then how to get it and what we get.

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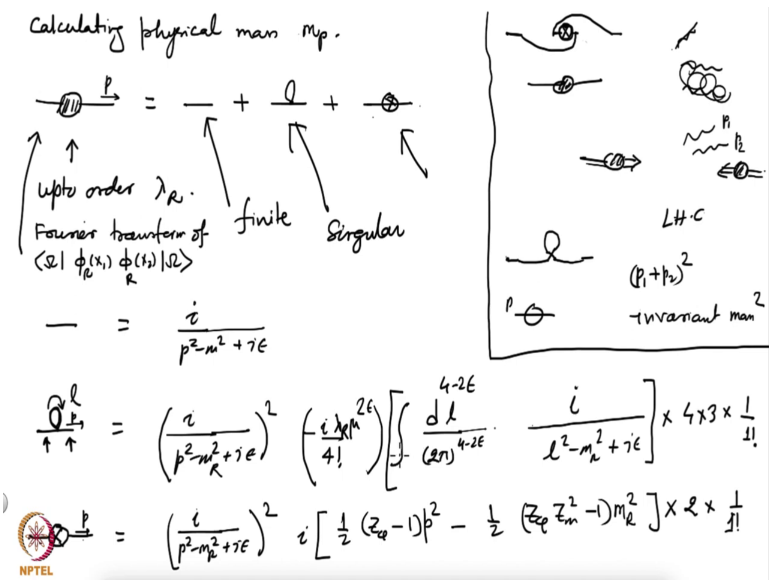

So, physical mass is something that you can observe right in an experiment like you can find out what is the mass of an electron or what is the mass of a h of any other particle recently I mean one of the fundamental particles which was discovered most recently is the Higgs Boson. So, what is the mass of the Higgs Boson that you find from experiment if you are aware of the story of it.

People were searching for a peak in the diboson di Photon Channel because you it is one of the easiest ways of finding it. So, you look at 2 photons in the final State the photons are usually denoted by curvy lines and you have 2 protons colliding. So, this is I am denoting proton they collide at LHC and then of course many processes happen. And you let us say are looking at a process in which you get 2 photons and you construct the invariant mass of these photons.

So, if you have this one has momentum p 1 and this one has p 2 then p 1 + p 2 whole square that is what is called as invariant mass and people found a peak in that invariant mass at some energy. So, from there you find out what is the Higgs Mass because this will give you the this invariant mass square. This has dimensions of energy squared or mass squared. So, this is invariant mass squared and from this you find out what is the mass of the Higgs Boson that is how it was done.

Now that mass of the Higgs Boson is something physical irrespective of what theory you have what M r and Lambda r etcetera you are choosing it does not care about those things it is an experimental outcome. So, you find that now once you have found m p okay or some number of observables you have observed in the experiments. Then you can using those values for the physical mass or whatever else you have calculated you can try to fit what m r and Lambda r would be.

So, that is how you can determine the values of m r and Lambda r and then if you have a new observable for which you want a prediction then to calculate that new observable you can feed in the values of m r and Lambda r and that is how this works. So, first let us calculate physical Mass MP assuming that some observations have given us what the values of m r and Lambda r should be.

So, some observables have been observed in the experiment they have been measured once these observables have been measured one feeds in some values of m r and Lambda are and try to see which values of m r and Lambda r reproduce those observed results and then you use those m r and Lambda r to calculate M p or any other observable. So, that is how you should think about it.

So, let us calculate M p and you know how to get this. So, you look at the 2 point function and look at the pole of it pole of the propagator. So, this is this plus this plus this why you what we are doing here is we are writing down this 2 point function up to order Lambda r that is what we are doing. So, I should have ordered Lambda R to the 0 term an order Lambda R term but no order Lambda R square term they are not needed.

