Introduction to Quantum Field Theory

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Lecture 45 : Feynman Diagrams Continued

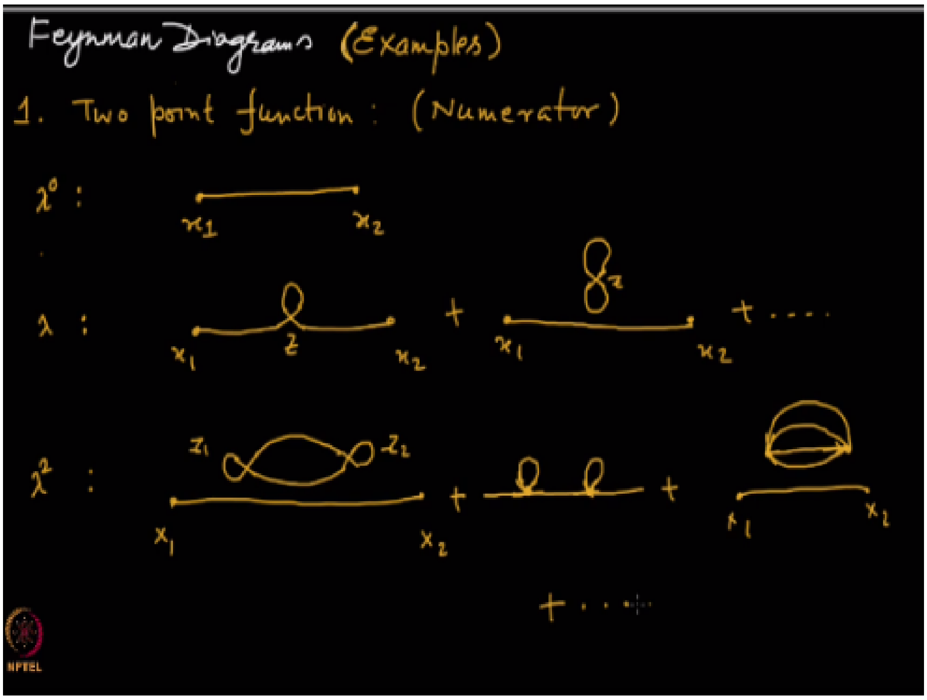

Figure 1: Refer Slide Time: 00:13

Okay, let us take a few examples of Feynman diagrams. So first I will look at two point function. Okay, and remember that I am only looking at the numerator in the master formula, okay. We will talk about denominators later. So two point function, so I am looking at, okay so let us look at order one term, any order lambda to the 0 term. So you have two external points x 1 and x 2.

And there is no vertex because you are at order lambda 0, okay. And there is one possibility that you connect these two, okay. And this is just the Feynman propagator. Now let us go to order lambda. So again, I have my two external points x 1 and x 2. And at order lambda, you have one vertex, okay. Let us call it z. So now I can connect this and I could connect it like this, and then I am left with this possibility.

So this is one possible Feynman diagram. There is another possibility that we do the following. So instead of connecting x 1 to a vertex and x 2 to a vertex, I connect x 1 and x 2, okay. And then these lines coming out of this vertex, I connect them among themselves. So that is another possibility, okay. So this one will have delta d of or D F z - z and this one also D F z - z okay?

So this is, these are some examples. Let us look at what kind of diagrams to get at order lambda square. So as before, always start with putting x 1 and x 2. I am not drawing this one tick out of it, because you know that you can only just take out one line out of these external points. So I am not going to put that tick every time. So at order lambda square, you have two vertices z 1 and z 2, okay.

And there are many diagrams that you can draw with these two external points and these two internal vertices. Let me draw a few of them. So of course you can, let me write z 1 here and z 2 here. Of course, you can draw a line between x 1 and x 2. So that is a Feynman propagator D F(x 1 – x 2). And then there are many ways in which you can contract these.

And one possibility is that you contract this with this. This one like this and connect these two. So this is one possible Feynman diagram at order lambda square, okay. They are more, some of them I will draw, so you can easily convince yourself that this is another possibility, okay. So one vertex here, one vertex here. So there is 1, 2, 3, 4; four lines coming out of this vertex and same here 1, 2, 3, 4, okay.

