Introduction to Quantum Field Theory

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Lecture 7 : Quantization of Klein-Gordon Theory

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1 Klein-Gordon Theory

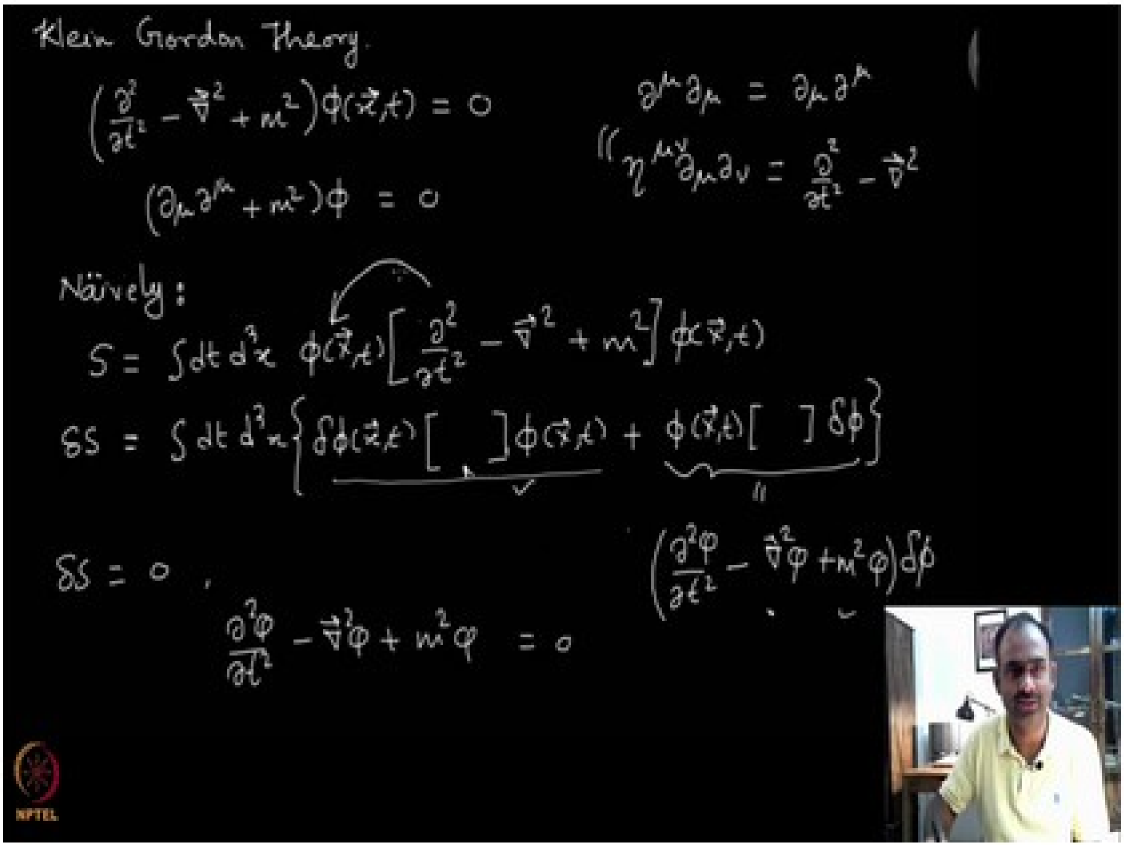


Figure 1: Refer Slide Time: 00:16

Last time we were looking at Real Scalar Field we were looking at Klein-Gordon theory. And we wrote down the equations of motion. So, the equation of motion for this theory is that is the operator you have. And let me specify the fields you have Phi xt is equal to 0 is the equation of motion which we can also write as the following. So, to make the Lorentz invariance explicit, it is explicit here but notationally more clearer.

I can write this as the following + m square phi is equal to 0 where del mu del mu, this is the following so del mu is same as I believe this will be familiar to most of the students and which is same as So as far as the equations of motion is concerned changing the sign of s will not change anything so that we can do. So all we have to do is put a minus sign here that minus sign will remove this minus sign and of course it will change other signs and then everything will be fine here your kinetic term will be positive So that is one thing so I will now correct for the minus sign so I will put a minus sign. And the second thing which I want to do is instead of having Q dot square I want to have half Q dot square canonical normalisation you have for kinetic terms I will have half Q dot square. That is the following. You have del mu del nu eta nu these are all same and which is if you expand it is del 2 over del t square - radiant square that it can easily see. When you are writing it like this. It is very apparent that the operator here you see the operator del mu del nu + m square that operator is a Lorentz scalar. Where you see term has in this is which are fully contracted and if you have all the indices contracted that object can only be a scalar m square is a scalar because it has no indices Lorentz quantity. So, scalar and Lorentz invariant object they are the same thing. And phi here is assuming to be a scalar field that is the field we have. So, these are either this way or that way these are the equations of motion.

Now like we did for the case of Schrodinger theory. We would like to write down the action which will give Klein-Gordon equations of motion. So we want to construct that action. And let us follow that procedure which we adopted last time. So what we do is; first I am going to but very naively without paying much attention. Of course, I am going to make a mistake with I will tell you what that mistake is.

So let me write down the action to be the following. So I take the action to be dt d cube x then phi xt taking the theory to be that of real scalar fields. And if you remember here, I will put down put the basically the left hand side of the equation of motion, so this entire piece. So that when I take a variation with respect to phi I will get this equal to 0. This is exactly the way we did before. Of course I put here.

This entire thing acting on phi, so now let us look at a variation of s and that is dt d cube x so I varied this first. So this field is varied. Then we have this operator. Let me just write a bracket. It is understood that I have to put that operator here plus this field phi and then we this operator again and then delta phi. So the operator now acts on delta phi. So here this is good. But in this case operator is acting on delta Phi.