So, this is the order Lambda R to the zero term there is no Lambda R here. So, there is the first term plus you have the self energy diagram one Loop self energy diagram it has a factor of Lambda R here. So, it contributes and this one you have seen this vertex itself is order Lambda. So, this also contributes at order Lambda r now this is finite there is just a propagated I over p Square - M r square + i Epsilon.

So, this is singular because of the loop diagram because it is a loop diagram it has ultraviolet divergent and this has this expression as we wrote before this thing times these 2 propagators okay so yeah good. So, remember now whenever you are writing any Greens function you have to always include the counter terms also that can appear up to that particular order up to which you are interested in.

So, this you cannot say that this is the diagram this is also a diagram that contributes and you have to keep it this cannot be omitted because your action now has such terms. So, they all have to be included. So, let us calculate this just to be a little bit clearer this object is now a 4ier transform of this is how we had looked at this 2 point function in the 4ier space in the final in the in the momentum space.

So, this is what I am looking at where the fields are having this subscript r these are correlators of renormalized fields. So, let us look at the first term the first term in the expansion is simply I over p Square - M square + Epsilon this one has 2 factors of I over p

Square - M square + I Epsilon right one coming from this propagator another coming from this propagator. So, you have a square of it times this integral.

So, let us assign a loop momentum l and this integral is. So, let me first write down the vertex it is - i Lambda over 4 factorial mu to the 2 Epsilon that is our vertex then I should write down the integral the fundament integral which is d power 4 - 2 Epsilon d l over 2 pi 4 - 2 Epsilon into i over l square - M r square + Epsilon this is because of this propagator see there is one propagator here right this which carries momentum l. So, that is what I am writing i over l Square - M square + Epsilon.

Now I should also multiply the combinatoric factor. So, that is easy to find you have here this is this point I am drawing this is this side and then you have a 4 point vertex. So, this one can go to any of the 4 then this one can go to any of the 3 I should have there is nothing wrong in what I have done but it is not very convenient. So, it can go to any of the 4 it can go to any of the 3 and this can only connect this.

So, it is 4 times 3 and then you have 1 over n factorial where that n depends on the order of Lambda right and because that is 1. So, it is 1 over 1 factorial for we were at 2 Loops or a order Lambda r square it would have been one over 2 factorial. So, this is the expression and also let me now write down this one just a second now the counter term or counter vertex or counter diagram better counter diagram.

So, this is what again this has 2 propagators which each of which carries a momentum p. So, you get i over p Square - M r square + i Epsilon squared and then you have I am writing this vertex here this one I times half Z Phi - 1 p Square where p Square is the square of this momentum - half Z Phi Z M Square - 1 M r Square now I should find out the combinatoric factor and what is that.

So, let us do it here you have again this and then you have a 2 point vertex these sticks coming out are from the vertex these are not propagators and I have to connect these propagators. So, this one can go to any of these 2 this or that let us say it goes here one then this has only one option. So, which is same as this diagram right I am just drawing it differently it is the same diagram.

So, first one had 2 options the second one had only one option. So, you total 2 and again one over one factorial. So, this is the contribution of this diagram. So, so here Lambda r let us check if everything is fine all looks good. So, far just one remark here look at this integral this contribution the in the integral that you have here this one it is a bit special because it does not contain the external momentum p.

So, you do not see p anywhere here but this is not a general feature typically Loop moment Loop integers will carry the external momenta it is just because this loop is such that there is no external momenta that enters into it that is why this is completely dependent of p or p Square but this is not generic this is something very special here. So, if you had a diagram of this kind then p will also enter into the loops and then the integral will also depend on p ok.

So, I should now do what I should calculate this integral and substitute it in here to see what I get. So, let me first do that.

