

Introduction to Quantum Field Theory (Theory of Scalar Fields)
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Lecture No # 03
Module No # 01
Scattering Matrix

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$$\begin{aligned} \text{Iw: } & |\vec{k}\rangle_{in}, |\vec{p}\rangle_{out} \\ & |\vec{k}\rangle_{in} = |\vec{k}\rangle_{out} \quad \leftarrow \text{For the case of} \\ & |\Omega\rangle_{in} = |\Omega\rangle_{out} \quad \text{single particle states} \end{aligned}$$

Let us start our discussion on quantum field theory we had been discussing about in and out states and we were interested in scattering in and interacting field theory. So, we will continue our discussion from there.

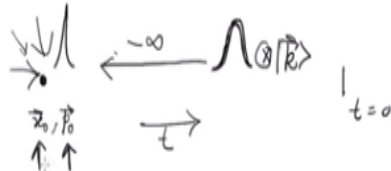
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Quantum Field Theory

- Existence of single particle states.
- 'in' & 'out' states

eg. $\rightarrow |\vec{k}\rangle_{in} = |\vec{k}\rangle_{out} = |\vec{k}\rangle$

$|\vec{k}_1, \vec{k}_2\rangle_{in} \leftarrow$



Let us do a very quick recap we had assumed that in the field theory that we are interested in those theories there exist single particle States. So, we had assumed the existence of single particle States and that's a good assumption to make because the world in which we live we know that there are particles and you can create a state where you have only one particle. For example you can have 1 electron or 1 positron or 1 Photon so this is a very reasonable assumption.

And then as I mentioned we were introducing in and out States these are the basis states in which we are going to use to describe scattering so we had introduced in and out states. So these are states that are defined at $t = 0$ if you are looking at the Schrodinger picture. And of course we are going to be working with Heisenberg picture and a state at $t = 0$ you can take as the Hilbert State and of course then the states do not evolve with time only the operators do.

Anyhow so you can for example have a state which is labeled by only 1 momentum k and I told you that if you have an n state which has only 1 particle there are nothing else that can happen. So that particle will just keep moving in the direction it was moving with the same momentum and it is going to keep doing. So the end state is same as the out state and there is no real meaning of putting these labels in and out in that case and we can just label them as k .

But of course if you have a state which in the far past looks like a state which has 2 non-interacting particles because they are very far apart from each other. And that state is not

necessarily going to evolve to a single 2 particle state it may evolve to a state containing 10 particles. So a state which you can label as $k_1 k_2 \dots k_n$ and if you fold it properly you can create a 2 particular state in the far past. But that doesn't necessarily evolve to a 2 particular state in the final state so I cannot drop this label in as I could in the in the case of single particle States.

My plan is that in this lecture I am not going to write a lot of equations I mean there will be but not too many and I do not want to do a lot of algebra. Because I would like you to have a picture of what we are going to do so that once I start being more accurate with the description of states and everything you already have a mental picture of what you're doing and you're not drowning in the equations.

So let us start by first having a single particle State so I am just creating cartoons now so I want to let us say have a single particle located here in the far past. Meaning I want to create a state which will be well localized in space at some position x_0 . So I want the particle to be here and I want it to have a fairly well defined momentum let me call it p_0 . So of course because we are doing quantum mechanics we cannot have the momentum exactly to be p_0 and position exactly to be x_0 that; is not possible because of Heisenberg uncertainty principle.

But you could choose it to make as sharp as you want in p_0 and then correspondingly it will be your uncertainty principle will tell you how sharp you can make it in x_0 but you know you can fairly localize the particle both in space and momentum. Now after all when you are for example when you are preparing a an electron that is going to run in some collider like LEP there was a machine called LEP where now LHC is host so there you used to have electron positron collisions.

So you know with what momentum you are firing the electron and from which place it is coming out so at that point where when it is coming out of the electron gun let me call it an electron gun its momentum and position are fairly well defined so you can create those states. Now this state is well localized in space and momentum and far past and as time progresses so along the horizontal line you should imagine time so time flows in this direction as time progresses.

