Introduction to Quantum Field Theory

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Lecture 51: Survey

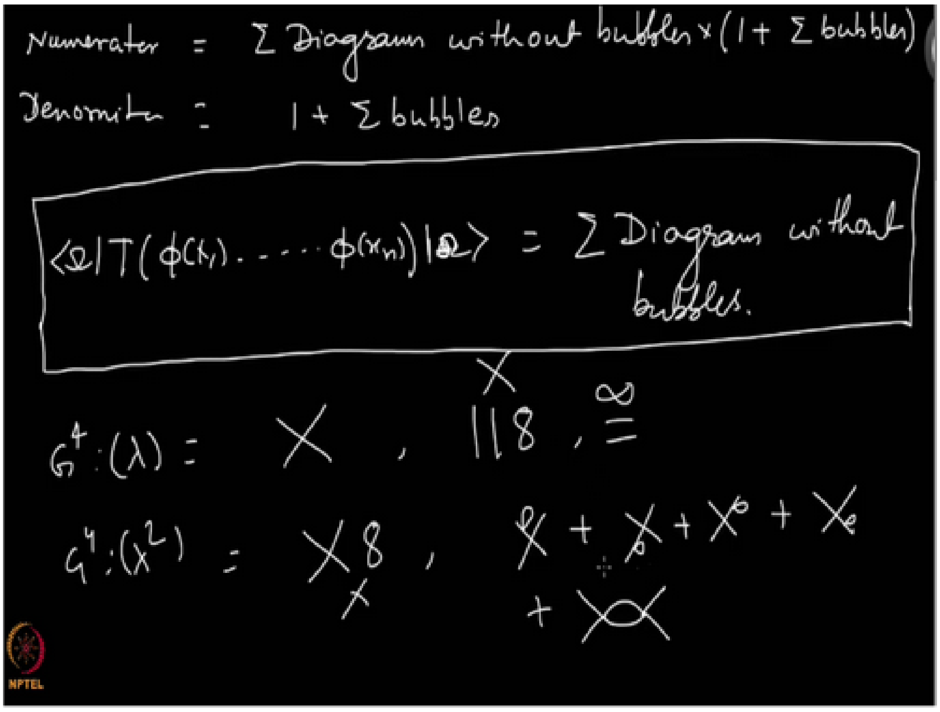


Figure 1: Refer Slide Time: 00:13

In this last 1 hour, we will do a survey of what we have studied and what things lay ahead and then we will close this course. First, I should remove this. So, in quantum field theory we have seen that the objects of interest to answer the questions that we can ask are the Green's functions and we have also been able to put them in the form of Feynman diagrams and we know the rules of calculating Feynman diagrams.

And you saw that there are diagrams which are containing loops and there are diagrams which do not contain loops. Maybe I should introduce a terminology. So, they are diagrams which are called tree level, tree diagrams. These are the diagrams we do not contain loops and then you have loop diagrams that you have seen already. For example, this is a loop diagram and formally we have written down the expressions of these objects. And to do a computation at let us say order lambda square.

You will need to calculate this diagram and other diagrams also and if you tried to calculate such diagrams you will have to do the following. So, let me draw a sample diagram. This is just representing what will happen in general. So, I will just make the point based on this diagram. So, here we saw that in addition to these propagators coming from external legs, we had an integral over the loop momentum that flows in here.

Let me label the momenta and this one will be p 1 + p 2 – l because this is what you get by using momentum conservation at this vertex and this will give you integral in the expression which we have seen before, it will give you an integral of this form apart from other factors and external propagators corresponding to these external lines you will be required to calculate this integral. Such integrals are called loop integrals.

And if you were looking at a diagram which was 2 loop diagrams, we have seen that will require doing 2 integrals. So, it will be some d 4 l 1, d 4 l 2 and a set of propagators over which you will have to do the integral. So, in principle that is what you have to do and get the answer but that is where things start becoming complicated, difficult and that is also what gives the quantum field theory or its structure because of these higher order calculations which involves such loop integrals.

So, let me show you one of the issues here first. So, let us say we want to do this integral and we are interested in the limit in which the loop momenta l loop momentum l is becoming large. So, I am interested in l becoming large. C l goes to infinity which we means basically that l 0, l 1, l 2 and l 3, they all become large. Because d 4 l is basically d l 0, d l 1, d l 2, d l 3.

