Introduction to Quantum Field Theory (Theory of Scalar Fields)
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Lecture No # 01
Module No # 01
Scattering Matrix

Welcome back this is the second course on quantum field theory, and I assume that you have taken the first course that I gave over in NPTEL. So before we proceed let me first tell you the books that I have used for preparing these lectures and you will also find them useful books and other sources.

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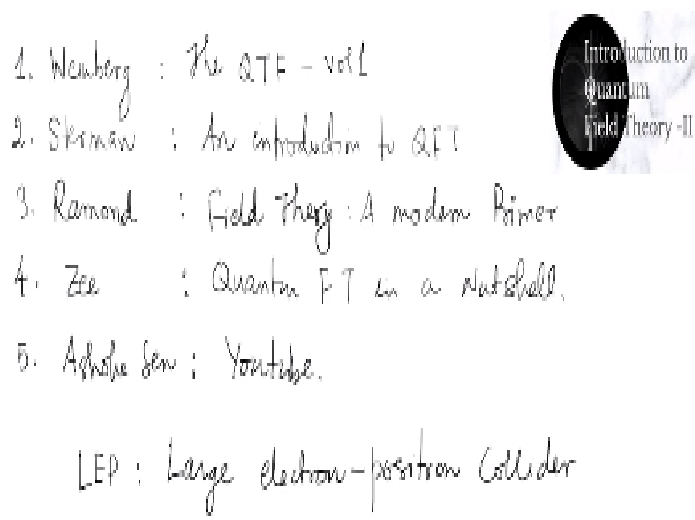

So first one is not necessarily I have used all of them, but it will be useful to know these books. So the first one is Weinberg this is Quantum theory of fields I am writing in short this is volume 1. Then second book is by George Sterman this is an introduction to Quantum field theory again I write in short. And then a few more I will write so book by Ramond and this is field theory of a modern premiere; or maybe primer I used to wrongly call it premiere.

And then also a book by Zee this is quantum field theory in a nutshell and there are many other books on the market and you can choose whichever you like the most. And also lectures by Ashok Sen his lectures you can find on YouTube, on conduct field theory and that will also be very useful and we will also refer to those lectures. So these are roughly the books which references which you are going to follow in this course and you would be benefiting by looking at them.

So as like last time we are going to have this course run for 12 weeks and you are going to get frequently assignments I will keep that those assignments very simple. So let us recall what we were doing last time in the previous course we were studying quantum field theory, and we looked at free theory and then we looked at interacting theories. We talked about their symmetries and what conservation laws they give rise to, and what conserved charges they give rise to.

Now the goal here in this course is to work out in detail what lies in quantum field theories or more precisely interacting quantum field theories. And we will be concentrating on scalar field theories but nevertheless the formalism that I am going to develop here will be useful for other fields as well so most of it actually. So you we do not lose anything by restricting ourselves to scalar field theories.

So let us start by asking let us put it this way what is the typical experiment that is done in high energy physics? I am going to phrase it in terms of high energy physics but the things which we do in this course are not limited to high energy physics, they are applicable to other fields as well like unless matter and quantum optics and these things. But I will restrict to a discussion to high energy physics as far as setting up the experiment is concerned.

So a typical experiment is the following you collide some particles, and you measure some particles you find some particles in the final state that is typically what you do? For example you might have heard of the large hadron collider, where you collide 2 proton beams or let us say 2 protons. And then you detect things in the detector in the final state and you must have heard of Higgs Boson being detected at the LHC and many other things are measured there.

Let me talk about a machine or a collider that was that existed before LHC and it was called LEP stands for large electron positron collider positron is the anti-particle corresponding to electron. So here what you do is you take an electron and a proton and shoot them at each other at very high energies. How a high is not so relevant for what I am going to discuss but very high energies and they collide and then they produce something in the final state which you see in your detectors it is a typical experiment.

And when you talk about LHC it is just little more complicated because actually a lot more complicated but one of the complications comes in because your initial state is not a fundamental particle but it is a proton which has many particles inside it. And that is why I am more inclined towards talking in terms of flap it is more, easy to talk. So suppose you are doing an experiment at the lab where you fire an electron towards the positron and they meet and collide and produce something.

What is that you would want to ask or even before that let us see what comes out of those collisions? So typically what will happen is you will get many different outcomes. So suppose you collide 1 electron and 1 positron let us say first collision happens; and you get 2 photons in the final state that is one possible outcome. And that you can detect in the detectors you will see to those 2 photons coming out with some energy and some momenta.

