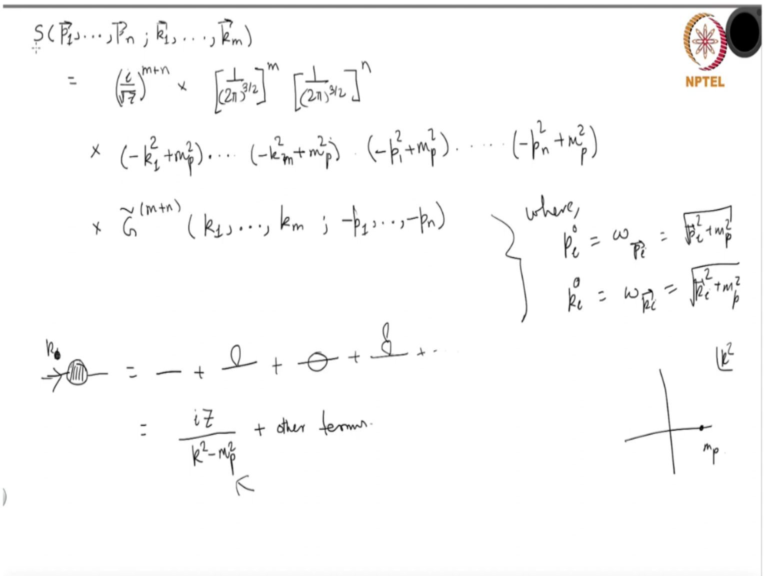
Introduction to Quantum Field Theory - II (Theory of Scalar Fields)

Prof. Anurag Tripathi
Department of Physics
Indian Institute of Technology - Hyderabad

Module - 5
Lecture - 11
S Matrix Continued

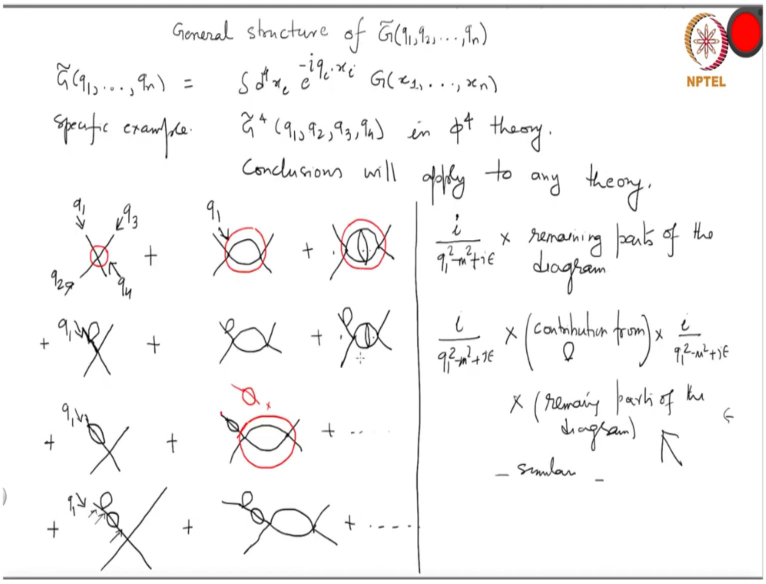
(Refer Slide Time: 00:17)



So, let us see what we were doing before. We were here. So, we wrote down S matrix element and we expressed it as a, apart from these constants, as a product of these factors which vanish when k 1, k 2, these are on shell, so that k 1 square is equal to m p square and this becomes 0, and same is true for other momenta as well. And then we had a Fourier transform of Green's function with these arguments.

This is a m + n point Green's function and we understand that this S Matrix element is going to be non-zero only if the G tilde here contains poles in k square plane. So, if you have terms like k 1 square minus m p square, then this pole will get cancelled and similarly other poles will get cancelled if G tilde has such poles. And in fact, it should, because this S matrix element should not be 0, otherwise there is no scattering. So, that is the expectation and that is what we wish to now figure out.

(Refer Slide Time: 01:45)



So, let us start by looking at the most general structure of this G tilde. I have put momentum labels as q 1 to q n. I am looking at this for any q 1 to q n and let me also remind you what G tilde is. This is just the Fourier transform of the Green's function when in the coordinate space. So, I transform each coordinate, each x i and the conjugate variable to that is q i. So, let me begin by taking a specific example of Green's function in 5 to the 4 theory which we are at present considering, but whatever I say will apply to any n point Green's function and in any theory.