So, we are right now, looking at what we call as ultra violet limit. It is called ultra violet limit because the momentum that is flowing here is very high and high momentum corresponds to ultra violet limit. Or low frequencies and low frequencies are violet or ultra violet that is what you said. So, we say we are interested in ultra violet limit and let us see how things behave in that limit.

I am not going to calculate this exactly. All I am interested in is the behavior of the subject in ultra violet limit which I have defined like this. So, this d 4 l which I will, I can turn to the following. So, one can go to Euclidean region where so, right now, we have, if I write l square that means l 0 square – l vector square but you can change variables and have everything with a plus sign. I will go to a new set of variables, where everything will be like this.

Numerator $= \sum$ Diagram without bubble $\times$ $(1 + \sum$ bubbles$)$
Denominator $= 1 + \sum$ bubbles

$$\langle\Omega|T\Big(\phi(x_1)\cdots\phi(x_n)\Big)|\Omega\rangle \tag{1}$$

$G^4 : (\lambda) =$

- Tree diagram
- Loop diagram

$$\int_{-\infty}^{\infty}\frac{d^4l}{(2\pi)^4}\,\frac{i}{l^2-m^2+i\epsilon}\,\frac{i}{(p_1+p_2-l)^2-m^2+i\epsilon} \tag{2}$$

Yeah, I am just sketching you, this is not difficult, you just have to do small change. But anyway, so, in the Euclidean space, your, this integral measure will become d 4 l E. All you have to do is change the sign and that is easy, you multiply a factor of i and that is how you get this. And you will have i over l e square. So, I am saying I am interested in large l limit. So, I can ignore the m square which is some fixed number.

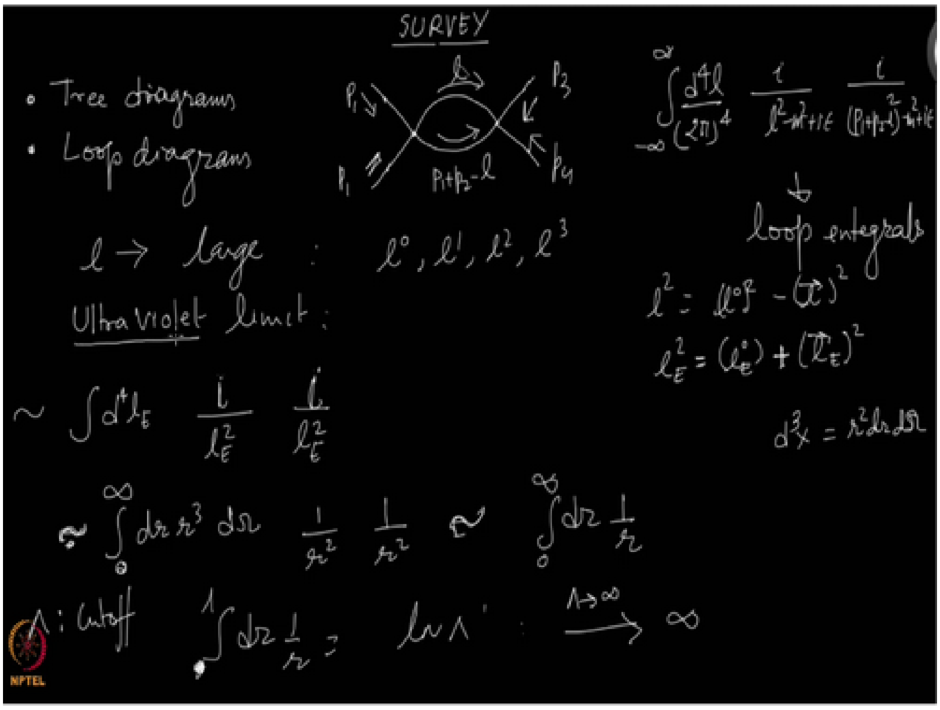


Figure 2: Refer Slide Time: 01:21

And because l is very large, I can drop m square in comparison to that l square or l e square. And here similarly, I can drop what p 1 and p 2 are. See p 1 and p 2 are fixed, they are provided to you because you took the Fourier transform with some p 1 into p 1 and p 2 specified by you. So that there are some fixed quantities and now, l is going very large. So, you can drop p 1 and p 2 in compression to l n, it will be of this form.