By momenta it is clear that depending on the moment they will be going in different directions. Now think of colliding the next set of electrons and positrons, so the second set comes in they collide and they might not produce 2 photons this time they might produce 2 z bosons in the final state that is a possible outcome. Let us collide, the third-one third set an electron and positron you might get again a final state which contains 2 photons, so you detect this time 2 photons in the final state.

So these electrons and positrons they disappear and they leave behind 2 photons that is another outcome. Let us collide the fourth-one and this time you might get a, 2 w bosons in the final state fifth- one you collide you may get 10 different particles in the final state. So you see there are many different outcomes that are possible given the same initial state, so I am not changing the initial state I still have the same electron, the same proton, positron coming in with the same energy; they both disappear but they produce something else in the final state.

So there are several possibilities and what you would like to know what is the following what is the probability that if I fire an electron with this moment and a positron with this momentum I do not have to specify energy because masses are known so mass energy gets fixed. And if I choose this initial state where these 2 particles have these specified momenta, what is the probability that I will produce a final state number one; or final state number 2; or final state number 3.

And this specification of final state is not just about telling which particles are produced it is also about with what momentum they are produced. So even though you might be producing n number of particles in the final state which have some momenta a second outcome will could be same particles again in the final state but with different momentum. So an experiment can measure this so and for those reasons you have those machines so experiment can measure.

And if you believe that you have a theory which is correctly going to describe electromagnetic interactions, electrodynamic interactions, or electro weak interactions then you take your model which you believe is a description of those interactions. Calculate these probabilities probability of these events which I have talked about. And see what you get and if your results agree with what experiments have to tell you, and then you can have a belief that you have gotten a reasonably good model of these interactions.

So that is the goal is to predict the probability of a given initial state to emerge as a given final state if we know these things then we know the dynamics of the theory, that is that is what the prediction would be from the theory side so that is what we want to do?

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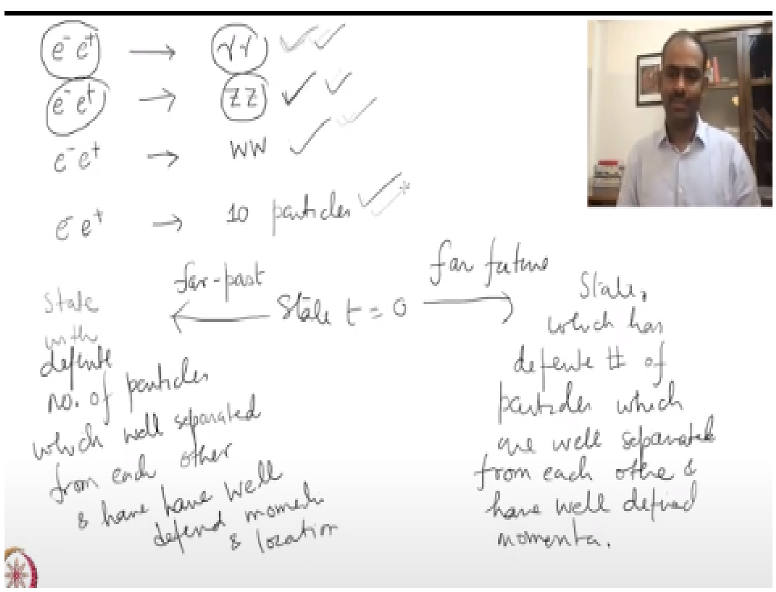

So let me just write down here so as I was saying you could take an electron I will write as e - and a positron the symbol for positron is e +. And these 2 collide and they might annihilate to give you 2 photons gamma is a simple result for photons. Or they could in another event give you 2 z bosons, or another possibility in fact there are many possibilities. I am going to list on only a few they may give 2 w bosons and they may give n number of particles whatever you wish.

Let us say for the sake of saying 10 particles moving in different directions these are all different possibilities and you would like to predict what is the probability amplitude or probability of this appearing as this state. Or this initial state appearing at this initials this final state and so forth. Once we know this entire set I can or even for a smaller set of final state and I can go and match with the experiment.

And then I can either conclude that what I have is a good description of the interaction; or what I have is pretty useless because it does not match with the experiments, so either way. So now that we have told what the goal is let us try to find out how to answer this question. So let me go to the next slide how are we going to find out so let me be a little bit more let me throw in more details about the scattering process i was describing. So typically it is like this; so you have an electron coming in from this side and a positron coming from this side.