So this is another order lambda square diagram. Maybe one more I will draw. So you can have this. You connect the external points x 1 and x 2. And then the four small lines that come out of each vertex, all of them can connect to the other vertex. So you can have like this. So let us say this is vertex z 1, z 2 and there are four lines coming out and I contract them like this, okay.

So at this vertex, you have 1, 2, 3, 4. And here 1, 2, 3, 4. And you have connected. So this is another possible diagram. And you can draw other diagrams as well. So this was an example for two point function. Let us look at some of the, look at two diagrams and look at their combinatoric factors, right? So I want to look at combinatoric factors. So let us look at this diagram. I think I drew it. Yeah, this is this one, okay. So in how many ways I can draw this diagram and get the same expression. So all the ways in which this diagram can be drawn, that will be the combinatoric factor, and let us count. So start with x 1, x 2, and then you have this vertex. So x 1 could go to any of these four lines.

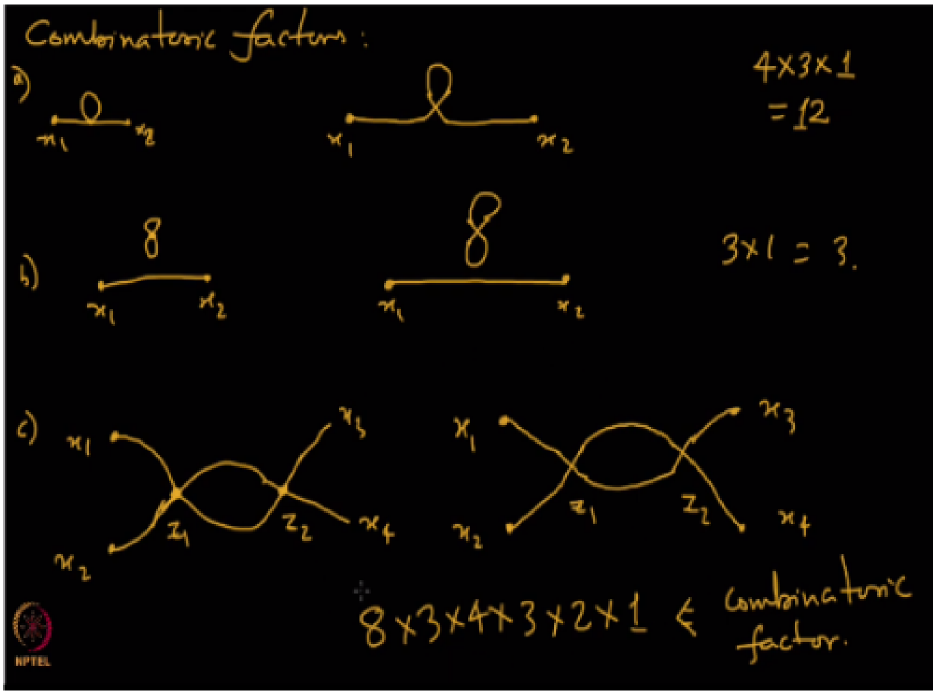

Figure 2: Refer Slide Time: 05:29

x 1 cannot go to x 2, because I am looking at this topology right, where x 1 is connected to the vertex. So x 1 could go to any of the four. So it goes, let us say to this one. So I have four possibilities here. Then x 2 could go to any of the three, right because they all are equivalent. So x 2 could go to any of the three. So there are three possibilities. Then this guy can connect only here, so there is only one possibility, okay.

So you have 12 such diagrams and they all contribute equal I mean same, they all have the same expression, okay. So the combinatoric factor is 12 for this case. Let us look at another example. This one. I want to look at this here. Okay, this one. So there is a vertex here. Let me draw it nicely, okay. So we want to again know the factor corresponding to this one. And that is easy. Let us see. So this one can go only to this one, because these are connected, right? So x 1, x 2, there is only one possibility. Now this guy, let us start with here. This guy could go to any of the three, remaining three. So let us say it goes here. So you have three possibilities. Then this one has only one possibility. So the factor is 3. Okay, let me give you a slightly more complicated example.