In this factor of i which we can forget about i, you can put 1, I am just interested in that limit. I am not evaluating it exactly. So that is good. Now, this is d 4 l e. So, you know that if you were looking at d cube x, you could write it as r square d r d omega, d omega will be taking care of the angular part of the measure. And this is the radial part, and you can also see that dimensionally, it is correct.

Because this is 3 powers of length on the right hand side also you should have 3 powers of length which is here r squared 2 d r 1, so that makes 3 and d omega is anyway dimensionless because that is related to angles. So, similarly, when you are doing a decomposition of d 4 l e into radial angular coordinates, you will have r square sorry, r cube d r or let me write as d r r cube.

So that makes 4 powers of length and then you will have your angular integrals. And then this one will be 1 over r square 1 over r square that is how it will go and your integration will be from 0 to infinity. The radius you will integrate from 0 to infinity. So, let us look at this angular integrals are not going to give you an in trouble. So, as far as u v is concerned, they just sit outside do not do much. So this one is basically d r and you have r cube and r 4 in the denominator, so, you have 1 over r, 0 to infinity. Instead of putting 0 to infinity and seeing that this is divergent, I will just do 1 thing, I will say 0 to lambda and then I will take lambda to infinity. Which is so, all I am saying is I am cutting off the integral at some very high value lambda. So, lambda is some cut off I introduced and of course, I should take lambda to infinity.

Eventually but for a moment, let us keep it like this and this gives you log of lambda. And I am not interested in the lower limit because this entire expression has been written in the limit where l is large. So, lower limit is not something which you should take seriously here. So, let us

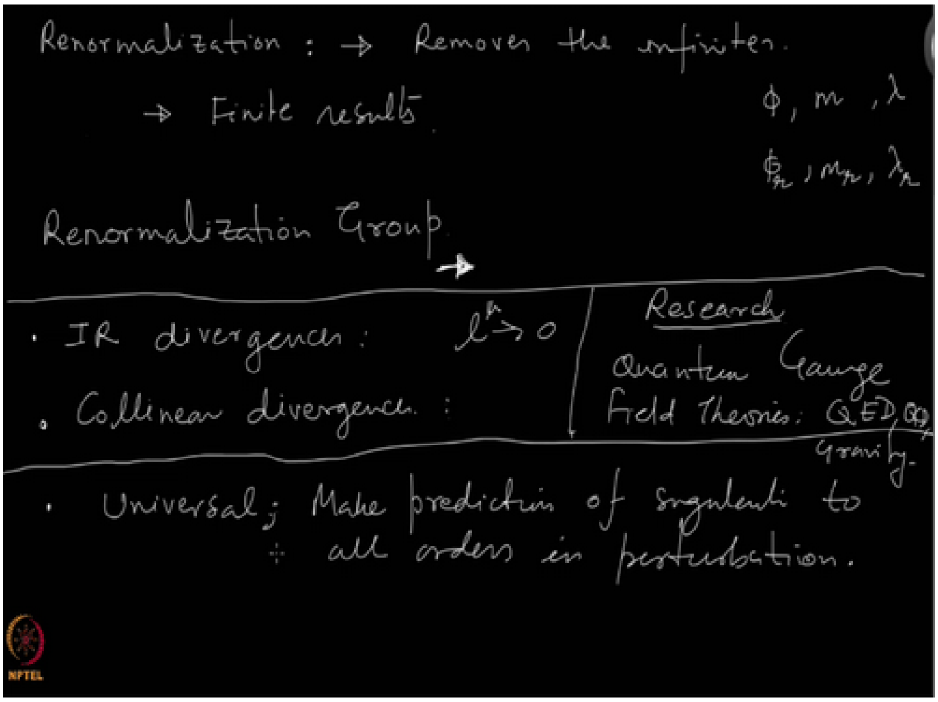


Figure 3: Refer Slide Time: 12:25

leave that part out to avoid any confusion because this is only true in large l limit.

So, you see that if you were to evaluate this integral by putting l cut-off lambda, you will get log of lambda and then when you take lambda going to infinity, this diverges. So, we had heavily written down expressions of Green's functions to a higher orders in perturbation theory and we were believing that now, we will have very precise calculations I mean precise up to the order in which precise to the level we want because I could go to whatever order in perturbation theory I want.

