Introduction to Quantum Field Theory

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Lecture 42 : Wick's Theorem

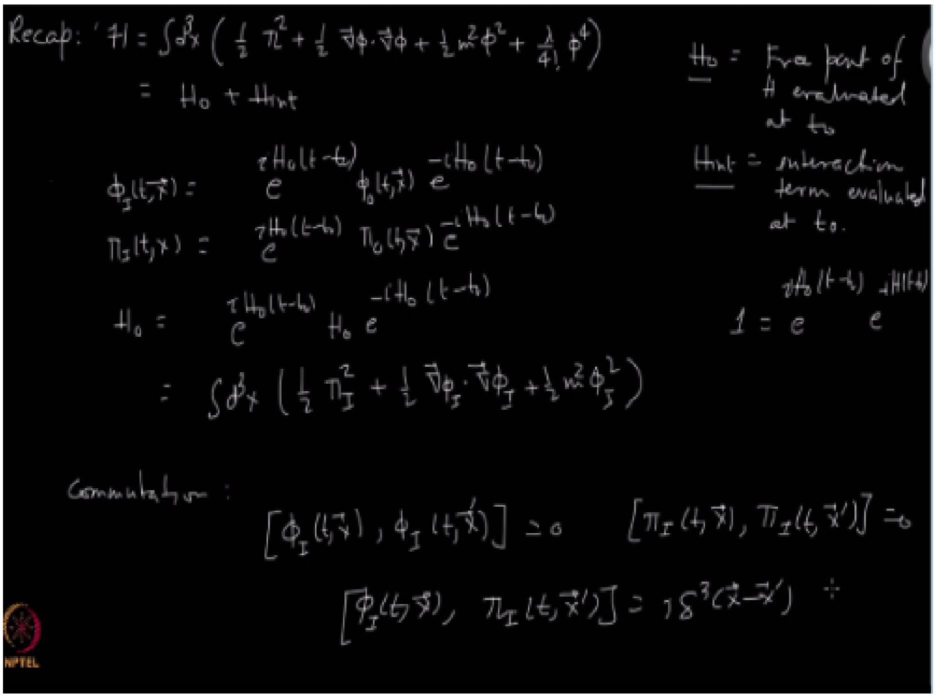

Figure 1: Refer Slide Time: 00:13

Okay, let us start with a quick recap. So here is the Hamiltonian of the phi-4 theory that we are looking at okay, which we wrote as sum of free part and an interacting part, okay. And we wrote down the free part and then interacting part at time t naught, right? Independently if you look at this piece this is time dependent and that is time dependent, but the sum is time independent because the left hand side is.

And this is what our H naught is, so it is basically the free part of the Hamiltonian evaluated at time t naught, okay. Similarly, H interaction is the interaction term evaluated at time t naught. And we had introduced the fields in interaction picture, the phi I's and pi I's which evolve according to free Hamiltonian, okay. And you can also write the H naught as follows. So you can just sandwich it between these two operators. Okay, they commute. So because this is H naught, this is H naught, this will commute and this kills this exponential. So H naught is equal to H naught. That is what you get. And if you take the H naught here okay and then from left and right you multiply with these two factors and if you insert 1 in this form okay, then you will be able to write H naught as follows, okay.

So that is clear. Once you use this you can convert all the phi's and pi's into phi I's and pi I's. And also you saw that the commutation relations that phi I and pi I satisfy are exactly the ones which free feel operators satisfy. So these are equal time commutation relations. So fields commute at equal times. The same is true for pi and the coordinate phi I and its conjugate momentum density pi I.

They have the usual commutation relation, okay. And then we further use this to write down Green's functions or correlation functions in the full theory which we write as follows. Now I will use a shorthand. I will write this as integral d 4 z. So d cube z and d tau I will combine to d 4 z, but I will write this limits, these limits as minus lambda to lambda, okay. Just to remind ourselves that the time component has integration limits which are given by these ones. So I write this as now lambda over 4 factorial, okay. So this is where we had stopped last time.

