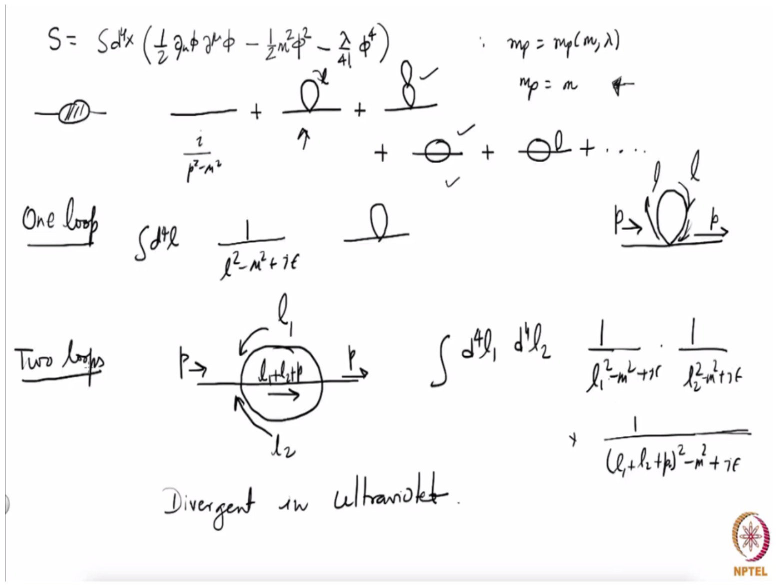
Introduction to Quantum Field Theory(Theory of Scalar Fields-Part 4)
Prof: Anurag Tripathi
Department of Physics
Indian Institute of Technology – Hyderabad

Lecture - 23
UV Divergences: Part 1

Let us continue our discussion further and let us try to calculate some physical observables in the phi 4 theory and see what we get. So, let us say we try to calculate a physical mass of the particles in this theory and see what happens.

(Refer Slide Time: 01:09)



So, remember that the action in phi 4 theory is integral d 4 x half del mu phi del mu phi - half m square phi square - lambda over 4 factorial phi to the 4 where m is some mass parameter. So, if I want to calculate physical mass that physical mass will be expressed in terms of these parameters m square and lambda that we call all observables will be functions of these parameters that appear in the action.

So, the physical mass which we have been calling m p will be a function of m and lambda and so let us see. So, I should look at 2 point function and find out where the pole of this 2 point function is and you remember that the location of the pole is the physical mass. So to lowest order is this propagator which is just i over p square - m square. So, m physical to lowest order is just m.

Now we want the effects of counter field theory in finding out what the physical mass is because this prediction is prediction based on a free theory because you have not included any interactions. This is a simple propagator which does include any interactions. So, let us include interactions and then you have this term. So, I have to calculate this loop and then take into account its contribution and we have these other diagrams as well.

We have seen this earlier in this course or maybe in the previous course I do not remember where we did and there are all these diagrams we have to sum up all of these and infinite of them and we have already seen that such integrals diverge. So, if you take this one it will give you d4 l where l is the loop momentum. Let me draw this clearly. I assign the momentum this way external momentum p is entering.

But you see at this point when it enters there is no external momentum that enters into this propagator because you have written down l for the loop momentum and momentum conservation does not allow anything else into entering into the loop because whatever enters in exits here. So, you see here is l going out and at this vertex from this side l is entering. So, momentum conservation says that p cannot enter into the loop.

But this is very special for this kind of vertices anyhow so you have 1 over l square - m square + epsilon and as I was saying last time if you look at you have four powers of l in the numerator and two powers in the denominator so this divergence. I will show you as I promised that I will show you how to do it properly, but you roughly see that this, is divergent and this is the situation at one loop.

The diagram which is this diverges let us see what happens at two loops. You have all this diagram, this diagram and others. So, let us look at this one. I am just doing it so that we get some practice with these Feynman diagrams so that the later discussion becomes a little easier. So, again you have p entering this is two loops, so you have two assignments of momenta let us call it l 1; l 1 enters here, l 2 flows this way.