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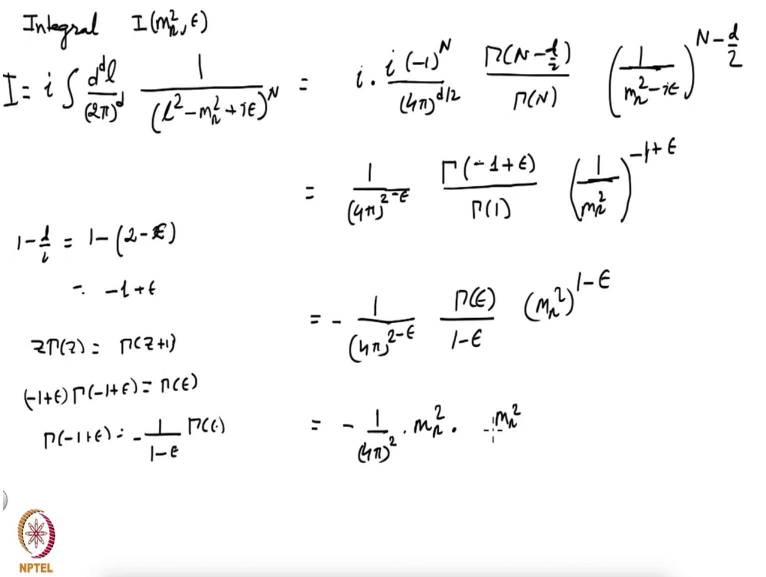

Integral let us call it i. So, let us define this integral to v i and it depends on what it depends on M r square and also on Epsilon let me also m r Square Epsilon this one let us be careful and write it like this. So, let us find out the loop integral we have already done this let me just write the result here integral d d l over 2 pi to the d 1 over l Square - M r square + i Epsilon to the n is i times - 1 to the n over 4 Pi d over 2 gamma of n - d over 2 over gamma n and then you have 1 over m r Square - Epsilon power n - d over 2.

So, let us put n equal to 1 yeah before I do that the integral is not really this there is a factor of I also here because that is what comes from the propagator. So, let me include that I I will not write it here I will write it outside. So, this is I capital I is this and then you have a factor of I also here. So, I Square gives you - 1 capital N is 1. So, there is a - 1 here that cancels the sign which you get from i Square.

So, you have 1 over 4 Pi power 2 - Epsilon d over 2 is 2 - Epsilon then you have gamma of n is 1 - d over 2. So, it is 1 - d over 2 1 - D is 4 - 2 Epsilon. So, 2 - 2 Epsilon 2 - Epsilon this is - 1 + Epsilon over gamma of 1. and this one is 1 over m r Square now you see M r square is positive. So, this is a fractional power of a positive number. So, there is no issue I there is no issue of knowing.

There is no issue of a branch cut appearing here and I have and that I should know on which side of the cut I am. So, I can just drop this if it was negative then I have to worry. So, I will just drop that i Epsilon and this will be again n - dual 2 is - 1 + Epsilon also another fact. So, you have gamma of n - d over 2 right. So, because d has an Epsilon and let us say we are not taking Epsilon to be integer because we wanted to take any arbitrary value.

So, unless d unless d is also integer such that n - d over 2 is a negative integer the gamma is not Divergent. So, gamma is analytic for all values of its arguments except for zero and negative integers. So, even though you are having a dimension d which is let us say which will lively give you a singularity because the by the power counting it appears to have more powers in the numerator than the denominator.

Even then it would not give a divergence gamma function will not be singular unless the power that you have the total power that you have d - n is integer. So, these are analytic functions for all non integral values okay good. So, we have this here and this I can write as I think I had already anyway nevertheless I should do it. So, we'll use gamma's Z gamma Z is equal to gamma Z + 1.

So, - 1 + Epsilon times gamma of - 1 + Epsilon is gamma of Epsilon. So, gamma of - 1 + Epsilon is 1 over or - 1 - Epsilon gamma of Epsilon. So, I will write this as 1 over 4 pi 2 - Epsilon is correct and then you have - sign one over gamma sorry 1 - Epsilon gamma of Epsilon and this is m r Square Epsilon. So, I will massage it a little bit more good hmm. So, this is - 1 over 4 pi Square Times m r Square times Mr Square this is in the denominator sorry times 4 pi over m r Square power Epsilon.