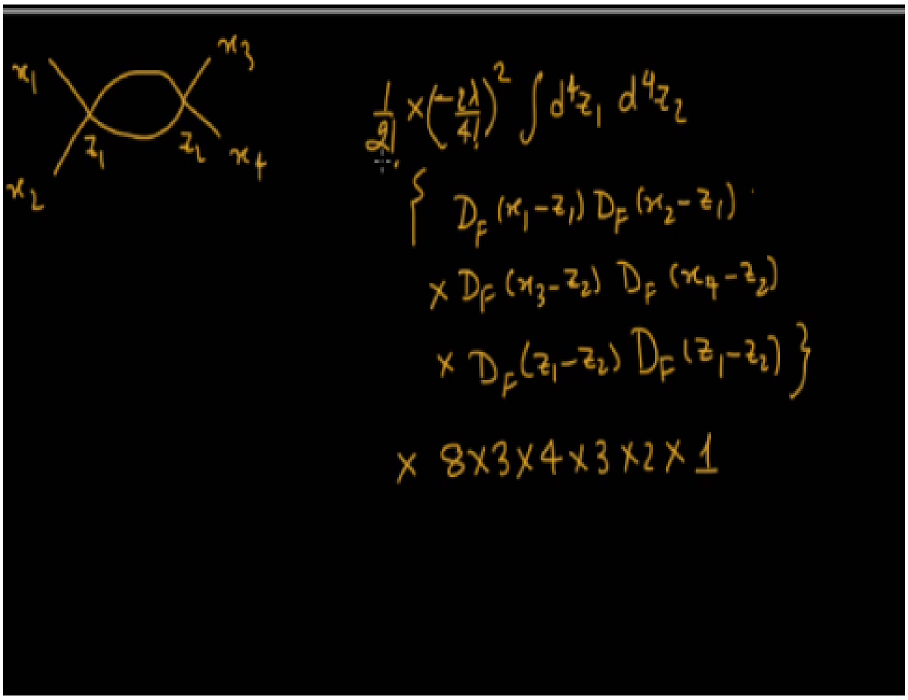

Figure 3: Refer Slide Time: 12:53

1. Two point function : (Numerator)
   combinatoric factor $= 4 \times 3 \times 1$

combinatoric factor $= 3 \times 1$
combinatoric factor $= 8 \times 3 \times 4 \times 3 \times 2 \times 1$

$$\left(\frac{-i\lambda}{4!}\right) \int d^4z_1 \int d^4z_2 D_F(x_1 - z_1) D_F(x_2 - z_1) \\ D_F(x_3 - z_2) D_F(x_1 - z_2) D_F(z_1 - z_2) D_F(z_1 - z_2) \\ \times 8 \times 3 \times 4 \times 3 \times 2 \times 1 \tag{1}$$

4-point function

Now I want to look at four point function. So we have x 1, x 2. Maybe I should first draw the diagram. So I am looking at this one. So there is a vertex here, there is a vertex here. Let us call it z 1. Let us call this one z 2. So this is the diagram for which I want to know the combinatoric factor. So let us see. So I start by again putting x 1, x 2, x 3 and x 4. So these external points are here.

And then I put two vertices z 1 and z 2, okay. Now x 1 could go to how many? Any of the 4 here or any of the 8 here. So x 1 has 8 possibilities, it can go to any of the 8 because see z 1 and z 2 are dummy. If you look at the expression here, I think I wrote it somewhere for, here. This z 1 and z 2 are dummy. You can, there is no distinction between this factor and that factor.

So when you are contracting phi x 1, with this you can contract or with that one you can contract, okay. And as long as the identical contractions are done, you get the same expression. So you get the same topology, same diagram, if you join at any, I mean if you start with any of these 8 points. So we have 8 possibilities to begin with, okay. That is what I want to say. So let us say it goes to this one.