So you have to derivatives of time which are acting at phi and 2 derivatives of space which are acting on phi, but I can use it integration by parts and pull out one after another the derivatives and put on this phi. I can transfer the derivatives to this one, so let me do that. This, part will anyway nothing has to be done for this piece. Let us look at this one. This will become; because we have 2 derivatives; every time you put a derivative get a minus sign twice you get -1 times -1 which is +1 and you will get delta 2 phi over delta t square.

So this derivative has now been transferred to this one minus again the same thing this gets transferred to that so we have plus m square phi square m square phi is no problem. As a constant so it is ok and then this gets freed up. So, you see this and that this term and this term are identical now. So, if you put delta is equal to 0 then this will work only if the term here and the term there they are identical anyway so you can add them up.

Unless this vanishes identically this cannot be satisfied delta s is equal to 0 cannot be satisfied which means that you have So as far as the equations of motion is concerned changing the sign of s will not change anything so that we can do. So all we have to do is put a minus sign here that minus sign will remove this minus sign and of course it will change other signs and then everything will be fine here your kinetic term will be positive So that is one thing so I will now correct for the minus sign so I will put a minus sign. And the second thing which I want to do is instead of having Q dot square I want to have half Q dot square canonical normalisation you have for kinetic terms I will have half Q dot square. That is the equation of motion which is what you had here. But there is something not so good here and let us look at that part. So here is phi and the Klein-Gordon operator in the PHI. Now what I will do is I will take only one of the derivatives and put on this phi. So, can do that and when I do so you have when this goes here to get del phi over del t and this one you still have one more derivative that acts on this phi we have still del phi over del t. So, del phi over del t times del phi over del t because you transferred 1 derivative here using integration by parts. You get a minus sign.

So to get here is minus del phi over del t whole square. So let me write down the thing; so let us look at only this piece. This first time will not worry about species right now.

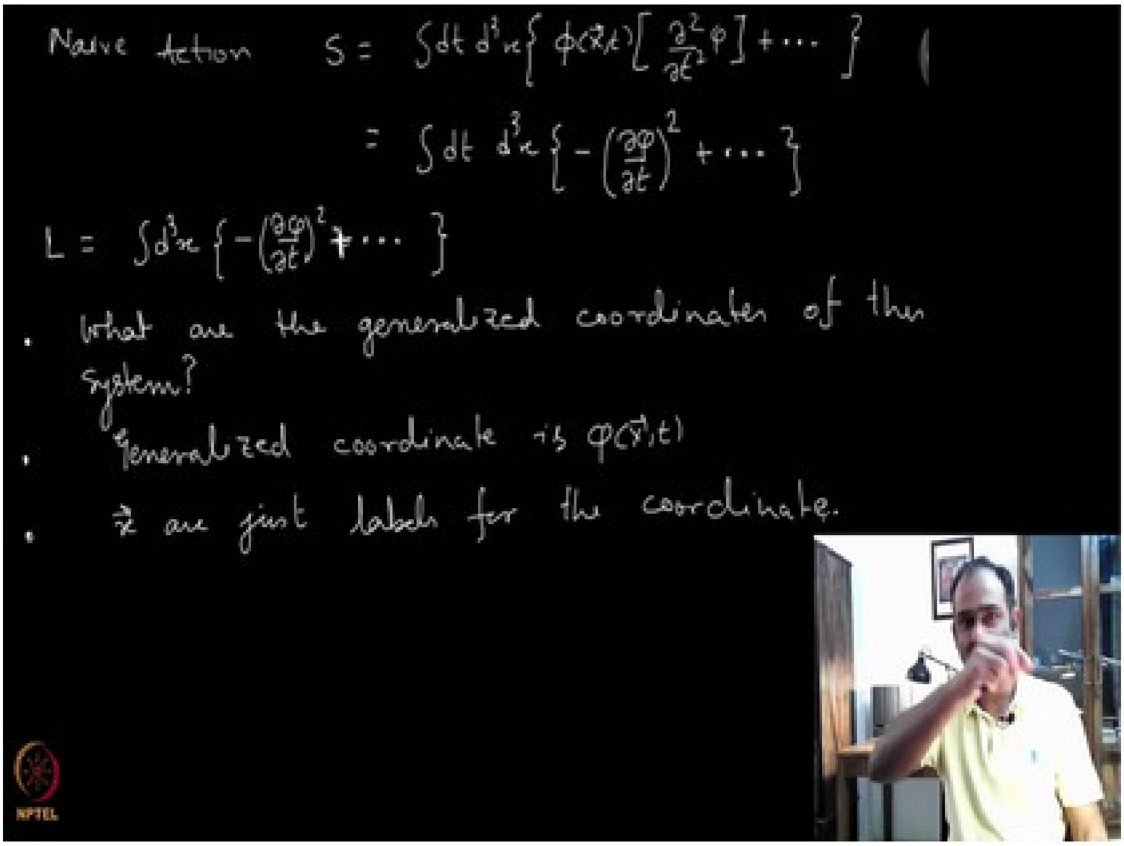
$$\left(\frac{\partial^2}{\partial t} - \vec{\nabla}^2 + m^2\right)\phi(\vec{x},t) = 0 \qquad (1)$$

$$(\partial_\mu \partial^\mu + m^2)\phi(\vec{x},t) = 0 \qquad (2)$$

where,

$$\partial_\mu \partial^\mu = \partial^\mu \partial_\mu \qquad (3)$$

$$\eta^{\mu\nu}\partial_\mu \partial_\nu = \frac{\partial^2}{\partial t^2} - \vec{\nabla}^2 \qquad (4)$$