So an experimentalist has prepared this state so where he or she knows the momentum with which this electron is fired the momentum with which this positron is fired that is known to you; you have prepared that state. Of course you know the masses of these particles and then it is come going to come out of some let us call it loosely as a nozzle, so it is going to come out from some nozzle so you know where that electron was located at some time get some point of time and same for here.

So you roughly know or you very precisely know where that electron was located and with what momentum it was moving, at some time and same for this one. So you have prepared a state in which you specify the momenta the energies and locations of those particles at some at some time. Consistent with Heisenberg uncertainty principles of course you cannot say that the momentum is precisely this; you have to take into account the Heisenberg uncertainty relations.

Now this state where it contains 2 particles and these 2 particles are far apart from each other and because they are far apart they are effectively not interacting with each other. So that is the initial state they do not feel each other they are far apart so non-interacting particles and I know there are 2 particles this is electron, positron. And they then start coming closer and closer because they have been fired so they come closer and closer at some point of time they start feeling each other.

And then something happens and that is what we want to know what happens and something happens some other particles might get produced. Let us say 2 photons but I cannot say that when I am in that region where the interaction is happening I cannot say that because remember for an interacting theory the number of particles is not conserved; that is true in free theory but not in interacting theory.

So this notion of how many particles are there all this is not applicable here it is lost so I cannot say that there are 2 particles or 3 particles or whatever but something has happened. This state where you had 2 particles and the way I described that state is evolving with time and remember; I am looking at showing your picture because now states are evolving with time. They this evolves with time and then it produces some state at time t = 0 and t = 0 is the time I am going to use for that time when these 2 start feeling each other appreciably.

So that we can say that the states are interact I mean this there is interaction going on. So they feel each other at time t = 0 and of course as you can guess it is arbitrary which one you call it t = 0. If you take a moment earlier a moment later it does not matter so sometime you choose arbitrarily. So they come closer and then the state evolves again and when you have waited long enough this state will evolve into a state which will have fixed number of particles.

Maybe 2 photons which; are far apart from each other so that you can say that you have gotten 2 photons, or 2 z bosons. So what has happened is a state with a definite number of particles which are far apart from each other has evolved into another state which has a definite number of particles which are far apart. So if you look at let us refer to time 2 = 0 so compared to time t = 0 this state, so t = 0 I do not know how many there is no such notion of how many particles these notions is left is lost.

But if you take the state at t = 0 and go backwards in time; when you go backwards in time long enough you will see that that state started appearing as a state with an electron and a positron. So the state at t = 0 when evolved backwards in times into far past is a state with a definite number of particles which are well localized in space. Remember it was in the nozzle at some point of time and well localized in momentum you fired it with some momentum.

So that is your there is the state in the far past. So let me write it down the state at t = 0 and remember t is arbitrary when you go backwards in time. It gives you a state with definite number of particles which are well separated from each other and have well-defined momenta defined momenta and location so this is something in far past. And similarly if you were to start with the state at t = 0 to which this initial state has evolved to.

And if you were to evolve it to the far future; that is you wait long enough again you have a state which has definite number of particles which are well separated from each other and have well-defined momenta. And the restriction given by Heisenberg uncertainty relations has to be taken into account so you cannot say that momentum is precisely this because then your particle could be anywhere.

But you can construct Gaussian wave packets and fairly localize both in momentum and position space. So I know that I am repeating a lot but that is worth doing because this otherwise it appears unnecessarily difficult. So what we have done is state t = 0 when evolved far past it looks like well separated particles with well-defined momentum and location. Similarly the same state evolves far in future looks like again well separated particles with well-defined momentum and position.

Now let us concentrate on the right hand side state at t = 0 evolving in far future and that is what I want to focus on first. If so let us again repeat what I have said so at the state that t = 0 could evolve into those many different possibilities which I was listing. It could evolve into this, or it could evolve into that, or is this could evolve into this or that and whatever is possible it could evolve into this and that. And of course when you measure you get only one of those.

So it makes sense that the state at t = 0 you decompose into a basis states into a set of basic states where a given basis state would evolve. If you were to take that basic state and if you were to evolve it into far future you are going to get let us say this one. So there would be some basis state which when evolved into far future is going to give you 2 well separated photons with this momentum and that momentum.

Similarly there will be another basis state which when you evolve into far future would give you only to z bosons similarly a third basis state which will give you this a fourth this and so forth. So the state at t = 0 is a state which is an in a linear combination of states which when evolved into far future will give you either this, or that depending on the coefficients which are multiplying those basis states right.