So, I will take specifically four-point function. Conclusions will be applicable in a general theory. So, let us draw some diagrams which contribute to G tilde 4. So, diagrams that contribute are the following. That is the lowest order diagram. Then we have this diagram and then many others. Then we also have these kinds of diagrams, and another diagram is this. I am arranging the diagrams in a particular way, but, as we have learnt before, we have to draw all possible diagrams but I am just doing some arrangement of this.

And one more set I want to draw, and they are infinite diagrams because you have to consider all terms of ordered lambda to the n, where n goes to infinity. So, we have this now. If you write the expressions; see, let us look at this first column; if you write the expression; let me first label the momenta, and the same thing here. It is also q 1 and similarly q 2 and others; and same thing here, so, q 1 and 2 is here.

So, when you write down this diagram and for the expression of this diagram for G tilde, you will write i over. So, you can visit the first part of this course and you will find that this is what we should write, q 1 square minus m square plus i epsilon times the remaining part of the diagram. When you come to the next diagram, it will have the form i over q 1 square minus m square plus i epsilon times, that is the propagator here, this propagator.

Then you have this bubble, and for bubble you should write down the contribution. So, contribution from the bubble and times, you remember you have momentum conservation at each vertex, so, the momentum that flows in this propagator is also q 1. So, we will have again i over q 1 square minus m square plus i epsilon times remaining parts of this diagram. And you see, the remaining parts of this diagram are exactly the same as the remaining parts of this diagram.

So, let me write remaining parts of the diagram. And remember, when you are looking at the remaining parts of the diagram, for example, let us look at this first diagram. What is the expression of this? You have propagator for each of these lines and then you have vertex, vertex which is -i lambda over 4 factorial and then you should supply 2 pi to the 4 delta 4 q 1 + q 2 + q 3 + q 4 that is the total momentum, that is a delta function which conserves the momentum; and then you also have to multiply with the combinatoric factor.

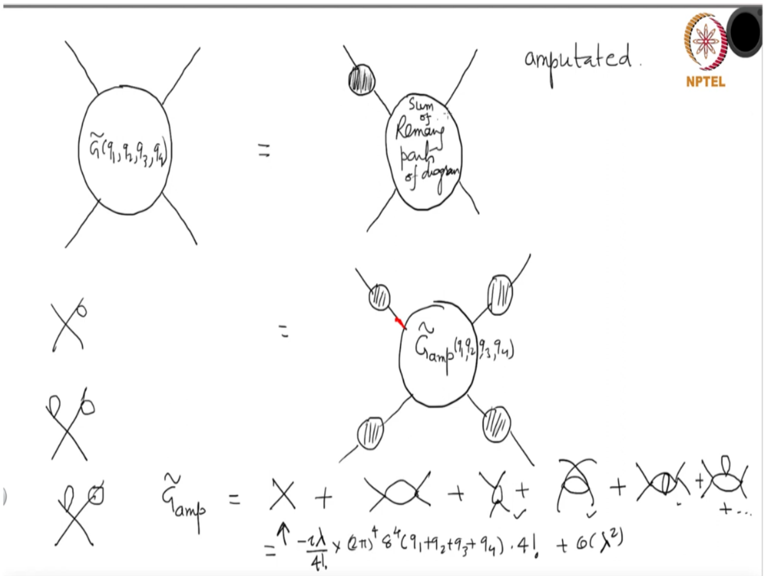
So, these remaining parts of the diagram contain vertex combinatorial factor times 2 pi to the 4 delta 4 of q 1 + q 2 + q 3 + q 4; and the same is true for this also. Let us look at the third one. Third one will also have exactly the same form as above. Instead of contribution from bubble, you will have contribution from this diagram, this part of the diagram. And, so, similar thing; I do not want to write it again.

And then for the last one, you will have here 3 propagators; one coming from here; second propagator again coming from here, which will also have a momentum q 1 flowing into this; and this one will also have a momentum q 1 flowing into this; and that is the case because we have momentum conservation at each vertex. And these are loops; they involve only the loop momentum, so, there is no other momentum flowing in.

So, this entire line will have, these 3 propagators will have q 1 flowing to them and you will multiply these 3 propagators and contributions coming from this bubble and this loop diagram, and then times the remaining diagram, which also includes combinatorial factor. Now, if you sum up the first column, you will get, you can factor out the remaining parts of the diagram because all of these, the way I have organised, all these 4 diagrams have identical contribution for the remaining parts of the diagram which I have written here.

Now, look at this second column. It has the same structure; that is how I have arranged everything. The only difference is that the remaining parts of the diagram here is different compared to the first column; but other than that, what happens on these external legs is identical. So, you can see that the general structure that you are going to get is of the following form.