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Naively,

$$S = \int dt\, d^3x \phi(\vec{x},t)\left[\frac{\partial^2}{\partial t} - \vec{\nabla}^2 + m^2\right]\phi(\vec{x},t) \qquad (5)$$

$$\delta S = \int dt\, d^3x \Big\{ \delta\phi(\vec{x},t) \Big[\frac{\partial^2}{\partial t} - \vec{\nabla}^2 + m^2\Big]\phi(\vec{x},t) + \phi(\vec{x},t)\Big[\frac{\partial^2}{\partial t} - \vec{\nabla}^2 + m^2\Big]\delta\phi(\vec{x},t)\Big\} \tag{6}$$

$$\begin{aligned}\delta S = & \int dt\, d^3x \Big\{ \delta\phi(\vec{x},t) \Big[\frac{\partial^2}{\partial t} - \vec{\nabla}^2 + m^2\Big]\phi(\vec{x},t) \\ & + \Big[\frac{\partial^2\phi(\vec{x},t)}{\partial t^2} - \vec{\nabla}^2\phi(\vec{x},t) + m^2\phi(\vec{x},t)\Big]\delta\phi(\vec{x},t)\Big\}\end{aligned} \tag{7}$$

If we put $\delta S = 0$, we get equations of motion,

$$\frac{\partial^2\phi(\vec{x},t)}{\partial t^2} - \vec{\nabla}^2\phi(\vec{x},t) + m^2\phi(\vec{x},t) = 0 \tag{8}$$

Naive action

$$S = \int dt\, d^3x \Big\{ \phi(\vec{x},t)\Big[\frac{\partial^2\phi(\vec{x},t)}{\partial t^2} - \vec{\nabla}^2\phi(\vec{x},t) + m^2\phi(\vec{x},t)\Big]\Big\} \tag{9}$$

Transferring the derivative and focussing on the kinetic term we can write,

$$S = \int dt\, d^3x \Big\{ -\Big(\frac{\partial\phi(\vec{x},t)}{\partial t}\Big)^2 + ...\Big\} \tag{10}$$

$$L = \int d^3x \Big\{ -\Big(\frac{\partial\phi(\vec{x},t)}{\partial t}\Big)^2 + ...\Big\} \tag{11}$$

So if I take my naïve action which is h plus other terms which have m square term and the gradient square term hat I am omitting for now. Now if you do that what I said in the our show in the previous slide transferring this derivative here you will get dt d cube x minus It goes one derivative is post here. So, it becomes del phi over del t and you still have one derivative left with it becomes this and again then of course you have other terms.

Now this might you might have already noticed what the problem is. We have a minus sign in the action. So if you are looking at the Lagrangian with me right in the more familiar setting. So, the Lagrangian is for the system d cube x - del phi over del whole square another terms which are really the potential terms. This is plus. Now this is a kinetic term, Lagrangian is kinetic energy minus potential energy and kinetic energy has to be positive. This is a square. So this is always positive but if the coefficient is negative, then the kinetic energy becomes negative, which means this is not good.

It is clear from what I have said, but let me nevertheless show it is slightly more detail what I am saying. So for that what I will do it is I will go from this continuum of space to discrete space. So let us say; so, let us ask what are the generalized coordinates of the system. So, what are the generalized coordinates? One thing is clear they are definitely not x. So, this x is not coordinate at all. To describe a system you need to specify phi. Once you specify phi over all the space then you have specified the configuration of the system. It is not the x, x is not at all a coordinate. So, your dynamical variable or the generalized coordinates is phi. So, your generalized coordinates is phi and x is just label x is labels for the coordinate. So let me make this further clearer by doing the following. So, instead of taking x to be the continuum space.

Negative kinetic energy

- What are the generalized coordinate of the system?

- $\phi(\vec{x}, t)$ is generalized coordinate
- $\vec{x}$ are just the label for coordinates

Let us take the system and put it on a lattice. So what I do is I will discretised the space. So instead of x taking continuous values here and there and there. We will take x will make; x to take discrete values. I put it on a lattice and then I let us will look at the same Lagrangian again. It will be easier to understand what is being said. So I will discretise space. So what I am going to o is instead of taking this as a space I say that I have discrete space. So this is two dimensions and you can imagine third dimension and then the squares becomes cubes. And any point if you take here this will have label i 1, i 2, i 3 so which for example here you mean you go away i 1 number of steps in that direction, i 2 number of steps in that direction and i 3 number of steps in the z direction. And let me take the length of each of these to be h in all three directions. Then if I am looking at this point I can think of a cube of side h around this point. So this point is in the centre of the cube. So that we will remember us, that there is a cube here. Now you look at phi xt it becomes when you are discretize the space. So, phi xt means the field at point x and that will become field phi at point x and the point x is now level as i 1, i 2, i 3 and of course it is a function of time.

Now I want to write down this Lagrangian in this discrete space. So, d cube x so you take a small volume element and integrating over it which means you are basically summing over all your small volume elements. In each volume element d cube x will turn into h cube small volume element we have and we are taking h to be very, very small, h is very small. It is so small that the field phi is effectively a constant within that cell so we can ignore any variations of phi which happens within the volume h cube.

So what will integral d cube x look like it will of course become h cube and you have to sum over all the lattice points which is i 1, i 2, i 3 so that is what the integral d cube x would look like in this discrete space and then you have a del phi over del t square. What is that becomes? It becomes phi i 1, i 2, i 3 that is what your point x is I should have it here x goes to i 1, i 2, i 3 and as I said this is function of time del over del t square.