And then you have gamma of Epsilon over 1 - Epsilon that is the integral that is the value of the integral. So, what is the value of that full filament diagram.

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$$= \left(\frac{i}{p^2-m_R^2+i\epsilon}\right)^2 \frac{i\lambda_R m_R^2}{2\cdot(4\pi)^2}\underbrace{\left(\frac{4\pi\mu^2}{m_R^2}\right)^\epsilon \frac{\Gamma(\epsilon)}{1-\epsilon}}$$

$$= \left[1+\epsilon\log\left(\frac{4\pi\mu^2}{m_R^2}\right)+O(\epsilon^2)\right]\left(1+\epsilon+O(\epsilon^2)\right)\left(\frac{1}{\epsilon}-\gamma_E+O(\epsilon)\right)$$

$$= \left(\frac{1}{\epsilon}-\gamma_E+1+\log\left(\frac{4\pi\mu^2}{m_R^2}\right)+O(\epsilon)\right)$$

$$= \left(\frac{1}{\epsilon}+\left(\log(4\pi)-\gamma_E\right)+1+\log\left(\frac{\mu^2}{m_R^2}\right)+O(\epsilon)\right)$$

It is this and then yeah. So, there is a factor of - I here there is a factor of - 1 here and it will combine with this factor of - 1 here to give a + and then I have Lambda over 4 factorial then you have mu Square power Epsilon mu to the 2 Epsilon which is Mu Square power Epsilon that I will combine with where is that with this Factor. So, it will become 4 pi mu r square mu Square over m r Square power Epsilon that is what it will give when you combine and then these factors.

So, let me write down that I should have a factor of i. So, let me write and then check. So, I will get i Lambda r m r Square there is a factor of half and then you have 4 Pi Square and this factor of half let me show you. So, 4 times 3 and you have divided by 4 factors. So, that gives a factor of half in the denominator effect of 2 in the denominator. So, that is the factor of half here 1 over 2.

Then yeah then I have taken care of Lambda r and this vector of i Lambda r n this Vector of i and then I have pulled out m r Square from the integral ok and this 4 Pi Square also I have already taken now I should write these vectors including the mu Square to the Epsilon. So, I have 4 pi mu Square over m r Square power Epsilon then you have gamma of Epsilon over 1 - Epsilon and Mr Square I have already taken here.

So, let me check I over 2 Lambda r m r Square 4 Pi Square 4 Pi mu Square over m r Square per Epsilon gamma Epsilon over one. So, that is correct that is the correct result. Now let us look at the poles explicitly this piece here is a 1 + Epsilon times log of 4 pi mu Square over m r Square + order Epsilon Square term just a second sorry again should work. So, this is this Factor then you know what gamma of 1 over 1 - Epsilon is this is 1 + Epsilon + or Epsilon Square.

When I write order Epsilon Square it means all the terms including Epsilon square and all the terms after Epsilon Square also because they are successively of lower lower orders then you have gamma of Epsilon which is 1 over Epsilon - Euler gamma that is that was the number I told you earlier + order Epsilon terms what does that give. So, in this product the most singular term will be produced when you multiply 1 over Epsilon to the one here and to the one there when you pick out that term 1 times 1 times 1 over Epsilon you get the most singular piece Epsilon to the -1.

Now let us look at Epsilon to the zero term. So, there are several ways in which you can generate order Epsilon to the zero term. So, let us take - gamma e here which is odd Epsilon to the zero multiply it with 1 here and that one there. So, that gives you - gamma I also if you yeah. So, that is one source if you multiply this one over Epsilon with this Epsilon the pole term multiplying the order Epsilon term gives a finite value finite term right.