So let me look at the numerator part here and the denominator part separately, okay. So denominator is easy, it is almost the same thing as numerator except for the fact that phi I's with external points x 1, x 2 these are absent there, but it is the same thing. So let us start looking at the numerator. So we have here this, okay. So what is this object? This, sorry I forgot the exponential which I should not factor of i is missing here. Before z phi I to the 4, okay. That is what we want to calculate. So I will just expand this, okay. This is an exponential. And you know what the meaning of this exponential is? See this exponential is an exponential of an operator, okay. This is an operator, the phi I's are operators.

And this was formally defined as the expansion with so e to the operator A is 1 plus A plus half, 1 over 2 factorial A square and so forth, right? That is how this object e to the A is defined, okay. And we have already discussed about this one in detail. So I will just expand it and write as follows. So the first term of the exponential will be 1. So that is what I write here.

So this is the first term you get. Then the second term comes with a factor of -i lambda over 4 factorial, okay. So it is a, let me see whether I want to write it differently. That is fine. So I write -i lambda over 4 factorial, okay. And then you have one factor of this operator. So I get 0, sorry vacuum time order product of phi x 1 up to phi x n and then integral d 4 z phi I z to the fourth, okay. So they are 4 factors of phi I here.

And similarly, if you proceed to nth term in this expansion you get -i lambda over 4 factorial raised to the power 4, 1 over n factorial. That comes because of the exponential, okay. This is 1 over 2 factorial, 1 over 3 factorial for the next term and so forth. And then you get this. Okay I am forgetting to put I here. And now I will have such phi-4 terms repeated n times, okay.

$$
\begin{aligned}
H &= \int d^3x \left(\frac{1}{2}\Pi^2 + \frac{1}{2}\vec{\nabla}\phi \cdot \vec{\nabla}\phi + \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4 \right) && (1)\\
&= H_0 + H_{\text{int}} && (2)
\end{aligned}
$$

H_0 = free part of H evaluated at t_0, and H_{int}= interaction term evaluated at t_0

$$
e^{iH_0(t-t_0)}e^{-iH(t-t_0)} = 1 \qquad (3)
$$

So what I will do is I will write d 4 z 1 and to save some space I will write d 4 z, I will write like this. So integration limits are of course from minus lambda to lambda. And then you have phi I to the 4 z 1 so and so forth phi I to the 4 z n and the vacuum, right?

And it continues like this. So if we are interested in calculating these objects, the numerator to some accuracy in lambda, let us say I am interested in calculating the subject to order lambda. Then I will just calculate the first term here and the second term, okay. But if for some reason I am interested in going up to order lambda square then I will calculate the first three terms; this one, this one, and the next one, okay.

So it will be up to us up to which order we want to calculate our correlation functions, okay. And accordingly we will get the results up to a certain precision. So our problem has boiled down