So, you have l 1 + l 2 + p in this propagator and then of course p comes out. So, this will what will be the expression? It will have first of all because it is two loop meaning you have two a loop momenta l 1 and l 2. You convince yourself that you cannot have only one loop momentum here it will not work out and you will need two undetermined loop momentum, but that is something you should check that cannot work without having two loop momenta.

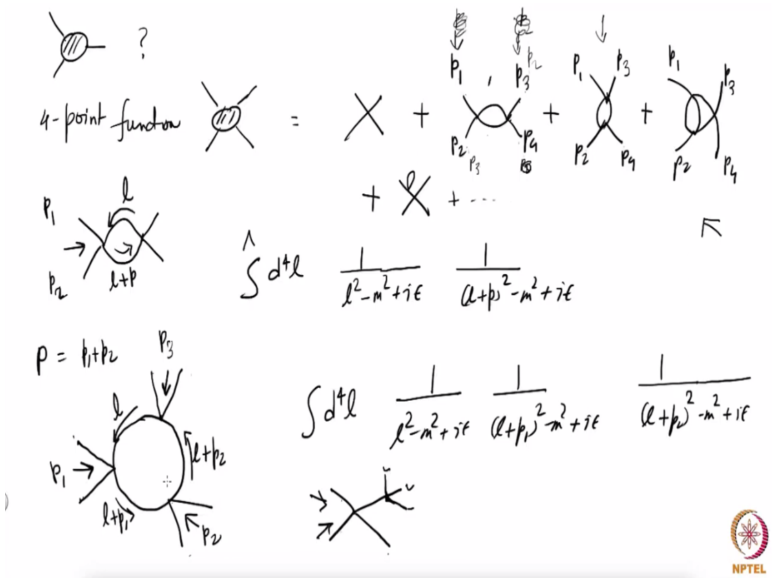
So, d 4 l 1 d 4 l 2 and then you have all these propagators l 1 square - m square + i epsilon and then you have 1 over l 2 square - m square + i epsilon then you have l 1 + l 2 + p square – m square + i epsilon. I am not being careful with the overall factors. I am just writing the important parts. So, you see this one is also divergent you have two powers from this factor, two powers of l from this factor and two powers from this.

So, when all these l's are large this will provide 6 powers of l you can forget about this external momentum p and m because they are of no importance when l is very large; l 1 and l 2 are very large. So, you see 8 powers in the numerator and 6 powers in the denominator so this will diverge and going to any other loop the same situation you will see that these are all divergent in ultraviolet.

This statement means that when you take large momenta this is divergent this is also called as ultraviolet divergence. So, you see that if\, you try to calculate this 2 point function to extract what the physical mass is you do not get anything because all these corrections are infinite they are all divergent. So, there is/has no prediction coming out from your theory. So, even though this is a very bad situation it is hopeless.

But just because I am drawing some diagrams and writing down some expressions let us continue that and even without any good motivation for this I will just draw some diagrams and see what happens, what kind of diagrams they are later that information will be useful, how about 3 point function; so I saw that 2 point function is ultraviolet divergent.

(Refer Slide Time: 09:43)



How about 3 point function in phi 4 theory. If you remember maybe we can see now itself that such Green's functions or such diagrams you cannot draw in phi 4 theory because you have a vertex which has four lines attached to it. So, you will never be able to draw any diagrams which has odd number of external lines. So, this is out anyway let us go to four point function now.

So, you have; so this will be plus this is one loop and we have drawn already several times such diagrams, draw them again and then you have always find this drawing this last one confusing so that is at one loop I mean you also have these ones and others, but these ones I will not consider because when we were calculating s matrix. We saw that this after LSZ reduction was done I could express the dash matrix in terms of amputated Green's functions.