Even if it goes to that one, you get still the same kind of diagram, the way I am going to construct. So let us say it has chosen to go here, out of these 8 possibilities. The next two has how many? x 2 goes to the same vertex or x 2 is connected to the same vertex to which x 1 is connected. So x 2 has to connect on z 1, and there are only three possibilities. So let us say it goes here. You can connect this to here also.

It is the same, it does not matter, these are identical. So let us say we have connected here. So you had three possibilities. So let us count that. Now x 3 can go to how many places? x 3 cannot go here, because the kind of diagram we are looking at, there x 3 goes to the vertex to which x 1 is not connected. So x 3 goes to z 2, it does not go to z 1. So x 3 should go to any of the four. So let us say it goes here.

Then x 4 should go to the same vertex to which x 3 is connected. And there are three possibilities. So you get 3. Now this guy could go to any of the two, these two. It cannot go here. Because that will be a completely different diagram if you connect like this, okay. It has go to here so that you generate this topology. So this one can go to any of these two, let us say it goes here.

So you have 2, this guy has only one possibility, okay. So this is 1, 2, 3, 4 okay, whatever that number is these are the, this is the combinatoric factor. So what you should do also is try to draw the same thing in different way and convince yourself that they are all identical. So you should be sure that this is really the combinatoric factor.

So for example, let me try to say this way. So suppose you start here, but instead of connecting here, I said in the beginning, you could connect to any of the 8. So let us see connect here. And then this one has to go to the same vertex, so it goes there. This one can connect like this, this one can connect like this, and then this here and that there. And this is looking bad, but it is the same diagram, okay. Now I want to write expression for this diagram. So let us do that. So I have this Feynman diagram, okay. So what is the expression? First I should write down all the propagators, okay. So you have 1, 2, 3, 4, 5, 6 six propagators. So let us write, I have D F(x 1 – z 1) this one. Then you have D F(x 2 - z 1). Then you have D F(x 3 – z 2). Then you have D F(x 4 – z 2), which is this one.

Then you have D F(z 1 – z 2) this one. Then you have this propagator, again z 1 – z 2, okay good. Then I recall that for each vertex, I have to integrate over the internal points, z 1 and z 2, okay. For each vertex, I have to do an integral. And each vertex comes with a factor of minus i lambda over 4 factorial. Let us go back and see here, minus i lambda over 4 factorial times an integration over the internal space time point, okay.

So let us write that down. So you have two factors of this. So you get minus i lambda over 4 factorial squared, D 4 z 1, D 4 z 2, okay. So this is the integral. Then I should write down, what should I write done? Yeah, I still have some factors to multiply, so I should write down first, the combinatoric factor or this one. The ways, the number of ways in which I can draw this diagram