And write down an expression for Green's function. But now, you see that this is badly divergent object. And it makes no sense to do this integral because the divergent integral, or more precisely, ultra violet divergent integral. But fortunately not, it is not a lost game.

$$l^2 = (l^0)^2 - (\vec{l})^2 \tag{3}$$

$$(l_E)^2 = (L_E^0)^2 + (\vec{l}_E)^2 \tag{4}$$

$$d^3x = r^2 dr d\Omega \tag{5}$$

l → large, ultraviolet limit

$$d^4 l_E = \frac{i}{l_E^2}\frac{i}{l_E^2} = \int_0^\infty dr \frac{1}{r} \tag{6}$$

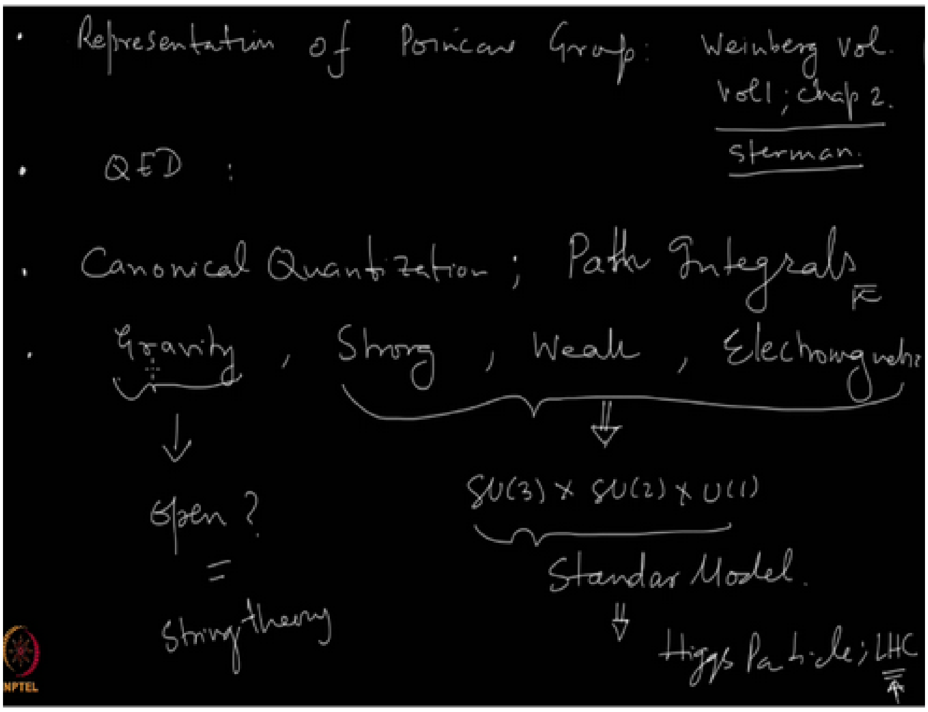
$$\Lambda : \text{cutoff} \qquad \int_0^\Lambda d\Omega \frac{1}{r} = \ln \Lambda \xrightarrow{\Lambda\to\infty} \infty \tag{7}$$

Renormalization: Remove the infinities → finite results.

What saves you is a procedure which is called renormalization. Actually at 1 point of time, it was believed that because of such divergences, it was believed that quantum field theory is a bad framework. And it is, one should abandon it. Because people were getting infinities everywhere. But eventually, it was figured out that what one needs to do is something called renormalization.

Will not have time to go into this but I will maybe I will just say a few words what it does it removes the infinities by hiding them in the fields and bear parameters that appear in the Lagrangian. So that m that you saw in the Lagrangian and that lambda that you saw in the Lagrangian and the field phi that you saw in the Lagrangian, these are called bare fields bare mass parameter and bare couplings.

And one hides these infinities which you are seeing here, in these parameters, there is a well-defined procedure for doing that and one gets rid of these infinities and then the entire result is expressed in some in terms of some new fields which are called renormalized fields and renormalized masses and renormalize parameters which are finite, these become similar.