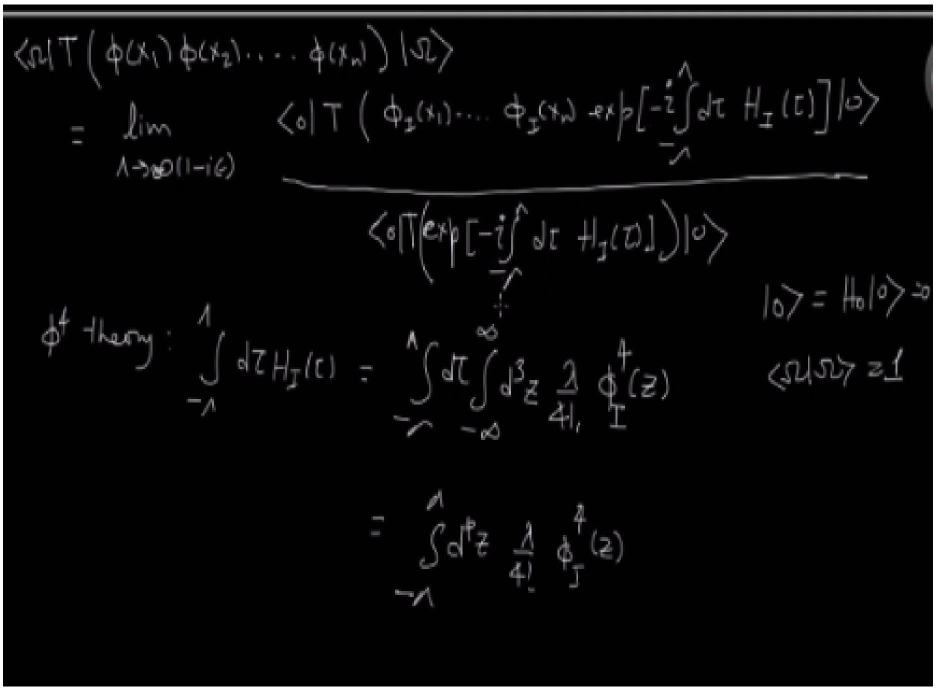

Figure 2: Refer Slide Time: 03:41

to calculating these kinds of objects okay, which is apart from these integrals and some factors it just expectation value of time order product of certain number of fields in which some fields are at some space time points which you decide, you choose the external points x 1, x 2 and x n.

And some others they are internal points, right? The z is not something you choose. See in the numerator or look at this one. So here this is what we are calculating right. So x 1, x 2, x n, these are the ones which you choose. But you have no control over the z here, right? That is an integrated over thing. You do not have control over z. So such points I will call internal points, okay. So let us try to calculate this.

$$\phi_I(t,\vec{x}) = e^{iH_0(t-t_0)}\,\phi_0(t,\vec{x})\,e^{-iH_0(t-t_0)} \tag{4}$$
$$\Pi_I(t,\vec{x}) = e^{iH_0(t-t_0)}\,\Pi_0(t,\vec{x})\,e^{-iH_0(t-t_0)} \tag{5}$$
$$H_I = e^{iH_0(t-t_0)}\,H_0\,e^{-iH_0(t-t_0)} \tag{6}$$
$$= \int d^3x\left(\frac{1}{2}\Pi_I^2 + \frac{1}{2}\vec{\nabla}\phi_I\cdot\vec{\nabla}\phi_I + \frac{1}{2}m^2\phi_I^2\right) \tag{7}$$

$$[\phi_I(t,\vec{x}),\phi_I(t,\vec{x}')] = 0 \tag{8}$$
$$[\Pi_I(t,\vec{x}),\Pi_I(t,\vec{x}')] = 0 \tag{9}$$
$$[\phi_I(t,\vec{x}),\Pi_I(t,\vec{x}')] = i\delta^3(\vec{x}-\vec{x}') \tag{10}$$

$$\langle\Omega|T\Big(\phi(x_1)\phi(x_2)\cdots\phi(x_n)\Big)|\Omega\rangle = \frac{\langle 0|\,T\Big(\phi(x_1)\phi(x_2)\cdots\phi(x_n)\mathrm{exp}\Big[-i\int_{-\Lambda}^{\Lambda}d\tau H_I(\tau)\Big]\Big)\,|0\rangle}{\langle 0|\,T\Big(\mathrm{exp}\Big[-i\int_{-\Lambda}^{\Lambda}d\tau H_I(\tau)\Big]\Big)\,|0\rangle} \tag{11}$$

Now eventually I want to put everything you know sandwich this operator between the vacuum, okay. So when I do that look at this one. See phi I plus kills the vacuum to the right and phi minus to the left. So this one has a phi I plus, it will kill the vacuum. This one has a phi I plus, it will kill the vacuum. This one has a phi I plus, it will kill the vacuum, okay.

This will kill the vacuum also because it has a phi I minus on the left either way. But this guy, this guy will not kill the vacuum, right this. So this is something which will give some non-zero value. So what I will do is I will leave these three terms as it is as they are and interchange the order of phi I minus and phi I plus in this term.

And for that I will just use commutation relation and write this as phi I minus x 2 phi I plus x 1 and then you have a commutator, right? So now if you see, I have this the following result. I have term number 1, term number 2, term number 3, number 4 and number 5, right? And in all the four terms 1, 2, 3, 4 you have always a plus to the right and minus to the left, okay.

And of course if you have both pluses then it does not matter because you see the pluses are what? Pluses are containing a's and a is commute. So the order does not matter here. You can write whichever way you want, okay. And but here the order matters. So anyway, we have eventually the four terms which kill the vacuum and one term which does not kill the vacuum which is a commutator, okay.