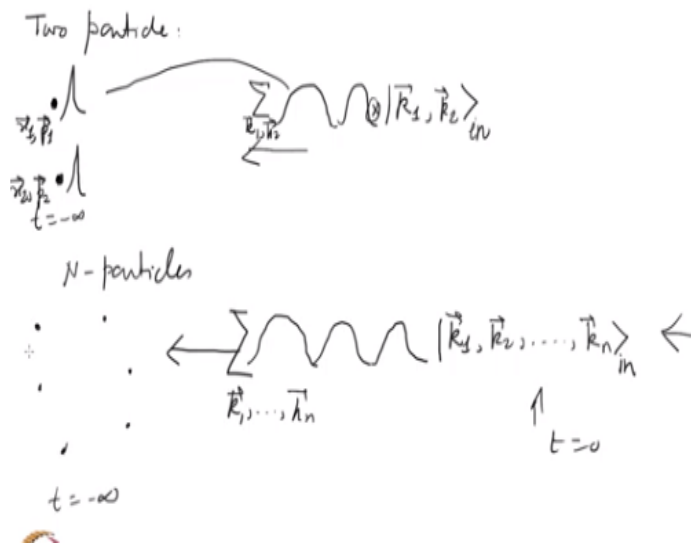
This will start spreading in the Wave packet which you have constructed it will start spreading. So you now look at $t = 0$ so here is time $t = 0$ which you we have chosen arbitrarily and I choose a basis State a basis momentum state which is an in state are in out state. Because for single particle it does not matter see this is a single particle state right there is only 1 particle so that state I can construct out only of single particle in states not out of a multi-particle in states.

So there is no role of this one for example in constructing this particle so I can only take this one so I take k so that is the basis state. And of course I should multiply it with some function and what should that function be let us take it to be a Gaussian. And what do I demand of that Gaussian this product when it is evolved back in time and we go to minus infinity this Gaussian should be such that. It picks out only these positions meaning it is only sharply peaked around x naught at $t = \text{minus infinity}$ and around p naught at t equal to minus infinity.

And how sharp you prepare this will determine how sharply defined the momentum and position of your particle at equal to minus infinity. So you go backwards so you construct the state evolve it backwards in time you have to choose appropriately a Gaussian and you get this state. Now let us see what happens when it runs forward in time so you have created that state now it goes further in time and of course the wave packet will start getting delocalized.

So it will not remain so very sharply peaked so let us say if it was very sharply peaked here it will start becoming a bit delocalized or actually this is it is going to become very delocalized and that is how that is what will happen. Suppose you now want to have a 2 particle state in the far pass what should you do?

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So let us construct now a 2 particle state so what do you want you want to have 2 particles which are very far apart from each other so you can imagine one in one Galaxy another in another galaxy. If you like or one in Delhi one in Bombay so that when they are because they are so far apart they are not going to interact with each other so you can treat these as 2 individual single particles. So this is a 2 particle straight now how are you going to create this 2 particle state at equal to minus infinity.

Well, let us start with the basis States and the relevant basis states are these ones which carry these 2 labels k_1 and k_2 . And what are we going to do we are going to multiply it with some function which will turn into 2 well separated Gaussians so let me for example have 2 Gaussians which you multiply with this one and then you evolve backwards in time. And when you evolve backwards in time these 2 Gaussians take the following shape so they are going to take a shape which is one which is very sharply localized at some position x_1 and some momentum P_1 .

And another one is going to get very sharply localized at some position x_2 which is Bombay or Mumbai and next one which is Delhi and some momentum you have chosen p_1 and p_2 for them. So they get very sharply localized here so these Gaussians should evolve into these Gaussians because if you do so then you have this localization so and of course you'll have to sum over all k_1 and k_2 .

So you have written down this state at t equal to minus infinity that you have gotten by writing down a linear sum of these in states so you have an integral over k_1 and k_2 which I am denoting as a sum for now. And you have folded with some functions which take you to these 2 particle states which are well separated from each other. And similarly if I want to create a state with n particles I have to just do the same thing I should take an in state actually all possible in states

And I should multiply with some function which will let us take it as Gaussians will give you foreign particles in the t going to minus infinity limit which are well separated from each other. And of course you have to sum over all k_1 to k_n so this this is a linear sum of these basis States I should put an n and you appropriately multiply with them with some functions which are called folding functions typically you take them to be Gaussian.