So, I will not include this one I will include only the ones without any external corrections and all the external corrections were in the z functions or wave function renormalization constants. So, let us see what we get. Here let us look at this one. So, I will draw it again p 1 p 2 let me tell you that here this moment are like this. So p 1 p 2 p 3 p 4 and in this one also it is the same assignment because if you change the momentum assignments then your diagrams will look different.

I can just also draw all these three diagrams in exactly the same manner and change the places where p 1 p 2 p 3 p 4 appear and then the all diagrams will look the same. So, what I am saying is if you take this one here p 1 p 2 p 3 p 4 let us say you change the labels here instead of p 1 you make let us say p 3. So, this one if you change to p 3 and this one to p 2 p 3 then it is this diagram because at this vertex p 1 and p 3 are meeting.

And at the other vertex p 2 and p 4 are meeting and that is what you will get if you were to make this an assignment. So, then at this vertex p 2 and sorry I should have said not this way, but here p 3 and here p 2 then it would be p 1 and p 3 meeting at this vertex which is what is here p 1 and p 3 meeting at this vertex and then p 3 and p 2 meeting at this vertex which is what is here p 2 and sorry this is p 3.

And then this should be p 2 I am repeatedly making mistakes because these (()) (14:00). So then p 2 and p 4 meet which is here. So, either you can choose to draw these diagrams like the way I have drawn or draw the same diagram repeatedly, but change the external momentum positions where it is more convenient if you draw it this way anyhow. So, at this vertex p 1 and p 2 enter.

So, the total momentum that is in injected at this vertex is p 1 + p 2 let us call it p. So, I define it p as p 1 + p 2 that is what enters in here and then you have a loop momentum l which I want to choose this way then you have l + p flowing this way and again let us write down the integral it is d 4 l 1 over l square - m square + i epsilon and then you have 1 over l + p whole square - m square + i epsilon.

So, this is also divergent 4 powers in the denominator and 4 powers in the numerator and this is a divergent quantity as you can see by this power counting. If you were to put a cut off lambda it will diverge logarithmically. So, no surprising this is also bad, but let us move further ahead and draw few more diagrams. So, I have looked at already 2 point function and four point function and found that they are all diverging at one loop order.

Let us look at six point function now. So, you have sum of these two momenta let us call it p 1 it enters here, sum of these two let us call it p 2 that enters at this vertex and the sum of these two let us call it p 3 and it is injected into the diagram at this vertex and let us assign momentum l like this then it becomes l + p 1 this becomes l + p 2 and this is anyway l + p 3. So, let us write down the expression or at least the integral d 4 l it is one loop.

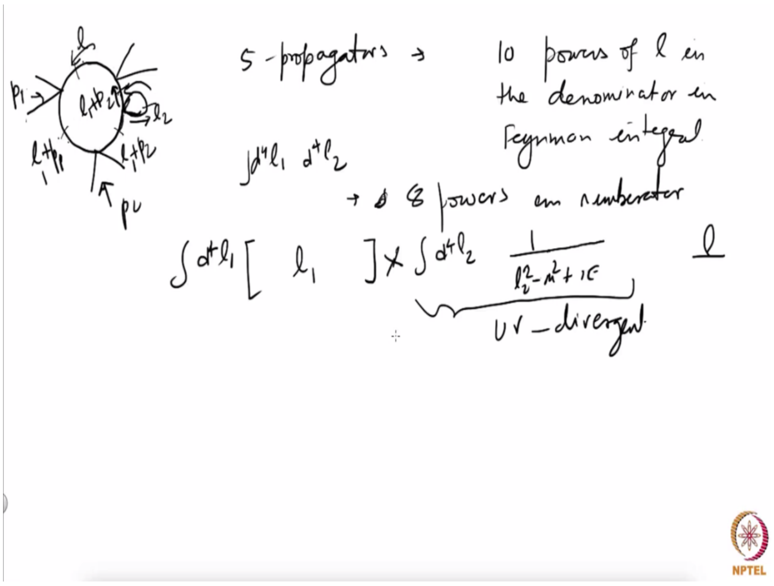
So, only one loop momentum 1 over l square - m square + i epsilon 1 over l + p 1 square + i epsilon. So, we have three denominators because we have three propagators. Each brings a power of two. So, we have a total power 6 in the denominator and in the numerator you have four powers. So, this is ultraviolet converging because the function is following sufficiently fast that in the limit l going to infinity this converges it does not diverge.