And commutator is just a complex number because this apart from the exponentials they contain a's and a daggers and that will give you a delta function. So together it forms a complex number and that when you send it should be in the vacuum. It will just pull out that complex number because that is not an operator and the vacuum will itself will give you 1, okay.

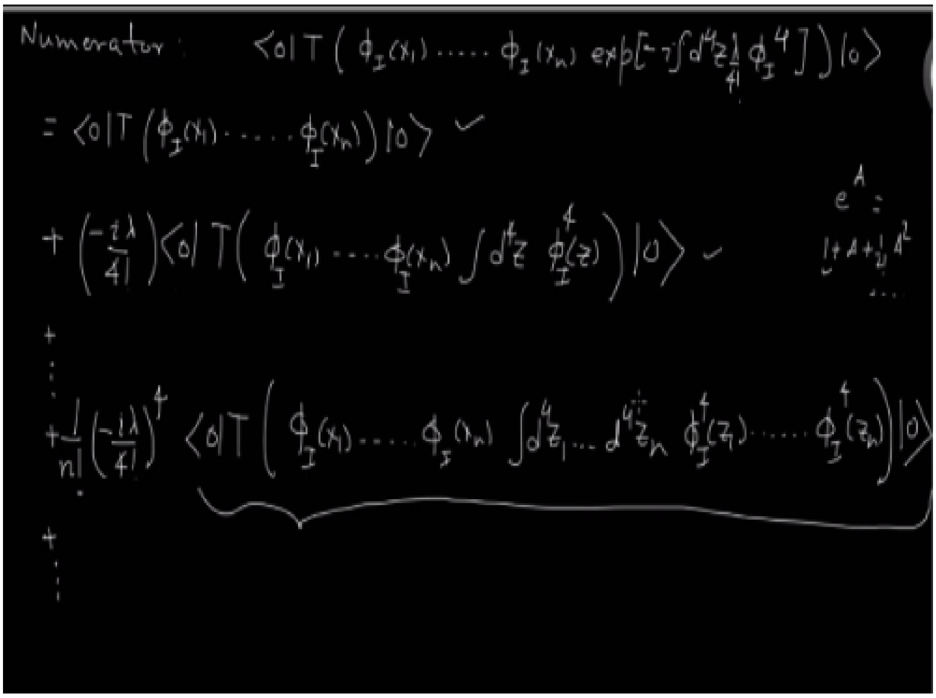

Figure 3: Refer Slide Time: 07:28

So you get a number out of this when you sandwich between the vacuum. So what I do is I define, no before I define something let me write the result for the case x 2 0 greater than x 1 0 okay and look at again the same object. And what would you get? You will get the same thing with x 1, x 2 interchange, right? But still the operators which kill the five pluses will be on the right and five minuses will be on the left. And the difference will come in this commutator piece, okay. So we will get exactly the four terms as before with x 1, x 2 interchanged okay plus this commutator and the commutator will be this time x 2 minus x sorry, yeah minus x 1. I have to interchange 2 and 1, okay. Now I see when I sandwich the time order operator between the vacuum okay, it will, all these four terms will go away because they give 0.

And the result will contain only this commutator and that commutator. And this commutator is present only when x 1 naught is is greater than x 2 naught. And this commutator is present when x 2 naught is greater than x 1 naught. So let me write down the result.

$$\begin{aligned} |0\rangle &= H_0\,|0\rangle = 0 \qquad (12)\\ \langle\Omega|\Omega\rangle &= 1 \qquad (13)\end{aligned}$$

ϕ^4 theory :

$$\begin{aligned}\int_{-\Lambda}^{\Lambda} d\tau\, H_I(\tau) &= \int_{-\Lambda}^{\Lambda} d\tau \int_{\infty}^{\infty} d^3z \frac{\lambda}{4!}\phi_I^4(z) \qquad (14)\\ &= \int_{-\Lambda}^{\Lambda} d^4z\, \frac{\lambda}{4!}\phi_I^4(z) \qquad (15)\end{aligned}$$