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But eventually you get finite results. So, you have a well-defined procedure for getting finite results which you can use to then make connection with what you see in the experiments. Then there is of immediately from this follows the idea of renormalization group and that is quite interesting subject and it has lots of the high energy physics and condensed matter theory. They have a lot of exchanges in this particular domain of renormalization group. And a field theory is quite used in condensed matter when people are studying things like critical exponents and other things. So, this is maybe I can tell one thing here for example, when you do renormalization group you start seeing that the couplings that you are seeing that the couplings that you have, they change with scale. So they are not constants but rather they change with scale and that has a very nice and simple and intuitive interpretation, if you were to see the same thing in condensed matter systems. Imagine you were looking at some system which has you know, if you are able to look at that system at a very fine scale, at very detailed level, let us say you are able to see that. So, at that level, the couplings that you will be looking at will be the couplings between those atoms, those interactions between those atoms. But, if you are to look at the same system at a slightly cruder level, rather than looking at atoms, if you could, if you were looking only at the molecular level then of course the coupling between molecules is different from the coupling between atoms, the strengths are different.

$$\phi, m, \lambda \to \phi_r, m_r, \lambda_r \tag{8}$$

Renormalizzation group

- IR divergence. : $l^\mu \to 0$

- Collinear divergence.

Research

And of course because now, you are looking at a different length scale, the couplings will change. And this is something you also see in this context of quantum field theory and this is one of the things which I can mention here in about renormalization group. So, it is very nice topic and it is fairly restarted subject. So, this is another thing. So, till now, I was mostly concentrating on the u v part.

But that is not the only place where things are becoming divergent, there are other places and one such place is the infrared region and where you again get divergences and by infrared I mean now, l going to 0. So, the components l 0, l 1, l 2 and l 3 they all become very small. And in that limit again, the integrals will diverge. There is 1 more region where you get divergences and that is what is called collinear regions or yeah.

And you get collinear divergences in those regions. And this is typically in massless theories where the momentum running in the loops becomes parallel to external lines and that is those configurations give again a set of divergences just like here, you also get here. So, you see you have ultra violet divergences, infrared divergences and collinear divergences. Again, it all seems not so, good because it looks like we cannot make progress.

But just like we could do something or actually we could get rid of all the divergences which were of ultra violet type. There is a way to deal with infrared divergences and these infrared divergences give a lot of structure to quantum field theories at higher orders. And this is a subject of research, it is not that everything is known about IR divergences but it is an active area of research especially in quantum gauge field theories.

So, you will find a lot of work in QCD, QED and QCD and gravity theories. And this is an active area of research and I will show you how to find some material in this direction or any direction. I will show you how to do that in a while and these infrared divergences are of course, you again get to rid of them and collinear divergences you get to rid of them because unless you do so, you cannot make any predictions which you can match against experiments.

So that you have to do anyway but even from purely field theoretic point of view they are very interesting one because these are universal. So, no matter which processes you are looking at, they have a universal feature which is same irrespective of what process you are looking at and also you have you can make predictions of certain or you can make predictions of similarities of these higher similarities to all others in perturbation theory.

That is another reason why these are objects of such interest. So, in general making any calculation to 1 loop or 2 loop or 3 loop is very difficult. But then here you see that if you are interested in infrared divergences then you are able to make predictions to all orders not just 1 loop, 2 loop but to all orders in perturbation theory. That is good. Now, in this course, we have been talking mostly of scalars.

In fact, we occasionally mentioned the words spinors and gauge fields corresponding to for example, photons and other things. But world is not really made up of scalars, it has photons, as you know; it has fermions like electrons and positrons and all other things. So, you would like to have a quantum field theory which deals with these objects also. (Refer Slide Time: 22:09)

So, one first step would be to first try to figure out all kinds of particles or fields that can exist and you can find a very good discussion about this which is basically looking at representations of Lorentz group or Poincare group. And you will find a very good discussion in Weinberg Volume 1, quantum theory of fields or yeah something like that is the title, volume 1 and chapter 2. I will strongly encourage you to read that. You will find also a good discussion in the book by George Sterman. So, I would recommend you to look at these. You will see that naturally they you find that there are spin 1 particle, spin 2 particles, spin half particles and other spin particles. And the next step is of course, to make a quantum theory of these and first you would like to make a quantum theory of electrodynamics which includes fermions and electromagnetic fields quantizing them.