So, this also that is why one has to be very careful in getting the finite results see getting singular part is fine you can just drop all the order Epsilon terms there is no problem. But if you want to have the finite Parts also correct you have to expand sufficiently in the orders of Epsilon depending on up to what Loop you are working. So, at one Loop this will be sufficient so, one by Epsilon times Epsilon gives you one.

So, that is + 1 then also when you multiply this one over Epsilon term with this Epsilon term here. So, 1 over Epsilon one from here and Epsilon term here that also gives you a finite piece and which is log of 4 pi mu Square over m r Square this is fine and did I miss something on our Epsilon other gamma square + 1 this is perfectly fine. So, I will just bring it a little closer this is what you get + order Epsilon terms.

And do not worry about order Epsilon because now when I take Epsilon going to 0 this will drop out and I have kept correctly all the finite terms I will rewrite it a little bit differently I will write it as one over Epsilon + log of 4 pi. So, that is log of 4 Pi coming from here - Euler gamma that is here this term + 1 + log of mu Square over m r Square + order Epsilon. So, that is the result for this now when I add this I think it will be but if I write it again here.

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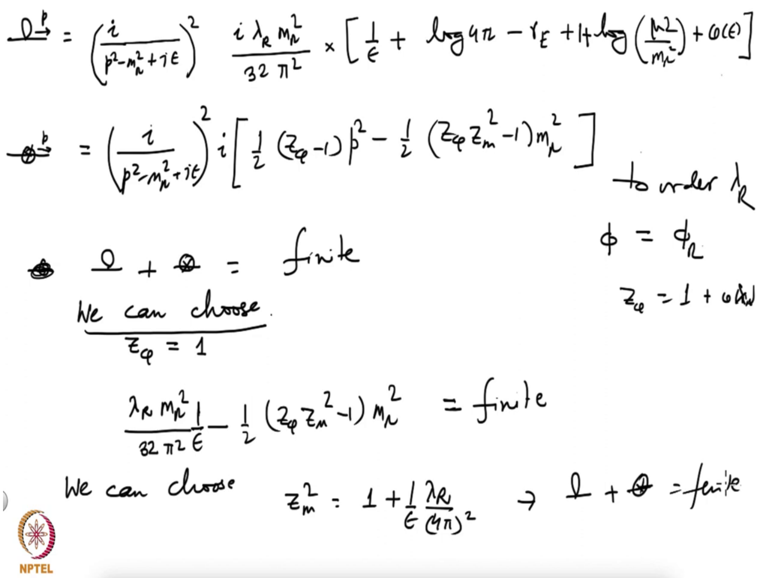

So, this piece what is that yeah I Lambda r or this vector let me write it. So, let me change this. So, 4 Pi 4 Pi square is 16 Pi square and there is a factor of 2 here that makes it 32. So, 32 Pi Square good so, that factor is there and then we have all these poles one over times 1 over Epsilon + log 4 pi - Euler gamma + log of mu Square over m r Square + order Epsilon terms. Let us check did I miss no I did not yes I did miss 1 + 1 and then I should add to this the counter term and of course the pure propagated term also.

And this was this one the counter term provides you this what are the factors I will write it here half Z phi - 1 p Square - half Z Phi ZM Square - 1 times m r Square. So, now I have the sum of these 2 is equal to the sum of these 2 expressions on the top now I want the following I want the sum to be finite because if the sum is finite then the mass which I am going to calculate at one Loop up to order Lambda r square the physical mass that will be finite I will so, first let me make the sum finite here that is what is called renormalization.

So, I am going to subtract Infinities now. So, let us look at this term this is what you had gotten if you had not split the lagrangian the way we have and just continued with the parameters Lambda and M if you had continued with the bare parameters Lambda and M and where field Phi this is what you would have obtained the only difference would be that instead of Lambda r and M r you would have Lambda and M.

But even after that you have used Lambda r and m r nothing has changed because it is still the same integral instead of Lambda you have Lambda r instead of M you have m r but the advantage is now that you have also this term but in this term the Z phi's and Z M's are still at our disposal there is nothing that fixes what Z phi should be and Z M should be in the steps that we have done there is nothing telling you what is the value of Z Phi and what is the value of Z M.