So that you are able to create such a state at equal to minus infinity and this is written at equal to 0 these states are defined at $t = 0$. So that is how you are going to construct these states I will write down the explicit expressions later and in fact we will turn these Gaussians to Delta functions in the far past. But that is something for later but I hope the picture is clear how you are going to use the in states to create states which look like particles which are not interacting in the far past.

And similarly you can use out states to create particles which are not interacting and far apart from each other in the far future all you have to do is replace the in states by out states. So that is how you create these states.

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single particle :

$$\vec{p} |\vec{k}\rangle = \vec{k} |\vec{k}\rangle$$

$$H |\vec{k}\rangle = \omega_{\vec{k}} |\vec{k}\rangle$$

$$\omega_{\vec{k}}^2 - \vec{k}^2 = \text{const.} = m_p^2$$

$$\boxed{\omega_{\vec{k}}^2 - \vec{k}^2 = m_p^2} \rightarrow \omega_{\vec{k}} = \sqrt{m_p^2 + \vec{k}^2}$$

Can a two particle state be identified as a single particle state?

$$\vec{p}_1 \omega_{\vec{p}_1} \quad \vec{p} |\vec{p}_1, \vec{p}_2\rangle = (\vec{p}_1 + \vec{p}_2) |\vec{p}_1, \vec{p}_2\rangle$$



$$H |\vec{p}_1, \vec{p}_2\rangle = (\omega_{\vec{p}_1} + \omega_{\vec{p}_2}) |\vec{p}_1, \vec{p}_2\rangle$$

Now let us first understand a little bit more clearly what I mean by a particle so you already have a notion of what a particle is you have a feeling of what you mean by a particle. But when you are doing this kind of filter you are doing these calculations how are you going to identify that a given state is a single particle state or not whether it corresponds to a particle or not. And that is what I want to again emphasize I have already told this in the previous course but let me repeat it.

So suppose you are given some state let me or what we really mean by a particle so suppose you are given some state in the theory which you are working with whatever theory you like some interacting theory. Now I am not going to write something inside right now because I want to leave it like this so this is some state in your theory it is given to you now you want to know whether this state is a particle or not.

So what should I do to determine whether this state corresponds is a particle or not well if it is a particle and let us say it is in some momentum Eigen state? So I am thinking of a particle which is having a precise momentum so it is delocalized in space but it is having a precise momentum. So if I want to create such if I want to if I take this state and operate on this with the momentum operator you know we have already in the previous course learned how to obtain the conserved quantities.

For example the momentum or the Hamiltonian or the premiums which is just h, p so let us take the momentum operator and act on this state and suppose I get the following suppose I get this meaning it returns a value k so it is an Eigen state of momentum. So it means that I can label it label the state with that's a good label so I will now Supply that label which I was not putting earlier so that is good now let me take the Hamiltonian and act on the same state.

Let us say I get the following I get some numbers so this state I am assuming that it is also it us a state of both momentum and energy so I get k for the momentum and I get some number which I call or some number which I call Ω_k that is what I have gotten. Now what I do is I take case what's up I take this Ω_k square it and subtract from it K square and try to find out what it is. Now if it so happens that what you get on the right hand side is a constant so you can always change the value of K that you have.

You can give it a boost the value of K will change so this k will change correspondingly when you find out the energy by acting with the Hamiltonian the Ω_k will be different and for each such for each of those different choices of k and Ω_k you find out what is the difference of these 2 squares. And if it turns out to be some constant then you say that this state K corresponds to a single particle state or that is what you are looking at is a single particle and that constant is called the mass let us call it MP square that constant is the mass square.

So energy square minus the momentum Square if it is a constant we call that constant as the square root of mass. And that is how you identify that a given state is a single particle state or not so this is the dispersion relation you already know this is the relativistic dispersion relation. Again from this you can see that if MP is a constant you are Ω_k or the energy is always going to be this. Now can it be that you are given some state and you find out its momentum.