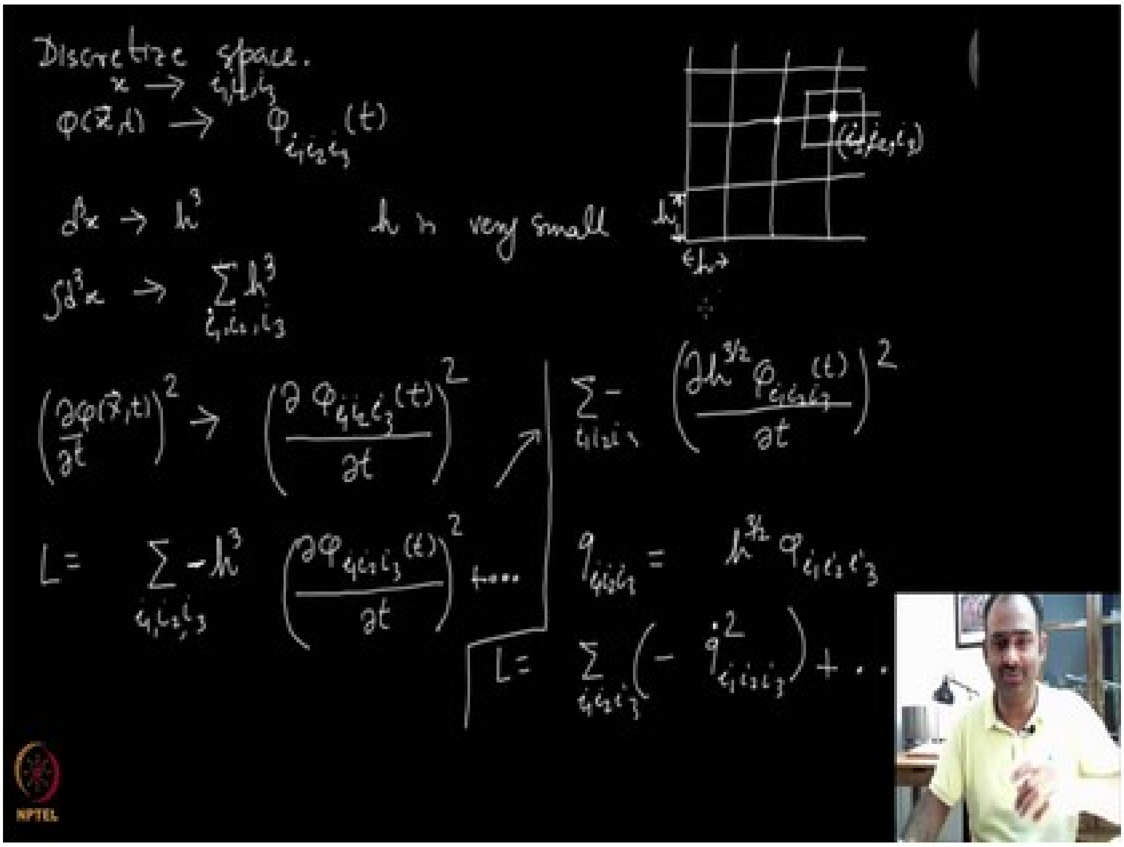
So see your Lagrangian is now summation over i 1, i 2 and i 3 h cube and then delta phi square I know there was a minus sign. Remember that that minus sign here. Now this is; so now this h cube I can put in here combined with the phi. So let me write that also, so that will be summation minus h power 3 halves. So see it looks like now if you regard the Q i 1, i 2, i 3, so that is what I am defining and defining Q as the coordinate which is h 3 half of phi i 1, i 2, i 3 then my Lagrangian is At least the first term of the Lagrangian is say to sum over all the coordinates minus Q dot square.

So that is what I was saying that the kinetic term here has a negative sign which should not happen when you see what are So as far as the equations of motion is concerned changing the sign of s will not change anything so that we can do. So all we have to do is put a minus sign here that minus sign will remove this minus sign and of course it will change other signs and then everything will be fine here your kinetic term will be positive So that is one thing so I will now correct for the minus sign so I will put a minus sign. And the second thing which I want to do is instead of having Q dot square I want to have half Q dot square canonical normalisation you have for kinetic terms I will have half Q dot square. That is the coordinates? Coordinates are the Q's, Q's are just h 3 half times phi so really the phi which is the coordinate h 3 half is the and overall factor you are multiplying is constant that you are multiplying and of course you have to sum over all the coordinates. This way it is very clear that i 1, i 2 and i 3 are just the labels for the coordinates.

They are not the coordinates themselves i 1, i 2, i 3 when you go to continuum they become x and x are the labels and the real fields are the real coordinates are the phi and this is what you waiting now and of course we have other terms now. Now the mistake that we have done is we took the action to be this thing here. But if I take instead of s minus as the action you still get the same equations of motion.

Discretizing space,

$$\begin{aligned} \vec{x} &\rightarrow i_1 i_2 i_3 \\ \phi(\vec{x}, t) &\rightarrow \phi_{i_1 i_2 i_3}(t) \\ d^3x &\rightarrow h^3 \\ \int d^3x &\rightarrow \sum_{i_1 i_2 i_3} h^3 \\ \left(\frac{\partial \phi(\vec{x}, t)}{\partial t}\right)^2 &\rightarrow \left(\frac{\partial \phi_{i_1 i_2 i_3}(t)}{\partial t}\right)^2 \end{aligned} \tag{12}$$



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The lagrangian in discretized space will be

$$L = \sum_{i_1 i_2 i_3} -h^3 \left(\frac{\partial \phi_{i_1 i_2 i_3}(t)}{\partial t}\right)^2 \tag{13}$$

$$L = \sum_{i_1 i_2 i_3} -\left(\frac{\partial h^{3/2} \phi_{i_1 i_2 i_3}(t)}{\partial t}\right)^2 \tag{14}$$

If we define

$$q_{i_1 i_2 i_3} \;=\; h^{3/2}\phi_{i_1 i_2 i_3}(t) \tag{15}$$

The lagrangian will become

$$L \;=\; \sum_{i_1 i_2 i_3}\Big(-\dot{q}^2_{i_1 i_2 i_3}\Big) + \ldots \tag{16}$$