The numerator of [11]

$$\langle 0|\, T\Big(\phi_I(x_1)\phi_I(x_2)\cdots\phi_I(x_n)\text{exp}\Big[-i\int_{-\Lambda}^{\Lambda} d\tau H_I(\tau)\Big]\Big)\,|0\rangle$$

$$= \langle 0|\, T\Big(\phi_I(x_1)\phi_I(x_2)\cdots\phi_I(x_n)\Big)\,|0\rangle$$

$$+ \left(\frac{-i\lambda}{4!}\right)\langle 0|\, T\Big(\phi_I(x_1)\phi_I(x_2)\cdots\phi_I(x_n)\Big)\,|0\rangle \int d^4z\,\phi_I^4(z)$$

$+$

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$$+\frac{1}{n!}\left(\frac{-i\lambda}{4!}\right)^4\left(\frac{-i\lambda}{4!}\right)\langle 0|\, T\Big(\phi_I(x_1)\phi_I(x_2)\cdots\phi_I(x_n)\int d^4z_1\cdots d^4z_n\,\phi_I^4(z_1)\cdots\phi_I^4(z_n)\Big)\,|0\rangle$$

$+$

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$$\langle 0|\, T\Big(\phi_I(x_1)\phi_I(x_2)\cdots\phi_I(x_n)\Big)\,|0\rangle \qquad (16)$$

$$\phi_I = \int\frac{d^3k}{(2\pi)^{3/2}}\frac{1}{\sqrt{2\omega_k}}\Big[a(\vec{k})e^{-ik\cdot x} + a^\dagger(\vec{k})e^{ik\cdot x}\Big] \qquad (17)$$

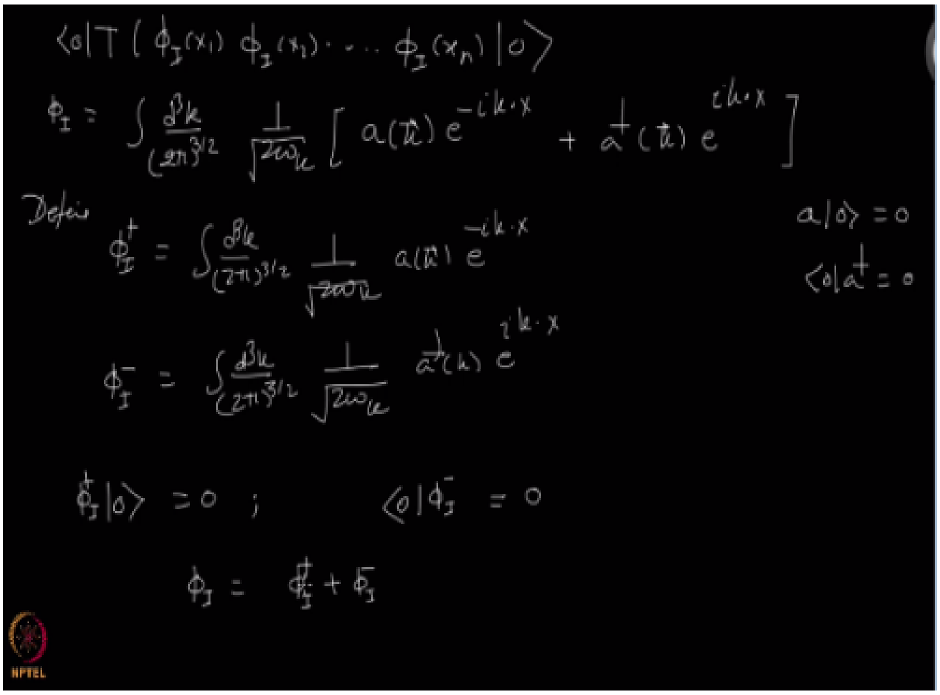


Figure 4: Refer Slide Time: 14:31

Define

$$\phi_I^+ = \int \frac{d^3k}{(2\pi)^{3/2}} \frac{1}{\sqrt{2\omega_k}} a(\vec{k}) e^{-ik\cdot x} \tag{18}$$

$$\phi_I^- = \int \frac{d^3k}{(2\pi)^{3/2}} \frac{1}{\sqrt{2\omega_k}} a^\dagger(\vec{k}) e^{ik\cdot x} \tag{19}$$