A super so you construct a superposition where you have this linear sum of all the basis states with some coefficients. And depending on the coefficients the probability of finding that state in the final state, I mean the probability of getting the state in the final state is determined by those coefficients. So the point I want to make is the state set t = 0 should be a linear combination of states which when evolved into far future give you these dates; that is all I want to convey.

Now I do not know at this moment much about what this basis states are what they look like and that is something which we are going to do in the next few lectures with a lot of care but I can say something at least I can put some labels on these states.

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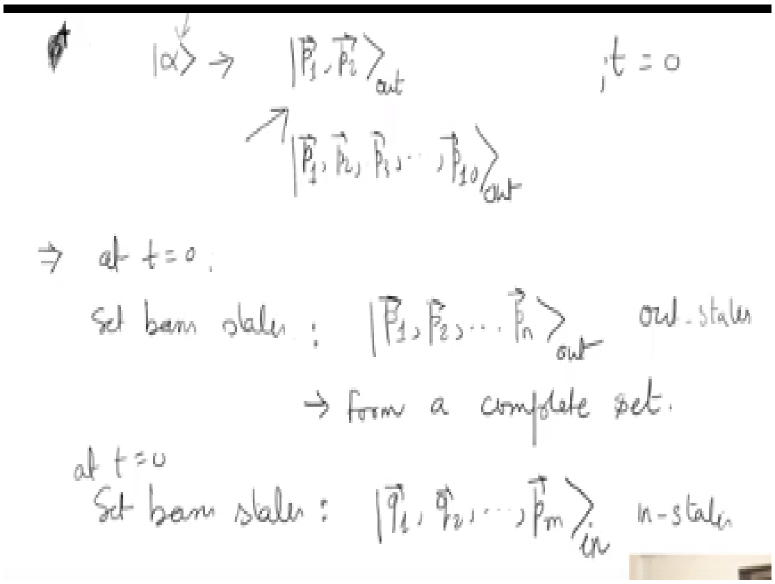

So let us say what these labels would be, so if I say that now what I will do is I will forget about not forget but I will not be really looking at electrodynamics because my interest is in phi 4 theory in this case. Or where all the same kind of particles are there it is not different kinds of particles you have only one field. So in the final state you will be measuring particles which are excitations of this field phi.

So but if you wish you can keep thinking in terms of electrons and positrons and z bosons and whatever, but let us let us keep it thinking that way, it is useful so let us not worry about this at the moment. So you will have some state alpha let us call that alpha which is a basis state, so and if depending on what are these different labels that alpha contains you will get one basis state or the or another base system.

So there will be some such states alpha which will evolve to those states in the far future so what could be the possible labels. Well we could label them like this we could say that since this state evolves to a state which has let us say 2 particles like 2 photons with momentum p 1 and p 2 I can label the state alpha like this. Now do not think of this as a state which has these 2 particles carrying momentum p 1 and p 2 that is not how you have to think about it.

As I said at t = 0 so I am you see basis states I am talking about at t = 0 so whatever I am doing is at t = 0 and at t = 0 there is no such notion right particle number is not conserved there is no such notion. So but since this state this basis state by definition when evolves to far future gives me 2 particles far apart with momentum p 1 and p 2 I will use this as the label. And in fact I will put a subscript out here which means that this is a state when you evolve in far future that is the outcome final state outcome the outcome which I specified earlier.

So now if your theory contains only scalar particles there is nothing else which you need to specify if you had electrons and cross electrons and photons then you know you can also you need to also put other labels like charges and other things. But since I am interested in only phi 4 theory I do not need to put any other labels it is the momentum which is the only label. And energy is not an independent label because the masses of the particles will be known. And once the masses are known the energies are fixed by knowing the momentum.

So what I have argued till now is that the time t = 0 there has to be a set of basis states which will be labeled, which can be labeled like this and these evolve to those states. And there will be lot of sub states right it is not just 2 particles as i said you could produce 10 particles also. So there will be another state which will have some momentum p 1 some momentum p 2 another particle carrying momentum p 3 and p 10 and we also put this subscript out.

And at this moment I am not telling you what this state contains what it looks like we do not know that is something we will figure out later. So in general we can say the following that at t = 0 we have a set of basis states which can be labeled as the following p 1, p 2, p n. Where and we will take different values and we have put a subscript out and these basis states should form a complete set right because any final state we should be able to write as a linear sum of these basis states.

So you multiply with appropriate coefficients but once you have taken all the basis states whatever final state you get can be expressed in terms of these basis states, so that they should form a complete set. So we have found one set of states which we call out states, and now let us look at the state at t = 0 and repeat the same thing which I have said for when you look at the thing at as time goes backwards so we want to see what happens at as time goes backwards.