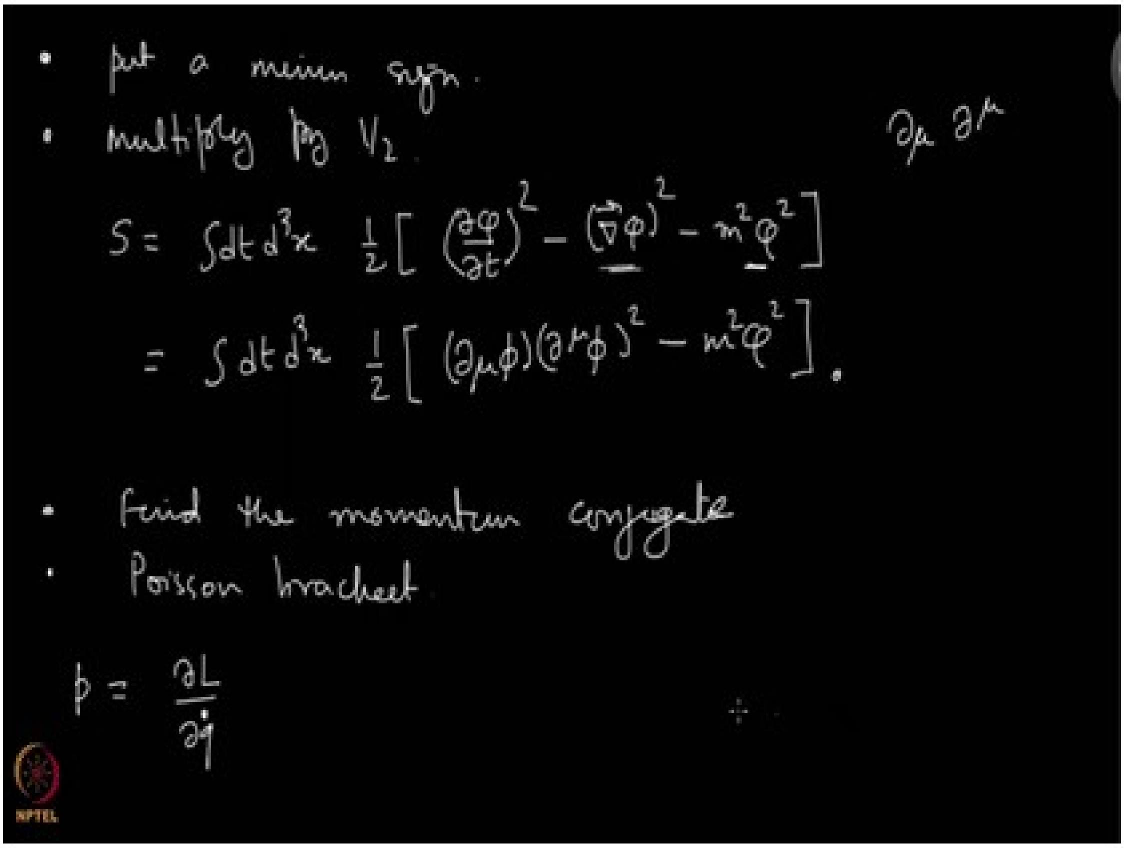
- Put an extra minus sign in action
- Multiply by 1/2

The action after these steps

$$S \;=\; \int dt\, d^3x \Big[\frac{1}{2}\Big(\frac{\partial\phi(\vec{x},t)}{\partial t}\Big)^2 - \frac{1}{2}(\vec{\nabla}\phi(\vec{x},t))^2 - \frac{1}{2}m^2\phi(\vec{x},t)\Big] \tag{17}$$

$$S \;=\; \int dt\, d^3x \Big[\frac{1}{2}\partial_\mu\phi(\vec{x},t)\partial^\mu\phi(\vec{x},t) - \frac{1}{2}m^2\phi(\vec{x},t)\Big] \tag{18}$$

1.1 Conjugate momentum



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So as far as the equations of motion is concerned changing the sign of s will not change anything so that we can do. So all we have to do is put a minus sign here that minus sign will remove this minus sign and of course it will change other signs and then everything will be fine here your kinetic term will be positive So that is one thing so I will now correct for the minus sign so I will put a minus sign. And the second thing which I want to do is instead of having Q dot square I want to have half Q dot square canonical normalisation you have for kinetic terms I will have half Q dot square. That is also easy you can multiply your action with half a constant that does not change the equation of motion. So, equations of motion are not changed.

So I will take my; this expression which I wrote earlier and multiply with minus half and then everything is fixed. So I get dt d cube x half del phi over del t squared it had a minus sign it had gone and minus gradient of phi whole square -m square phi square. If you never worry about the relative science, you can easily remember that the action has to be Lorentz invariant and unless you have a minus sign here. You will not have a Lorentz scalar quantity.

Because you have del 2 over del t square del over del t and gradient. So if you look at the del mu this or even better, let me write it simply this way del mu this when you contact with that. That is going to give you a raise to minus time this one. Let me also write it in a slightly different form not different but using the notation of Lorentz indices it becomes del mu phi this is what I was saying above, del mu phi square phi square.

Because that is the term which belongs to a potential, potential term Kinetic term is in here one of the terms is the kinetic term and these all are potential terms so they come with their relative minus sign. So that is the action that we have now and this is what we will work with. Now we have the action with in front of us. We know what the coordinates are. Now what we should do if we should find out what their momentum conjugate; momentum conjugate to the phi or more appropriately h 3 half times phi because I am going to use discrete space to find the conjugate Momentum.

So we will find out the conjugate momentum and I think I want to also; and then we should you should find out what the Poisson bracket would be of the conjugate Momentum which were going to find. Because then you can quantize by simply replacing the Poisson bracket by the commutator and putting the ih bar factors at the right place that is what we want to do. We want to find momentum conjugates and then we want to find the Poisson brackets. So as far as finding the conjugate Momentum is concern you need only the Kinetic piece kinetic term because other terms do not matter. I hope we have talked about this already, but if you have q s your coordinate then taking a derivative with respect to Q dot is what gives you the corresponding moment so this what we want to do and that is easy. So I am going to discrete space my q is this and Lagrangian was the plus sign here and half was this. Let us write L for my Klein-Gordon theory half summation over i 1, i 2, i 3, q i 1, i 2, i 3 dot square. Let us calculate del over del q dot let me simplify the notation a bit instead of putting all the i 1, i 2 , i 3 every time let me put let me define i to be this, this pair is i 1, i 2, i 3 vector i so here I can simply write this what would that be? So here this becomes q i dot that is at the wrong place q i dot summation I should use not i but something else so j half q j dot square which is half summation over j 2 q j dot del q j dot over del q i dot.