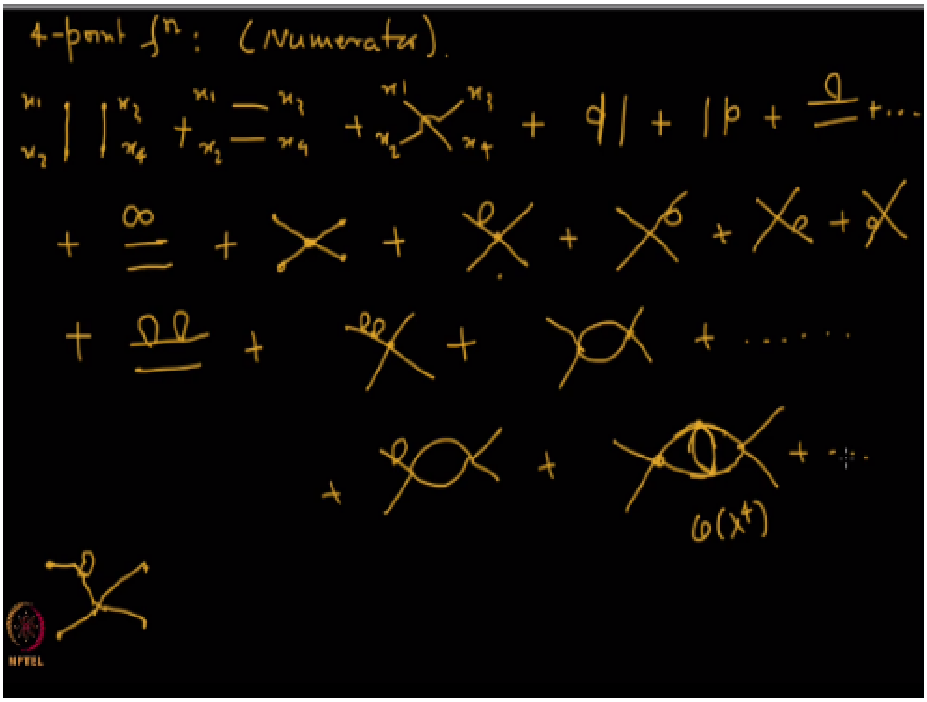

Figure 4: Refer Slide Time: 17:49

and that was 8 into 3 times this is 4 factorial.

So let us write, okay is something still missing? Yes, there is one thing missing, which is the, this is order lambda square diagram. So when you look at the pre-factor of this because you are expanding the exponential to order lambda square, it comes with a 1 over 2 factorial, right? I am talking about this factorial. So this is 1 over 2 factorial that I should multiply.

And this is the expression for this 4 point, this diagram that contributes to 4 point function. Okay, looks fine. Hopefully there is no mistake here. Okay, good. I will give you some more examples. So I will give you an example of 4 point function, okay. And basically the numerator, I am not looking at the full thing. Denominator I will worry later, okay. So if I am looking at 4 point function, what are the possible diagrams? So I start with order lambda zero then proceed like that. So this is one possibility, okay. Plus you have another possibility like this. Plus you have another possibility like this.

Okay, these all are added plus, so I think at order lambda to the 0, this is all. Now I go to the other terms, order lambda terms. At order lambda you can have the following, okay. So there is a vertex here. Now I will not write x 1, x 2. It is understood this that I am writing always this way. Then there is another possibility at order lambda, which is this, okay.

So basically what I have done is I have taken this diagram and done what you call a order lambda correction here, okay. So a order lambda vertex has been connected here. And similarly here and the same thing you can do for other ones. So you can have, okay. So these are all other diagrams, which are order lambda. And then there are other kinds, so you can have this, this.

So let us say you have taken this one. And the vertex you do not connect to any of these ones. But vertex you, vertex lines coming out of vertex that you connect among themselves. So you get one possibility like this, okay. And then there is, of course this possibility, which you have seen that you connect like this. So vertex is here. So this one is connected to this, this one here, this one here, this one there.

Okay, so that is another possibility. Note that I am adding up all of them because that is what it is in the numerator. Then you can have, I am not drawing all diagrams, I am drawing some diagrams. So now I am looking at order lambda square. So this is order lambda, there is one vertex. This is order lambda one vertex. This is order lambda one vertex, but I can put a correction here, okay.

I attach a vertex like this. So this is order lambda square diagram. You see how you get this one, right? This one you get by doing the following. So you have external point here. Then there is a vertex. Then there is one more vertex. So you connect like this, okay. This is this diagram. And this is called external leg correction, and this kind of diagrams.

And of course, you can put corrections here or on other legs also. Not or and, okay. Now if you go to order lambda square, there are still more diagrams, many more. Some of which are the following. Of course, you can do if you go further order, so this is 1 vertex, 2, 3, order lambda cube, this is order lambda cube diagram.

I should have drawn at order lambda square before I draw this. So this is order lambda square, okay. And there are many such possibilities. You can, maybe one more I will draw the last one. You can also have this, okay. Okay, maybe one more. Now that I am drawing, I can draw many more. So from here, I can connect like this. So this is 1, 2, 3, 4. So that is a 4 point vertex, 1, 2, 3, 4.

So that is another 4 point vertex. So this is 1, 2, 3, 4, order lambda to the 4 diagram, okay. So this way, you can draw all possible diagrams and one has to be systematic. Otherwise, it is very easy to miss diagrams and get wrong results. So care has to be taken in doing that, okay. So we will stop with these examples here, and we will continue discussion in the next video.