So, what we can do is we can choose the values of Z Phi and Z M such that they cancel the singularities that are present in this expression in the in the loop diagram. So, what I should do is make to make this finite I make the following choice first let us look at if there is any singularity in this Loop diagram which is proportional to p Square because there is a term proportional to p Square in the counter term which can take away any Singularity which is present here that is proportional to p Square.

But there is none there is no term that is proportional to p Square. So, there is no such Singularity that needs to be cancelled see this p Square term cannot interfere with this term because you keep Mr fixed it is a value you have chosen. So, you can fix the value of m r and then p can be varied right you can you can look at the 2 point function for different values of P. So, changing that p will change the value but this one will not change.

So, any Singularity that is proportional to p Square here cannot be utilized to cancel something which is independent of p Square here. So, we can make the following choice we can choose Z Phi to be equal to 1. So, if Z phi is equal to 1 this goes away and that is fine I do not need any such term to cancel any poles here. So, that is a choice I can make and it is good Z phi does not. So, to this order the field Phi is equal to Phi r right because we had all these Z's always start with one.

So, 2 order Lambda r Phi is same as Phi r but this will change if you go to second order when you have different other diagrams at 2 loops this statement will change but up to order Lambda r Phi has remained Phi r or Phi r is Ms Phi the normalization has not changed. Now let us look at this term this is proportional to m r Square you have an m r Square here. So, I can kill this Singularity.

So, for that I can choose we can choose Z phi to be one and Lambda r is a issue of vector of I how come yeah because I missed the factor of I here this comes with the this factor vertex as a factor of I here this one that is what I had missed. So, Lambda r m r Square over 32 Pi Square this is this factor I have see this is common in both the I factor is common in both and I am not now looking at this part m r Square - because you are adding these 2 up minus.

So, this one gives you half - half Z Phi Z M Square - 1 M r squared. So, when you add these 2 up you get one term which is this and I want this to be finite. I am not specifying what I what exactly finite number I want to be but I want it to be something finite no there is a pole that I have missed this one over Epsilon I have missed. So, now if you choose so, this is finite. So, now we can choose Z M Square to be 1 + Lambda r 4 Pi Square.

If I make that choice let us see what happens see Z Phi is already equal to 1 + something F order Lambda r. So, Z phi is because we are doing everything correct to order Lambda r only I have Z equal to 1 so, that is one and if you compare then the sum of these 2 should be zero. So, Z M Square should be equal to 1 + 1 over Epsilon Lambda r over 4 Pi r square or 4 Pi Square M Square cancels and this factor of half divided from 32 gives you 16 which is 4 Pi Square 16 times Pi square is 4 Pi Square.

So, that is what you get and if you do this then this makes the sum finite. So, so here is the summary let me write it again.

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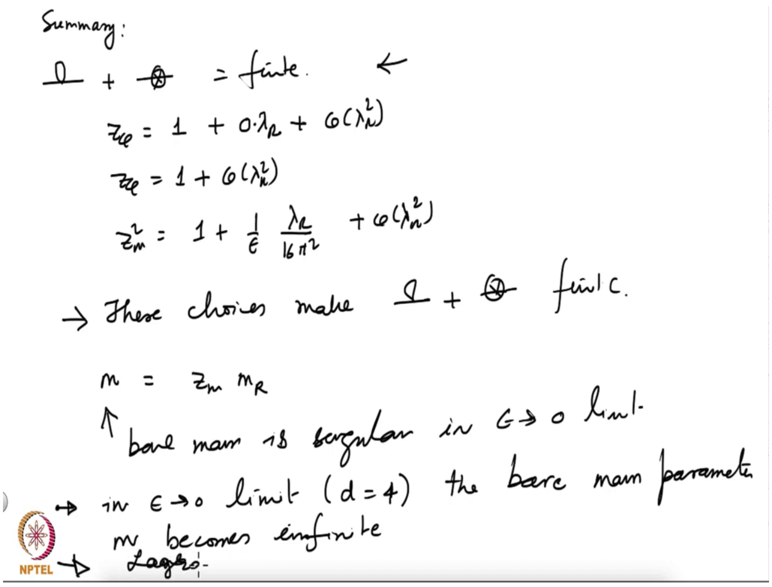