(Refer Slide Time: 12:08)



So, this is my G tilde. Let us look at this is G tilde q 1, q 2, q 3, q 4 four-point function. So, I denote it like this and this is equal to; I am looking at; let us look at the first line, first leg in which momentum q 1 is entering. When I factor out all this, we have contribution of this kind, of this kind, of this kind, of this kind multiplying the entire diagram. And the same is here identical.

So, if you sum up these 8 diagrams, then you see that you can factor out this i over q 1 square minus m square plus i epsilon times these 3 factors. And then, remaining parts of the diagram will be a series expansion in which the first term will be just -i lambda times 2 pi to the 4 delta 4 momentum preserving delta function coming from here together with the combinatorial factor plus contribution coming from here which will have these 2 vertices, again a momentum conserving delta function and its corresponding combinatorial factor.

So, the structure would be of this form. This is what we can see from the following discussion and this contains the remaining parts of the diagram. This is what is remaining parts of the diagram, and of course you have sum over all the diagrams. You have to sum over all; I should add sum. And what is that sum? That sum is, that sum contains this vertex, then this diagram here.

And if I had drawn another one, let us say this one, then it will contain this remaining part of the diagram. So, let me draw one more; it will help to understand. You see, here also I can put exactly this. So, this is also possible diagram in G tilde. So, that is how I am arranging everything. And of course, then you realise that there is nothing special about first leg; I can do the same thing on all legs, meaning I can look at also the contributions of this kind; or let us start with the lowest order diagram.

So, you could have a bubble here; you could also have a bubble here; and also this; there is also this diagram of course and all possibilities. So, you see that if we go through the argument which told us that we should have, structure should be of this form where this contains all corrections on a single line, these are all corrections on this q 1 line here, here, so, all those corrections I am including in this by, and all those corrections are denoted by this blob.

So, this blob contains all the corrections. And if I repeat the argument and it is clear that then I should be able to write it as this where this contains now not the contribution from all the remaining parts of the diagrams; that is true, but this contains these kinds of parts. What goes into this, let me call this as G tilde amp and of course, with the same arguments; amp is the short form of amputated, and to amputate means to cut off the limbs.

So, this part is what you get after you have removed all the limbs. Let us see what it means. So, G tilde amputated contains contribution coming from this, from this vertex. So, first term will be just this vertex, then this, this and so forth. So, how do we understand this? So, you see here G tilde amp, the way we have arranged everything, it does not contain any line which upon cutting will give you an external point removed from the diagram.

So, here for example, if you look at this; let us look at this one. There is no line in this which upon cutting can isolate any external points. So, let us say I want to isolate this point. Is there which any line which I can cut to isolate this? Well, yes there is one. If you cut here, then this gets isolated; this becomes this times the remaining diagram, but that is not what goes into the amputated diagram. What goes into the amputated diagram is this.

Now let us look here. Is there any line which you can cut to isolate this point? No, there is none. That is how it has been arranged. So, you see all the diagrams that enter into G tilde amputated have a property that upon cutting any single line, you cannot isolate an external point. So, for this G tilde, these are external lines, external lines and you can say this is an external point.

You cannot remove; you cannot isolate this external point by cutting any one single line. That is important. It should be one. It should be about one point or one line; you cannot cut one line. So, that is the structure and this contains all the corrections to the external leg, and similarly here and here and here. So, let me just write down one time again the expression for G tilde.

So, G tilde amputated or in short amputate amp is equal to a contribution; did I write already? No; will be like this. This, that is order lambda term, then this is an order lambda square term. Then of course, you have other diagrams which you have seen in the previous course. I will not put the labels q 1, q 2, q 3 and q 4; you understand what I am writing, plus we have drawn these diagrams several times in the previous course.

And also let me draw one more here. And here, let me write down the expression for this. This term will have -i over lambda 4 factorial times 2 pi to the 4 delta 4, and then you have combinatory factor which will be 4 factorial. There are 4 factorial ways in which you can get this; plus order lambda square terms. This contributes to lambda square; this contributes; this also contributes to lambda square; this contributes to lambda 4 and so on.

Let me try to explain why you have a 4 factorial here. See, after all, we are just writing these diagrams, rearranging them in a particular way. So, look at this one, this had a 4 factorial multiplying it, right, because that is the combinatorial factor. And once I have written it as a product of limbs and where limb is; for example, this limb is giving 1 over q square minus m p square; similarly, these ones; where should I include the 4 factorial factor?

So, that I am including in the amputated Green's function. In the amputated Green's function G tilde amp and not in the external lines because that will not be convenient. So, those factors are in here. So, now let us analyse this in more detail and we will write down the precise form of this Green function as product of these external leg corrections and the amputated Green's function. That is our next task to have a precise expression for it, but we already understand what it should be like, based on this discussion.