This is simple it is half summation over j 2 q j dot in this will be a delta function that unless i and j are same this is 0 and only when I and j are equal this becomes 1 that is delta ij and the summation will hit only when i is equal to j. So half get cancelled and you get q i dot. Now what I could have done already or I can still do I had q i to be h 3 half of phi. Let me define the conjugate momentum p i so here this is p i, i because we have label i here so p i to be h 3 half so this is just defining a quantity Pi.

Only kinetic term will contribute in the expression of conjugate momentum, thus

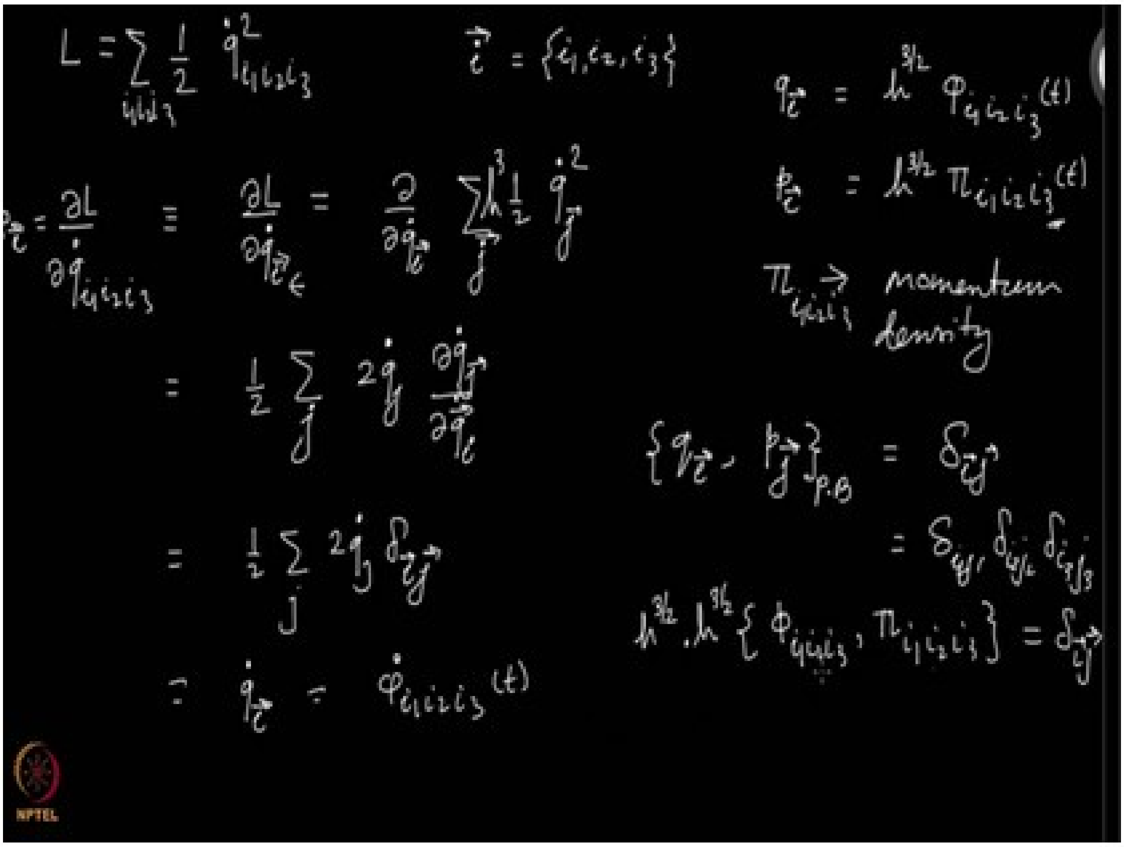
$$L = \sum_{i_1 i_2 i_3} \frac{1}{2} \dot{q}^2_{i_1 i_2 i_3} \tag{19}$$

We define $\vec{i} = \{i_1, i_2, i_3\}$

$$\frac{\partial L}{\partial \dot{q}_{\vec{i}}} = \frac{\partial}{\partial \dot{q}_{\vec{i}}} \sum_{\vec{j}} \frac{1}{2} \dot{q}^2_{\vec{j}} \tag{20}$$

$$= \frac{1}{2} \sum_{\vec{j}} 2 \dot{q}_{\vec{j}} \frac{\partial \dot{q}_{\vec{j}}}{\partial \dot{q}_{\vec{i}}} \tag{21}$$

$$= \frac{1}{2} \sum_{\vec{j}} 2 \dot{q}_{\vec{j}} \delta_{\vec{i}\vec{j}} \tag{22}$$