So, summary of what we have done is the following we took this one Loop diagram + took the counter term and we made this finite and to make this finite we have seen that we can take Z5 to be one + something at order Lambda r square. So, the term which is proportional to Lambda r turned out to be zero so, zero time Lambda r + order Lambda r square. So, it is really one that is Z Phi is equal to 1.

It is a it is a very special thing that has happened that the Lambda r term is vanishing. It will not be true in general but in some in some cases certain terms might just vanish for whatever reasons but they can vanish it is not necessary that each term has to be non-zero. So, Z phi is one + order Lambda r square and Z m Square I have found that it should be 1 + 1 over Epsilon Lambda r over 16 Pi Square + order Lambda r square.

So, if I make these choices make this + this finite now you see you can take Epsilon going to Infinity limit because when you take Epsilon going to Infinity limit these Z's they become infinite you see this Z phi did not become does not become at one Loop because this is zero but in principle if you were to calculate 2 order Lambda r square and other terms they will diverge but and also Z M has diverged.

So, what has happened is that the bare Mass see bare mass m which was written in terms of the renormalized masses is Z M times m r this has become singular the bare Mass is singular or infinite in epsilon going to zero limit ok. Because Z M becomes infinite but Mr is sorry Z M becomes infinite but Mr is something finite we have fixed M r we have said Mr take some finite value whatever value you wish to choose the cost of making Mr finite is that the bare parameter becomes infinite because Z M is finite.

But I am fine with that because if I look at this sum which is what really enters into the calculation of physical Mass which is something observable that is finite. So, I am going to get finite results for physical observables but the bare parameters and bare fields are going to become infinite. So, your action or lagrangian density is going to become infinite but you are still going to get finite results.

So, this is what is the renormalization program we make the bare quantities which are unphysical non observable objects we make them infinite and we still get predictive power because the observables become finite because they are expressed not in terms of where parameters and where fields and not in terms of bare parameters M r and Lambda r. But they are expressed in terms of finite parameters Lambda r and M r but remember these are also not physical observables Lambda r and m r

physical observable is something like physical mass m p that can be expressed in Lambda r and m r let me show you exactly what you will get if you calculate the physical Mass but before that let me make this marks. So, n Epsilon I am going to zero limit that is D equal to 4 4 dimensions your physical world the bare coupling constant in this case you are just seeing Z M.

So, let me make statement only about that the bare Mass parameter m becomes infinite which means that your lagrangian density also becomes infinite.

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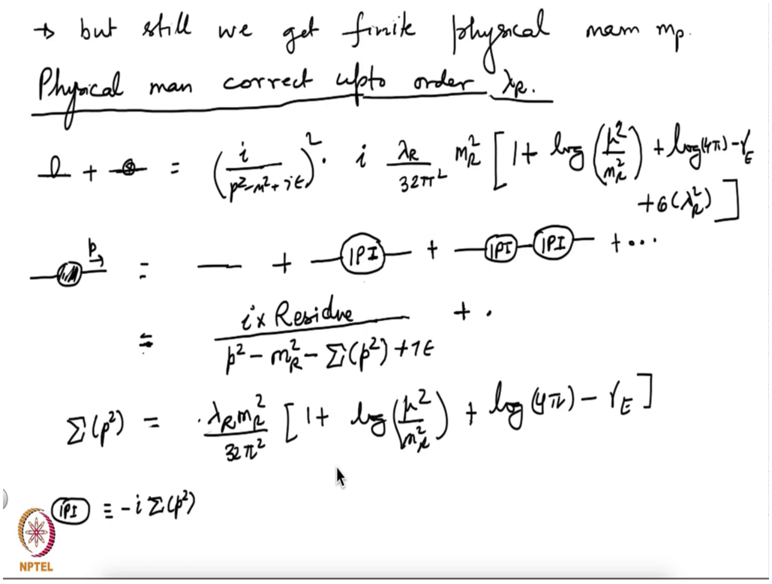