So till now the experiment I have been describing had only 2 particles an electron and a positron in the initial state. But that is not necessary you could imagine 10 particles coming together and colliding right or 20 particles coming together and colliding we could have you could collide 5 electrons and 17 positrons together. So that is another one thing to keep in mind it is not just 2 particles in the initial state. So all these particles would come and interact and create a state at t = 0 and we start from that state at t = 0 and let us look let us move backwards in time and see what happens?

So whatever state you have at t = 0 when you go backwards in time, it will correspond to some state it will give you some state which contains definite number of particles with those definite momenta and definite location. So repeating the same arguments, exactly the same arguments we can argue that again at time t = 0 we get another set of basis states which we can label as q 1 that is the momentum of one particle momentum of another particle and so forth.

I am putting m which is distinct from n because the number of particles in the initial state and the final state could be distinct and I say and I also put a label n. So you could the state at t = 0 you could write as a linear combination of these in states, so these are called in states. This is all standard terminology out states. So you could express the state at t = 0 in terms of the in-states and also in terms of out-states.

And of course these states are these basis states are not belonging to different Hilbert spaces they belong to the same Hilbert space and you are talking about the same system at equal to 0 good. So now that I have at least given a give I have at least labeled the in and out states even though I have not provided any description. Now I can ask my question again how do I what is the probability of some initial state at in far past or t equal to minus infinity to appear as some other final state in far future at t going to plus infinity what is that probability?

Now that answer is contained in the following if you could answer what is the probability of a given in-state or the probability amplitude of a given in-state to appear as a given out state. Then your job is done because once you have this information you can answer the question what is the probability amplitude of an initial state in the far pass to appear as a final state in the far future.

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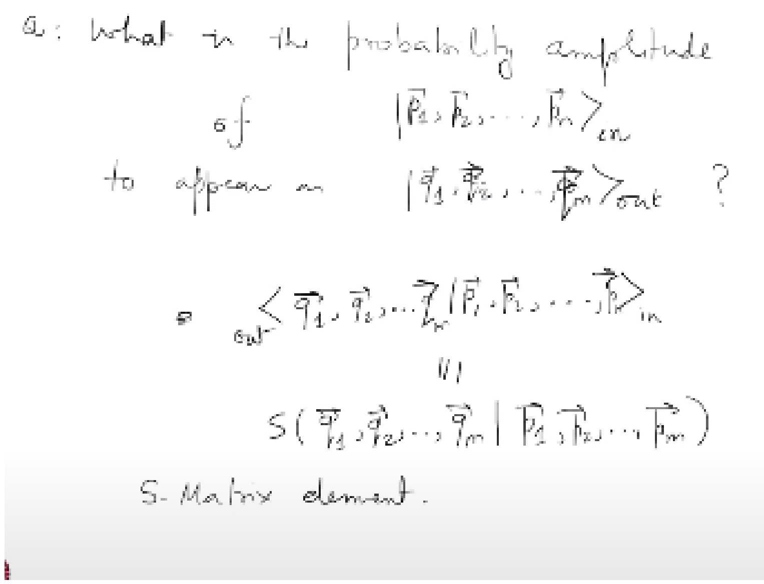

So our question about those probability; probabilities of the events in the collider can be now cast into the following question. And the question is this; what is the probability amplitude that this in state let us call it p 1, p 2, p n of this to appear? As I am not being very careful with whether I have used p 1 for the in-states or out state does not matter these are labels this p m. Or more precisely what I want to know is this following object so I start with this in-state and I want to know this object so this should be q out state.

So that is the object we want to calculate and this is called the asymmetric element. Which is you we can write it as the following. So this is what we want to know and this I will define as q 1, q 2, upto q m, p 1, p 2, up to p m. So this is the asymmetric element that is the definition of asymmetric element and that is what we want to calculate.

I hope it is clear why we want to calculate such an object because once you know about this object you can just easily calculate what will be the quality amplitude of an initial state appearing as a final state. Because apart from these matrix elements there is nothing else right other than is just the how you create wave packets those functions that you multiply in front of these in-states and out states the basis states.

So the question is clear and this is what we are going to spend a lot of time with and we will try to derive an expression for the s matrix. And the next goal would be to after that would be to find a formula for a cross section where all this information will go in so that you can come out with a prediction for experiments and you can measure there. So we will stop here and we will talk more about these things in the next video.