Let us say it is a in some momentum Eigen State and you find out its energy of that state and you confuse it with a single particle state. Can a 2 particle State be confused with a single particle state is it possible so what I am asking is can 2 particle state be identified as a single particle state. So let us look at t equal to minus infinity and suppose I have created 2 particles which are far apart and let us say this one has momentum p_1 this one has momentum p_2 and this 1 will have also some energy ω_1 ω_{p_1} and this will have some energy ω_2 ω_{p_2} .

So let me label this state for now as $p_1 p_2$ this is the state I am not saying this one is p_1 that particle number one has momentum p_1 particle number as momentum particle number 2 has momentum p_2 . I am saying one of them has momentum p_1 one of them has another one has momentum p_2 because these are indistinguishable I cannot tell which one is which but anyway I have 2 particles and note that I am not putting in here because that's a state at equal to minus infinity.

So I take a 2 particle state and act with the momentum operator well I will get the following and if I act with the Hamiltonian of the theory I will get omegas are just the energies of these individual particles. That is what you are going to get now you might think that there is some momentum some energy so maybe I can think of this also as a 1 particle which is having total momentum $p_1 + p_2$ and total energy $\omega_{p_1 + p_2}$. Now let us see whether we can do that so what how did we define that.

How did we tell that something is a single particle what you have to do is take the energy of that state square it and from that subtract the momentum of that state square the square of the momentum of that state and if that difference is a constant meaning independent of the values of p_1 and p_2 you take or k here you take then that is a single particle state.

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$$(\omega_{p_1 + p_2})^2 - (\vec{p}_1 + \vec{p}_2)^2 \stackrel{?}{=} \text{constant} \checkmark$$

Ex: R.H.S \rightarrow not a constant ∇

$|\vec{k}_1, \vec{k}_2\rangle \rightarrow$ can not assign a mass

Bound states: \rightarrow ~~can~~ assign a mass

Hilbert space: Vacuum
 single particle state
 bound states
 multi particle state



So let us check whether $\omega_{p_1} + \omega_{p_2}$ that is the energy of that state right that that sum so square that minus the momentum of the state is $p_1^2 + p_2^2$ square that. And ask whether this is a constant if it is a constant then of course yes then you can identify it as a single particle but it I leave it as an exercise to you to show that right hand side is not a constant. So thus you cannot think of this state as a single particle State you cannot assign a mass to such a state.

So what I am saying is this state cannot assign a mass because the signing a mass would mean that this square minus the momentum square should come out to be a constant and you know it's not going to be coming out to be constant once you have done this exercise. It is a simple exercise to do so please do so and similarly any number of particles even if you have more than 2 you will not be able to treat it as a as a single particle state because for the same reason as here.

It will not come the differences will not come out to be a constant but now let us think of let us say electrodynamics where you know you can have a hydrogen atom. Now hydrogen atom is a proton and an electron bound state so these 2 are bound together now you know what is the mass of the hydrogen right meaning you can assign a mass to a bound state so you can hydrogen atom is moving with this much of momentum and it has this much of energy and this is the mass of this atom.

So bound States you can assign masses and also note that a bound state so let me write it down so bound state the bound State you can assign a mass we can do that. Now if you have a bound State and you wait for a lot of time to pass let us look at hydrogen atom and you just keep waiting unless you do something to it will not change right the hydrogen atom will remain hydrogen atom it is not that its components are going to run away from each other.

So a bound state is going to evolve into the same bound state in far future and similarly if you take the hydrogen atom and you start going backwards in time it remains the hydrogen atom it does not change into something else. So these bound states are also part of your Hilbert space so not only you have the vacuum and the single particle states and the multiple multi-particle States you also have these bound States in your theory. They may be there they may not be there but that's one possibility.

So good and I am not sure whether I said but it should be clear that once you assume that there exists single particle states it is automatic that you also have multiple particle states in your theory. Right because if you are able to create a particle at some place one particle at some place nothing will prevent you from creating another particle far away from it right because your theories are local and they do not prohibit from doing something far away from the first particle.