But still we get finite physical Mass okay. So, this is nice let us see how to get the physical Mass it is almost there. So, we want to get physical Mass correct up to order Lambda r remember we are doing perturbation theory. So, you will be able to calculate observables up to some fixed order in the coupling constant which is your expansion parameter. So, what do you get you get the following this loop diagram plus the counter term.

After cancelling everything is i over p Square - M square + i Epsilon Square this is coming from the 2 external legs times the remaining part which is I Lambda r over 32 Pi Square 2 Pi r square Pi Square then M r Square times 1 + log of mu Square over M r squared + log of 4 pi - Euler gamma + order Lambda r square terms. So, I will drop them or maybe just write it like this that is your finite result for these object this sum.

So, how do you get the physical Mass we have seen earlier that we should look at the pole of the 2 point function and this has the form I times the residue divided by I should have first drawn the diagrams. So, this is this propagator the lowest order contribution plus all one PR diagrams one particularly reducible you remember this we have discussed about this earlier plus it has stopped working.

Now I am finding it difficult to write. So, this and of course all other such diagrams and here this is equal to how should I right where this one particle irreducible this does not contain the legs. Let me put this vertical arrows to say that you do not have to include these propagators these are these are amputated ones. So, for amputated object I will write this or or simply even I can remove that we had defined this earlier as - i Sigma p Square.

It will be a function of the scalar p Square and with this it becomes i times the residue over p Square - M r Square - Sigma of p Square it also depends on M r square and of course Lambda + Epsilon and of course other terms which are as the structure which are not singular. So, that is the 2 point function and what you have here is really this object right one Pi. So, this sum is one Pi up to order Lambda r.

So, this 1 p i up to order Lambda r is i times C if I remove the propagators meaning I remove the these 2 this Square then whatever is left is i Sigma. So, this is how should I write i Sigma so, because there is a factor of I already. So, whatever is left here is Sigma. So, Sigma is Sigma p square up to order Lambda square is one no Lambda r m r Square over 32 Pi Square times 1 + log of mu Square over M r Square + log of 4 pi - already gamma this is correct up to order Lambda.

So, you see what the physical mass is physical mass is M P is equal to or M P square is equal to M r Square see this is the pole right this is the physical mass M r maybe I will stop here it is becoming difficult to write. So, you can calculate the physical Mass here and we have the finite result and you see that the result depends on mu as I was saying earlier that the result will depend on mu even though you know that it should not but that is an artefact of doing a perturbation Theory because you have stopped at some order.

There is a mu dependence and then you also get these constants. So, these are also part of your result these are all auto Lambda r terms but here maybe I will tell more in the next video but I will just say in words instead of writing here yes, here. When we were subtracting Infinities we were choosing what the value of Z M should be we chose to subtract only the pole terms we chose Z M such that only one over Epsilon gets removed.

Because that did the job of making the some finite but to get a finite result you have freedom to also subtract some finite pieces right. So, in addition to this one over Epsilon if you were to remove the log 4 pi and this gamma - gamma Euler gamma also you still get a finite result and you have that freedom right. No one can say can tell you that you are not allowed to subtract this you are allowed to because that is that supply that additional subtraction is also going to give you a finite result.

This is what is called a renormalization scheme how much finite pieces you subtract is a particular choice of a scheme. I will write in these things in next video because my pen is not writing now and we will talk about a little more about these things in the in the next video.