So, good for us at least one diagram is finite, but that is not going to change anything because we still cannot determine physical masses in this theory or even 4 point scattering function. I mean suppose you are looking at a cross section calculation of two particles scattering to two particles you cannot calculate it because it will come out to be infinite because these diagrams are divergent.

You might think that if you were to calculate two particles let us say this and this going to four particles 1, 2, 3, 4 this will come out to be finite because this diagram is finite. So, lowest order diagram you can draw like this 2 I hope I can find yeah this one. So, if let us say this one is these two are incoming and these 1, 2, 3, 4 these 4 are outgoing then this is three level diagram there are no ultraviolet divergences because there are no loops involved.

So, you can calculate this one and then you look at this one loop which has turned out to be ultraviolet finite. So, it looks like you can calculate 2 going to force scattering that is scattering matrix element and that will come out to be finite and if you were to calculate cross section for 2 going to force scattering you might lively think from this that it will come out to be finite, but that is/has not the case and let us see why.

(Refer Slide Time: 19:38)



And the reason is the following. So, let us take this one again which is convergent and you understand why it is convergent because now you have larger number of denominators see the numerator power is still fixed it is d 4 l so 4 powers in the numerator and what we have done is increase the number of denominators by going to more number of external lines. So that the number of denominators has increased and it brings 6 power so it is converging.

But now when you are calculating cross section for 2 going to force scattering this diagram will also contribute, but now this is this convergent? Well you might think that this is convergent because you have 1, 2, 3, 4 and 5; 5 propagators. So, that brings 10 powers of l in the denominator in the Feynman integral and you have 4 powers in the sorry not 4. So, this is how many loops?

This is 2 loop you see 1 loop here, 1 loop here or you can just try to also check the momentum assignment so this is loop momentum it goes here it is determined by whatever momentum comes in from here that adds up let me write it p 1 so it is l + p1 p 2 enters here so this is l + p 2, but at this vertex this momentum you cannot fix because you have 3 lines going from here.

So, one momentum enters and you have 1, 2 and 3 lines you cannot fix momentum in all the 3 lines. So, let us assign this one as l 2, l 2 flows in like this and now this one is determined and then you have this and yeah and this one more propagator. So, you have 10 powers of l and how many powers in the numerator? You have d 4 l 1 d 4 l 2 let us call it l 1, let us call it l 2 so 8 powers.

So, (()) (22:43) it looks like this will be convergent because you have more powers in the denominator compared to the number of powers in the numerator, but then when you are doing this integral this see here let me just write here it is l 1 + p 2 in this because this cannot change the momentum that is going out it will just be the same. So, you will have d 4 l 1 let me write it down d 4 l 1 and then all these propagators which have only l 1.

And then this one d 4 l 2 and this will be 1 over l 2 square - m square + i epsilon. So, this now look at this one even though the way we are counting it looks like it is finite, but if you look at this piece which is multiplied here this is ultraviolet divergent we have already seen this because this is ultraviolet divergent. So, you have a diverting piece in here and this is called a sub divergence.

So, the diagram looks like if you do just a power counting it looks like UV finite, but it has a sub divergence sitting in it. This is UV diversion part. So our hope of getting 2 going to force scattering as a finite thing in this phi 4 theory even that is gone. So, there is nothing that we can really calculate in this theory if we were to proceed the way we have set up things right now.

So, what should we do next and what is the and you already know that people do quantum field theory and they get predictions for scattering cross sections at these different machines that we have built over time like large electron positron collider or tevatron or LHC. So, clearly we can get some finite results and the thing that makes it possible to have finite predictions from this theory is called renormalization.