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Where

$$\dot{q}_{\vec{i}} = \dot{\phi}_{i_1 i_2 i_3}(t) \tag{23}$$

and

$$q_{\vec{i}} = h^{3/2} \phi_{i_1 i_2 i_3}(t) \tag{24}$$

$$p_i = h^{3/2} \Pi_{i_1 i_2 i_3}(t) \tag{25}$$

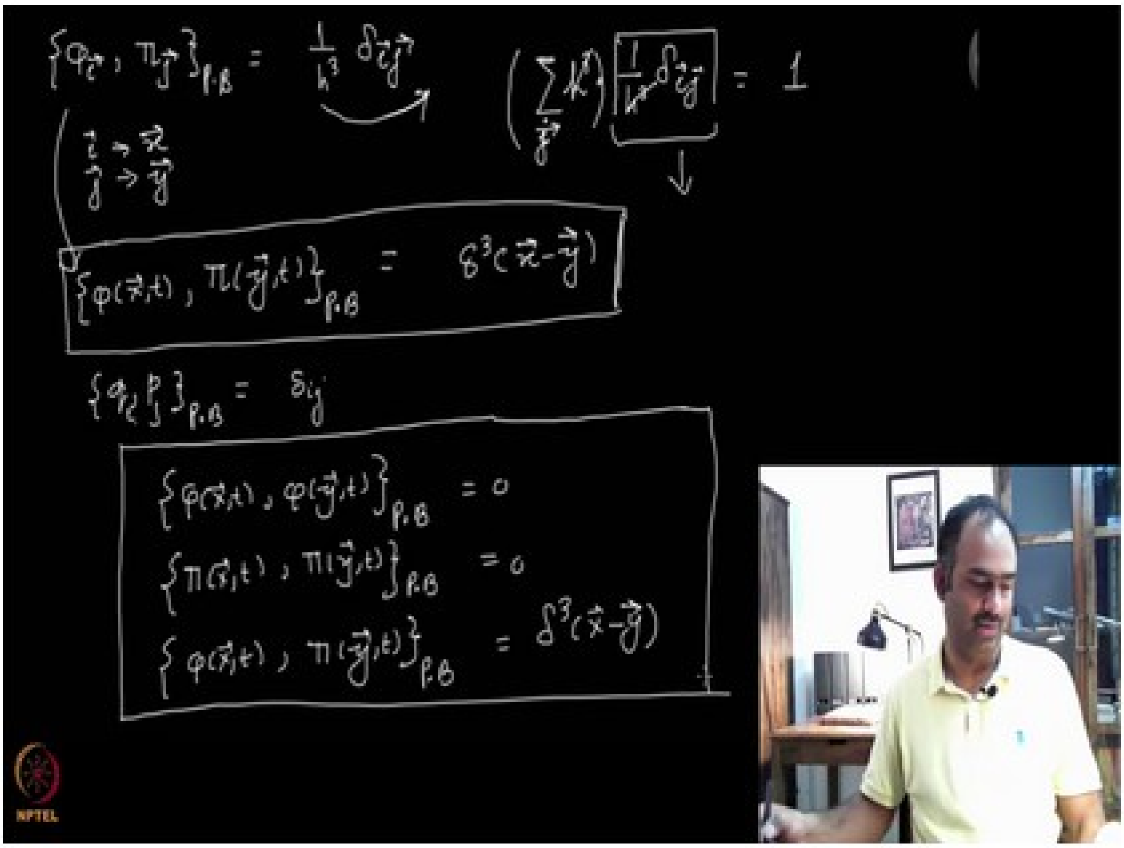
So just like your coordinate q is phi times something some constant and conjugate momentum is also other than this factor Pi. Pi is usually called momentum density. It is called momentum density, because this is a momentum and you are dividing by the volume of the cube. So it is really the momentum density. So here in this case you have i 1, i 2, i 3 dot. So that is good that some notation I have introduced. We have found out what the conjugate Momentum is and this turns out to be phi dot good so I can find out now something is missing. Yeah there is something that is missing is here.

I should have a h cube here that is missing you do not miss on the previous one. That is why here it should have h cube, where else here also. That is why it is here, this has been already define so it is fine here. And all is good, because our q and p are canonically conjugate pairs. If you calculate the Poisson bracket, you will get the following. If I take a q i and then this p j and this Poisson bracket the fundamental bracket will be how much delta ij.

Which is just a shorthand notation for the following it is i 1, j 1 delta i 2 j 2 delta i 3 j 3 that is what delta ij. Now this thing I can substitute, what is it this place q i h 3 half phi so I can put here instead of q i will have 3 half and p j, I can write h 3 half pi so that will again bring a factor of h 2 half. So it will be h 3 half and then you have q phi i 1, i 2, i 3 with pi i 1, i 2, i 3 and Poisson bracket will be delta ij or you can divide volume on that side and write the following. I will write on the next sheet. So I have delta ij I have to divide by the volume and on the left Poisson bracket of phi and pi.

$\Pi_{i_1 i_2 i_3}(t)$ is the momentum density.

1.2 Poisson Brackets



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So phi I should have been just writing phi ij simpler to write this way Poisson bracket is 1 over h cube delta ij. Now if I go to the continuum limit from discrete to back to our original space and i corresponds to x And j corresponds to y so when I am going back to the continuum limit. This will become phi of xt that will become pi of yt and I should figure out what the right hand side becomes.

What does it goes to this one, now it is not very difficult to realise what it is. So, if x and y are different meaning i and j are different than it is 0. So unless x is equal to y this quantity here is 0. And when x is equal to y then it has a value 1 over hq. So let us see what it becomes in the limit h going to 0. That is the limit of going to continuum. So that is easy, I let us take 1 over hq delta ij look at the all, as some over all i or j let us say j does not matter hq times this.

So, this is summing over all the cells. Each cell has a volume hq when you are summing over all of them. So when you summing over all of them this quantity what do you get? You get the following hq cancels and delta ij will hit for only when j is i and in that case you will get 1. So, this quantity which you have here and this box is something which is 0 unless x and y are same. And it integrates over I am summing over all the cells it integrates over to 1 which is what delta function. They are all delta function.

Clearly this quantity goes over to delta cube x – y. So we have found the poison bracket of phi with its canonical conjugate momentum density, and it comes out to be dealt cube xy and if you compare with what qp for any conjugate pairs, they are if we say i and j then they would be delta ij see this is what you have here is a generalization of finite number of variables or are coordinates. So, here in the continuum generalise to this one that is good. Let us see what else I want to say.

I will give you rather trivial exercises actually it is nothing to be done. So let me just list on here. So I found this Poisson bracket and obviously has the following also so if you have phi xt with phi yt this Poisson bracket is 0 pernicious if you have pi xt, pi yt that Poisson bracket is also 0 and of course we have our third one phi xt, phi yt Poisson bracket is delta cube x - y that is good. Now I can if I wish to quantise all I have to do is to promote the phi's is to operator and pi's to operator and replace the Poisson brackets by commutators.