So these 2 particle 2 particle states can be created once you assume that single particle states can exist in your theory. So existence of multiple particle states is not an independent assumption it is a consequence of your assumption that single particle states exist. So what; are all the states in your Hilbert space from based on our physical intuition and we realize that of course we have vacuum.

Then we have single particle states then we can also have multi particle states then of course we can also have bound states and these are all the states that are there in your in the Hilbert state.

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$$\begin{aligned}
 & |\alpha\rangle_{out} \quad \& \quad |\alpha'\rangle_{out} \quad \alpha \neq \alpha' \\
 & \langle \alpha' | \alpha \rangle_{out} = 0 \\
 & \langle \alpha' | \alpha \rangle_{out} = \delta_{\alpha\alpha'} \\
 & \langle \beta' | \beta \rangle_{in} = \delta_{\beta\beta'}
 \end{aligned}
 \left. \vphantom{\begin{aligned} \langle \alpha' | \alpha \rangle_{out} = 0 \\ \langle \alpha' | \alpha \rangle_{out} = \delta_{\alpha\alpha'} \\ \langle \beta' | \beta \rangle_{in} = \delta_{\beta\beta'} \end{aligned}} \right\} \delta(\alpha - \alpha')$$

Completeness relation

$$\sum_{\alpha} |\alpha\rangle_{out} \langle \alpha|_{out} = \mathbb{1}$$

$$\sum_{\beta} |\beta\rangle_{in} \langle \beta|_{in} = \mathbb{1}$$

So let me do 1 small thing you remember long back somewhere here now you see I had written down the completeness relation that these out states and in states satisfy it just means that any out state can be written as a or any state can be written as a linear combination of out states or in states. So I will take these relations and try to write them using what I have talked just now.

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Completeness relation

$$\sum_{\alpha} |\alpha\rangle\langle\alpha| = \mathbb{1} \quad ; \quad \sum_{\beta} |\beta\rangle\langle\beta| = \mathbb{1}$$

$$\mathbb{1} = |0\rangle\langle 0| + \sum_{\vec{k}} |\vec{k}\rangle\langle\vec{k}| + \sum_{\vec{k}_1, \vec{k}_2} |\vec{k}_1, \vec{k}_2\rangle\langle\vec{k}_1, \vec{k}_2| + \dots$$

\downarrow drop
 \uparrow "multiparticle states"

we will normalize the \vec{k} states in ϕ^+ theory as

In free theory $|\vec{k}\rangle = \sqrt{2\omega_{\vec{k}}} a^{\dagger}(\vec{k}) |0\rangle$
 $\rightarrow \langle\vec{p}|\vec{k}\rangle = 2\omega_{\vec{p}} \delta^3(\vec{k}-\vec{p})$

Ex: show that $2\omega_{\vec{p}} \delta^3(\vec{k}-\vec{p})$ is a Lorentz invariant object.

So let us look at the completeness relation which was alpha and alpha n and if you sum over all this alphas you get identity. And similarly if you take the entire states beta out and form this combination then you get all and then you get identity if you sum over all the out states. And remember alpha is not necessarily discrete it could be continuous and actually you know because there are single particle states so these labels are also continuous.

So let us take this identity and let us write does not matter which one it is identical so let us identity is so let us take this what are all possible states I have already labeled them here so first is vacuum so let us write down vacuum. I will use omega for interacting theory vacuum and 0 for free theory vacuum so that is one state of course plus you have single particle states. So k n and k out so let us write down k n we are looking at in in this relation and you have to sum over all case plus you have these in states which have 2 labels.

And of course I have to sum over all k 1 and k 2 and because k 1 and k 2 are continuous labels we really have to integrate but I will keep it as a sum. And then of course you have all other multi-particle states all other states which have more than 2 labels which correspond to the states that in that you can with which you can construct multiple particle states so I will loosely call it multi-particle states.