The best way to do a work with QED is not the way we have been doing. What we have been doing is called canonical quantization. The best way to work when you have gauge fields in your problem like you have in QED or QCD is to work with what is called path integral quantization or path integral approach. These are much more suited and eventually, you would like to write down a quantum field theory that describes all the interactions that you see around yourself.

Which includes gravity, strong interactions and weak interactions and electromagnetic interactions. Now, it turns out that you can use quantum field theory and write down a meaningful theory or describe these in a consistent manner, well defined manner by writing down by taking the symmetries in addition to Poincare symmetries, you look you take su 3 cross su 2 cross U 1 symmetry.

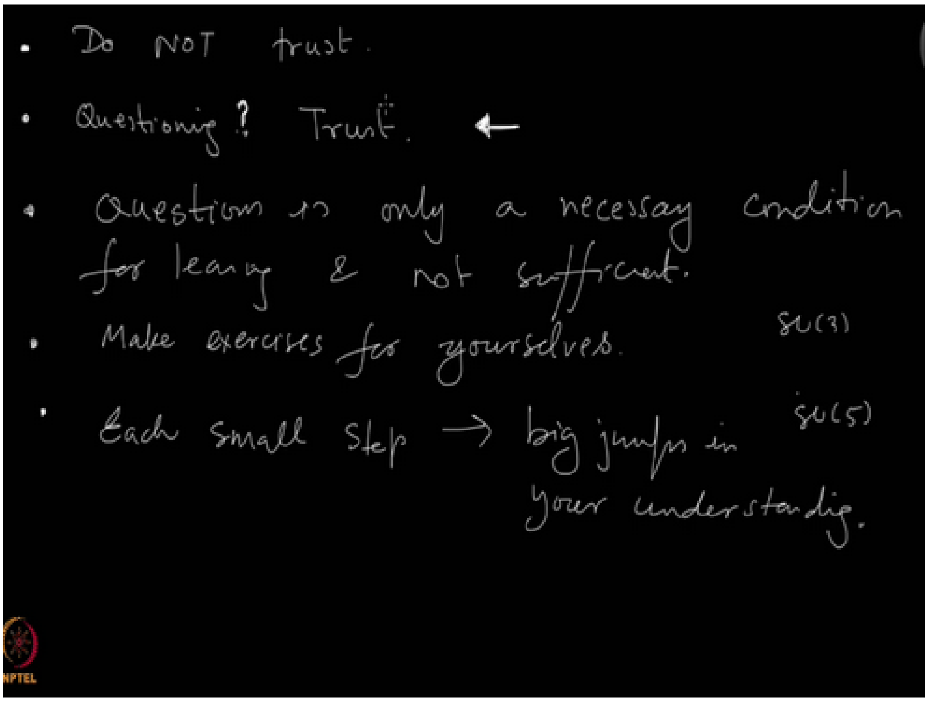
And you construct a condor field theory and that will provide you a theory of strong, weaken, electromagnetic interactions and this theory is called Standard Model and it is a very successful theory when which describes all the strong, weak and electromagnetic forces within this framework of quantum field theory. And in fact, it is so, good that it predicted that there should be a scalar particle.

Which was which has been called Higgs particle and as you might be aware that we have been searching for long for this particle and we have found it. Note that it was it was a prediction coming out of the theory rather than some input going into the theory. So, this tells you how neatly this entire framework has been built so that it required to have such a particle for everything to make sense. And in fact, it has been found by you know, experiments at LHC which is quite an amazing feat. But interestingly and not so, nice thing is that the gravity does not fit in the same framework. So, this does not allow itself to get quantized by the same principles by which we could quantize strong, weak and electromagnetic interactions and this remains an open question.

And among other candidates string theory is one of the candidates which tries to deal with this and merge them together. And we will try to look at the further development of the subject of quantum field theory in the second part of the course.

- Quantum gauge field theory. QED, QCD, gravity.

- Universal, make prediction of singularity to all orders in perturbation theory.



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- Representation of Poincare group: Weinberg Vol 1, chap 2.

- Canonical quantization, path integral.

- Gravity, strong, weak, electromagnetic.

- Strong, weak, electromagnetic $\rightarrow SU(3) \times SU(2) \times U(1)$, standard model, Higgs particle.

Now, before I close, I would like to say certain things which usually one would say in a classroom and give some game to students but I am not someone who has lot of games.