And the commutator for this one would just become ih bar delta cube because you to insert ih bar delta cube So that is what you would do to quantise I will replace the Poisson brackets by commutators. So that I would get let me instead of phi i let me use phi hat for the operator Pi is Pi hat and the

$$\begin{aligned}\{q_{\vec{i}}, p_{\vec{j}}\}_{P.B} &= \delta_{\vec{i}\vec{j}} \\ &= \delta_{\vec{i}_1\vec{j}_1}\delta_{\vec{i}_2\vec{j}_2}\delta_{\vec{i}_3\vec{j}_3}\end{aligned} \tag{26}$$

$$h^{3/2}h^{3/2}\{\phi_{i_1i_2i_3}(t), \Pi_{j_1j_2j_3}(t)\}_{P.B} = \delta_{\vec{i}\vec{j}} \tag{27}$$

$$\{\phi_{\vec{i}}(t), \Pi_{\vec{j}}(t)\}_{P.B} = \frac{1}{h^3}\delta_{\vec{i}\vec{j}} \tag{28}$$

In the continuum limit

$$\begin{aligned}\vec{i} &\rightarrow \vec{x} \\ \vec{j} &\rightarrow \vec{y}\end{aligned} \tag{29}$$

$$\{\phi(\vec{x}, t), \Pi(\vec{y}, t)\}_{P.B} = \delta^3(\vec{x} - \vec{y}) \tag{30}$$

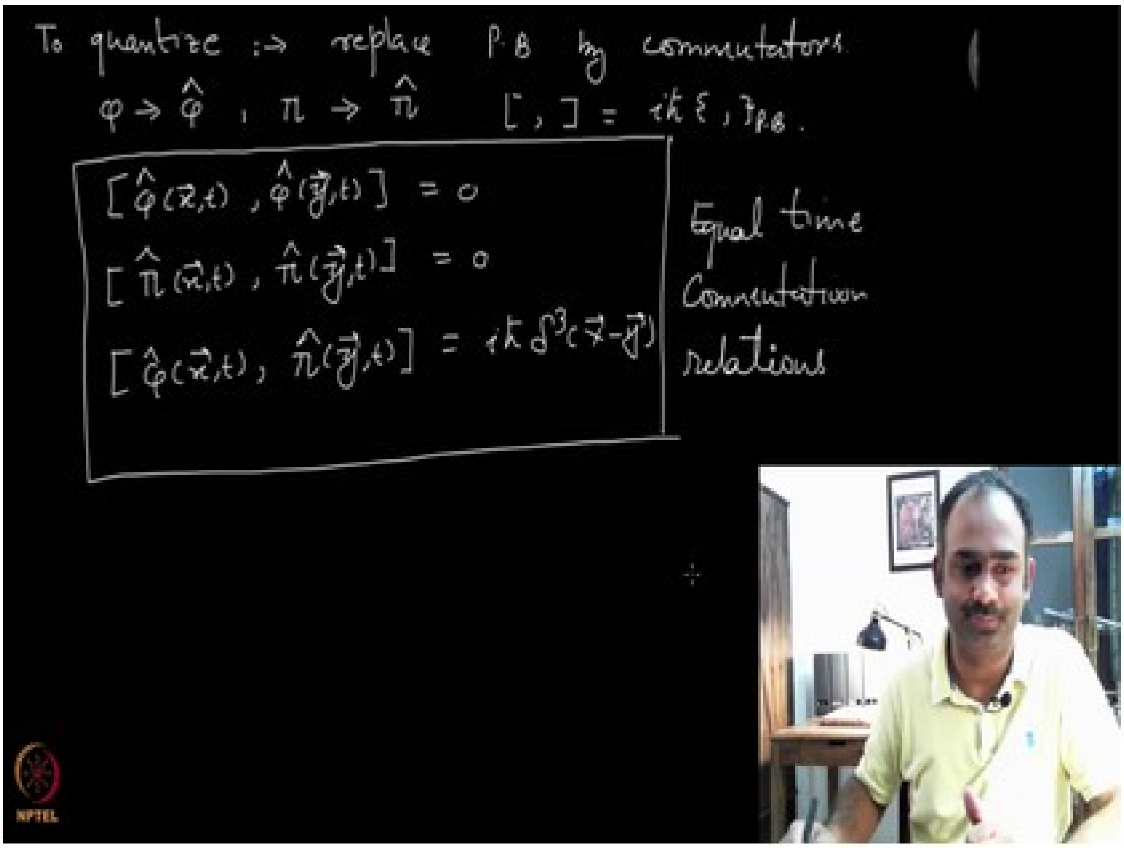
Similarly,

$$\{\phi(\vec{x},t),\phi(\vec{y},t)\}_{P.B} = 0 \tag{31}$$
$$\{\Pi(\vec{x},t),\Pi(\vec{y},t)\}_{P.B} = 0 \tag{32}$$
$$\{\phi(\vec{x},t),\Pi(\vec{y},t)\}_{P.B} = \delta^3(\vec{x}-\vec{y}) \tag{33}$$

To quantize

- Replace Poisson bracket with commutators
- $\phi \rightarrow \hat{\phi}$
- $\Pi \rightarrow \hat{\Pi}$
- $[,] \rightarrow \imath\hbar\{,\}_{P.B}$

1.3 Equal time commutation relations



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$$\begin{aligned}
&[\hat{\phi}(\vec{x},t),\hat{\phi}(\vec{y},t)] = 0 \\
&[\hat{\pi}(\vec{x},t),\hat{\Pi}(\vec{y},t)] = 0 \\
&[\hat{\phi}(\vec{x},t),\hat{\Pi}(\vec{y},t)] = \imath\hbar\delta^3(\vec{x}-\vec{y})
\end{aligned} \tag{34}$$

Poisson bracket will become the commutator will be ih bar times Poisson bracket. So that is the prescription so what will get his phi hat xt phi hat yt this commutator would be 0. Note that even though the space points are different.

These are different coordinates when you have x and y different means different coordinates and you are of course looking at equal time commutation relation now. Then you have phi xt phi yt that they also commute and phi xt and pi hat yt pi h bar x times hat you had for the Poisson bracket and that was that delt cube x – y. So these are our commutation relations. Let us put them in a bracket and box.

And because the time is same here in both think these are cal equal time commutation relations. That is good; we will stop here and we will continue further next time.