But you understand what I mean that you can fold it with appropriate function and evolve backwards in time and that will give you a multi particle State at t equal to minus infinity. So

right now I am just being a little a bit careless in writing this and then of course you can have all the bound States there may be more than one. So that is how this identity is made up so now let us make this more precise at least up to here.

So of course as I said this in is unnecessary there is no requirement of this in label because there is no distinction between in out so you can just drop these labels. So let us look at now this piece only because I want to replace this sum by the integral so I just want to look at how it looks like when you replace the sum up by the integral. For that it is important that I should know how I want to normalize the states in this theory.

And I will choose the same normalization that I chose in the case of free field theory when which we did in the last course and the normalization will be the following maybe I will write down so that you can recall easily in free theory. In free Klein Gordon theory we had done the following case was defined to be $2\omega_k$ square roots a dagger k acting on the vacuum. And we had this normalization you can replace this p by k because this delta function it can change the p^2 it does not matter right whether you have p or k because of this delta function because this delta function ensures that k is equal to p otherwise you do not get any contribution.

So this is what we had in free theory so what we will do is we will choose the same normalization of states like this one in this case also. So I will choose a normalization of states in this theory we will normalize the states or single particle states in phi 4 theory to be phi 4 theory as the following it will be again the same. See it is up to you can you can choose to normalize it differently you may choose to drop the ω_p here.

That is a normalization that you can take and it will not affect anything if you are consistent with all the calculations that you do. But we choose this normalization and it is a nice normalization because the right hand side is Lorentz invariant. So that is an exercise for you show that the right hand side is Lorentz invariant object and that is the reason we want to keep this normalization meaning if you were to take the state and boost it so if you boost all the states by some amount.

So the state k will become the label will change to some k' p will change to some p' because the momentum will change. But the new states that you get their inner products will still

be the same because this factor on the right hand side is not going to change and that is what you have to show that this is Lorentz invariant. So we will keep this normalization and now let us see with this how I write this piece.

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$$\begin{aligned}
 |\vec{p}\rangle &= \sum_{\vec{k}} |\vec{k}\rangle \langle \vec{k} | \vec{p} \rangle \\
 &= \sum_{\vec{k}} |\vec{k}\rangle 2\omega_{\vec{k}} \delta^3(\vec{k} - \vec{p}) \\
 &= \sum_{\vec{k}} 2\omega_{\vec{k}} \delta^3(\vec{k} - \vec{p}) |\vec{p}\rangle \\
 &= \int \frac{d^3k}{2\omega_p} 2\omega_p \delta^3(\vec{k} - \vec{p}) |\vec{p}\rangle
 \end{aligned}$$

$\mathbb{1} = |\emptyset\rangle\langle\emptyset| + \int \frac{d^3k}{2\omega_p} |\vec{k}\rangle\langle\vec{k}| + |\text{bound states}\rangle\langle\text{bound states}|$
 + contribn from n states with more than one label.

So if you have a single particle state of some momentum p or let us say p then I should be able to write it as a linear sum of other single particle states and of course it will pick out the delta. Let me say it again so let us say I want to write this p as the following so I write s so what I am trying to say is that if you look at this combination this is an identity if you are only restricting if you are restricting yourself only 2 single particle states. And because any single particle state can be written as a linear sum of this right.

If you look at the identity over the entire Hilbert space then you have the vacuum single particle States and these in states with 2 labels and instead with n number of labels and bounded at everything but if you are interested only in the subspace of single particle States then in that subspace this is identity. So now let us construct this so this is identity and I want to write $\text{cat } p$ so $\text{cat } p$ is this. So this is a correct equation now what I am wanted to do is I want to remove the δ and have the appropriate integral so that is what I am trying to do now.

So I will keep δ for a while so I will use our normalization that we have chosen and that is $2\omega_{\vec{k}}$ or p whichever you like $\delta^3(\vec{k} - \vec{p})$ and that is it. This I can write as $\delta^3(\vec{k} - \vec{p})$ because of this delta function \vec{k} is forced to be \vec{p} so I will put this \vec{k} to be \vec{p} and you have I can

write it here $2\omega_k \delta^3(k-p)$. So on the left hand side you have p right hand side you have p so this should be identity.