So, I will limit myself to a few simple things which I think are most likely correct. So, first thing I would say is that do not trust at all, all that I have said in this course.

So let me just emphasize this point. Do not trust: And that is the worst thing that you can do that is to trust. So, statements could have been wrong, what I have said, proofs might be incomplete, might be wrong as well, who knows? So, you have to, if you really want to know and go into research, you have to develop your own way of questioning and analyzing and not just believe or trust whatever has been said by me in this course or anyone in any course. No matter how great that person is in the research or in teaching. We have to be skeptical of the arguments and there is only way to make progress. And you should be questioning a lot. I mean, it does not necessarily mean that you start questioning every question you put to the person who is teaching. But you should definitely have lots of questions in your mind and that can happen only, if you do not trust.

So, you should have lots of questions do not just try to understand what is being said. Try to also see whether what is being said makes sense. And of course, it is always not very easy to have questions when things are not so, easy. So, if you found asking simple questions even simple questions, if you found them, I mean, if you found it difficult that now, you come up with simple questions in this course.

It was mostly because it was first round when then you go to the next round. When you read or look at the lectures again. I am sure you will start having more questions because you already have some experience some things with you and then asking becomes easier. But questioning or having questions is only a necessary thing. It is only a necessary condition for learning and not sufficient.

We should be able to also find answers to our questions. And one way to do it is have simple questions you create questions yourself. So, there can be 2 kinds of questions, you do not understand something that is why you have a question or you understand something very well and now, you are trying to phrase a small question around it. What if I do this? What happens then?

This is another variety of question one variety of question is, How have you done this part or how does it follow? This one, trying to understand what is being said? Other variety is you understand something and now, you take that something from the familiar context into slightly different context or maybe a small extension. One example could be we looked at SU(3), you say that is something I have understood.

I know how many generators there are these things and that things now, you say let me look at SU(5). How many generators there are? What will be their you know, how they would look like? Once you start asking such simple questions, soon you will be asking much nicer and much better questions. So that is one thing I would definitely encourage. So, learn to make exercises for yourselves.

It is not always a lot of fun to solve exercises created by others they are important you should do them. But then it is a lot of fun, if you are able to create exercises on your own because then that really means that you have understanding of something and then your exercise is going to take you beyond what you already know. Because it is coming out of you. So that is something we should try.

And whenever you do such a simple thing, each such small step, each such simple exercise that you create that will lead to big jumps in your understanding. I would definitely encourage you to try this. Now, I will before I stop, let me show you 2 websites where you can find the research that is happening especially in high energy physics, and of course other fields as well.

I am hoping that the screen is still captured. So, here is one. So, this is called , arXIV.org. So, here is the address. So, you can see that every day, papers are uploaded here. So, for example, let me take you to this one, phenomenology, HEP phenomenology or HEP theory. So, these are or HEP experiment, HEP lattice. So, these are the ones which are related to high energy physics.

So, let me take you to one of these. So, this is a new listing. So, all the papers which have been newly put here they are here. So, you can find the latest research here. So, these are not published ones already these are preprints. So, before people publish their work, they first put it in the arXIV so that others can see and comment and they can get feedback before they submit to a journal.

So, this is a very good resource. If you want to see what is happening in research, this is how you try to stay updated. Then the other source which is very useful is inspire and I have here done a search for the works by Ashoke Sen. So, you see F stands for a find ea I do not know exact author Sen, Ashoke so that is I think one can do more simply these days but I am doing it old style.

So anyway, so, if you are trying to search for papers by Ashoke Sen or by anyone, you go here, you type this, or maybe just type Ashok Sen probably that will also work. And you will find all the papers that he has recently written. For anyone so, you can also search for papers here and most of the things are available here. So, these are some 2 very important resources.

And of course, if you are someone who is interested in, let us say condensed matter, I think I am not sure let us check. I do see something here. So, condensed matter for example and astrophysics and yeah all other things are also here. So, these are 2 very useful resources and you can try to see papers in here to get an idea of where things are? What things are happening? And where things are going?

And what might be of interest to you for your research? I think that is all I wanted to say. So, I will thank you for attending the course and say bye.

- Do not trust.
- Questioning trust.
- Question is only a necessary condition for learning not sufficient.
- Make exercises for yourself.
- Each small step → big jump in your understanding.