And you know you know that I have to replace σ by an integral over k so I have to replace it by integral over k right because here itself it is integral d^3k so I have $d^3k 2\omega_k \delta^3(k-p)$ get p . So clearly if I just write d^3k this is not going to give you 1. That is not going to give you identity and if you instead of d^3k you write $d^3k / 2\omega_k$ then this works out.

Because this $2\omega_k$ will cancel this $2\omega_k$ and then you have just a $\delta^3(k-p)$ on the right hand side so that is fine which means that our normalization that we have chosen forces us to use this when we are looking at single particle states. So let us write down the identity for the full Hilbert space it is vacuum plus $d^3k / 2\omega_k$ and let me just see if it is all $\delta^3(k-p)$ that is fine and then you have what plus of course bound states and contributions from multi particle states or more appropriately contribution from states in states with more than one label.

This is all good and nice but we realize that we have not done the most elementary thing yet we have not told how we are going to create a single particle state in an interacting theory.

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How to create a single particle state in an interacting theory.

Free Theory:

- $|p\rangle \rightarrow a^\dagger(\vec{k})|0\rangle$
- $\phi(x)|0\rangle \rightarrow$ particle at (t, \vec{x})
- ϕ, π

Method worked because of harmonic approx. Quadratic terms in the action.

Naive expectation $\phi(t, \vec{x})|\Omega\rangle \neq$ single particle state.



I have not told you how to create a single particle state in an interacting theory see unless I know how to create a single particle state I will not know how to create multiple particle states then I

cannot scatter them and find out what is happening right so the first thing to do is to create a single particle state. Now this is something which we will do in the next lecture but let us at least understand what we should be looking for.

So let us go back to free theory what we did there free theory in free theory in free client garden theory when I wanted to create a state with momentum k all I had done was taken a dagger k and acting acted on the vacuum apart from some normalization which is here. That is what; we did we hit with a dagger on the free vacuum it is free theory or equivalently if I wanted to create a particle at position x i hit with ϕ on the vacuum and I create a particle at x where this x without any vector symbol is this four vector.

So a dagger of course is something you can construct out of ϕ 's and ϕ 's because after all what do you have in your theory you have the operators ϕ 's and ϕ 's. Where Φ is the Φ Dot the conjugate operator conjugate to Φ these are the 2 operators that you have and everything has to be constructed out of these there is nothing else right. And you have the vacuum so you hit on the vacuum with Φ you create a single particle state that is what we saw in free theory.

Or if you want to create a state which is of a definite momentum then you hit with some combination of ϕ 's and ϕ 's which is what we call a dagger and create that state. So you might think that we can just hit with the ϕ of interacting theory on the interacting vacuum and create a single particle state but that's not going to work. Because the reason all this all these things worked here why we could define a dagger and why this created a single particle state was because we were in the harmonic paradigm.

We were working in harmonic approximation meaning the action that we had was a quadratic action in the fields. So here it was the reason this method worked because of harmonic approximation. Meaning we were having only quadratic terms in the action. So what you can do is you can try to repeat the steps which we did in the in the free theory where you had only quadratic terms and see that it does not work you are unable to get anything out of it.

I mean if you in addition to having your free clan garden action you also have the 5-4 term or any other interacting term the steps will not yield a state which you can identify as a single particle states that is not going to happen. So the naive expectation that ϕ this is interacting

theory ϕ . ϕ of let us write t, x acting on interacting vacuum it is not going to give you a single particle state it does not work.

But still you know that if you want to create a particle all that you have is ϕ and π right those are the operators in your theory and you have vacuum and hitting on the vacuum you are going to create a particle so somehow it still has to work. Even though I am saying it is not going to work but there is not much that can happens it still has to work and that is what I will show you how to construct single particle states in interacting theory.

And once we have done that creating multi-particle states will be easy you have to just hit it several times at several different places and you create multiple particles. And then you can let them evolve in time and scatter them and we can try to ask what we get when these things scatter. So I think the question is clear what we want to do now and I will show you in the next lecture how to do these things.