$$\phi_I^+ |0\rangle = 0 \quad : \quad \langle 0| \phi_I^- = 0 \tag{20}$$

$$a|0\rangle = 0 \quad : \quad \langle 0| a^\dagger = 0 \tag{21}$$

$$\phi_I = \phi_I^+ + \phi_I^- \tag{22}$$

$$\langle 0| T\Big(\phi_I(x_1)\phi_I(x_2)\Big) |0\rangle \tag{23}$$

case $x_1^0 > x_2^0$;

$$T\Big(\phi_I(x_1)\phi_I(x_2)\Big) \tag{24}$$

$$= \phi_I(x_1)\phi_I(x_2) \tag{25}$$

$$= \Big(\phi_I^+(x_1) + \phi_I^-(x_1)\Big)\Big(\phi_I^+(x_2) + \phi_I^-(x_2)\Big) \tag{26}$$

$$= \phi_I^+(x_1)\phi_I^+(x_2) + \phi_I^-(x_1)\phi_I^+(x_2) + \phi_I^+(x_1)\phi_I^-(x_2) + \phi_I^-(x_1)\phi_I^-(x_2) \tag{27}$$

The third term in last expression can be rewritten as

$$\phi_I^+(x_1)\phi_I^-(x_2) = \phi_I^-(x_1)\phi_I^+(x_2) + [\phi_I^+(x_1), \phi_I^-(x_2)] \tag{28}$$

case $x_2^0 > x_1^0$;

$$T\bigg(\phi_I(x_1)\phi_I(x_2)\bigg) = \cdots (x_1 \leftrightarrow x_2) + [\phi_I^+(x_2), \phi_I^-(x_1)] \tag{29}$$

$$\langle 0| \, T\bigg(\phi_I(x_1)\phi_I(x_2)\bigg) \, |0\rangle = \theta(x_1^0 - x_2^0)[\phi_I^+(x_1), \phi_I^-(x_2)] + \theta(x_2^0 - x_1^0)[\phi_I^+(x_2), \phi_I^-(x_1)] \tag{30}$$

:: Normal ordering

So the result will be the following. So first term was let us go back phi I plus and the one with a larger time argument phi I minus one with the smaller time argument okay and this is the one which hits when you have x 1 naught greater than x 2 naught, okay. So this is present only when x 1 naught is greater than x 2 naught. So that is what the unit step function is doing. Plus the other term is present when x 2 naught is bigger.

So here you have phi I plus x 2 phi I minus x 1. Okay, that is the result. Okay, this is good. We have got some result for this expression. And I will now introduce a notation. So here you see the 4 pieces 1, 2, 3 and this fourth, they were in a particular fashion which I told you okay, that the pluses are to the right and the minuses are to the left. And when operators are in this form, we say that they are normal ordered, okay.

So that is the definition of normal ordering. Because and this is useful because if you have a operator which, operator or operators which are normal order, and if you were to sandwich those between vacuum, you will get a 0 result, okay. So it is useful to define something called normal order and the symbol is this. This is a symbol used for normal ordering. So if you have operators a k 1 and a k n.

So let us say you have series of a's and then you have another series of a daggers and then maybe you again have another series of a's okay and so forth, maybe another series of a daggers. And if you were to normal order these operators and what you get is the following. You should get a's to the right and a daggers to the left. So all the a's should appear on the right. So you will get a (k 1), a (k n).

I keep forgetting this and then you will have a q 1, a q n. Not n, I have used s, okay. So all the a's here and all the a daggers here, okay. And if there are more a daggers you can put them here, okay. And between a daggers it does not, the order does not matter, right because daggers commute. And between a's the order does not matter because they commute.

So this is the definition of normal ordering, okay. It just means push all the a's to the right and pull all a daggers to the left, okay. That is what normal ordering means and this is the symbol okay. So if you put between two dots all these operators, this is what you mean, okay. Now as I said this is useful because the moment you put this between vacuum that will give you 0, okay.

So let us see what it means. It means that the time order product of the fields phi I x 1 and phi I x 2 that we had, we wrote it as the following. You write phi I x 1, phi I x 2 okay and remember phi I x 1 and phi I x 2 you can write as phi, in terms of phi plus and phi minus. But then the

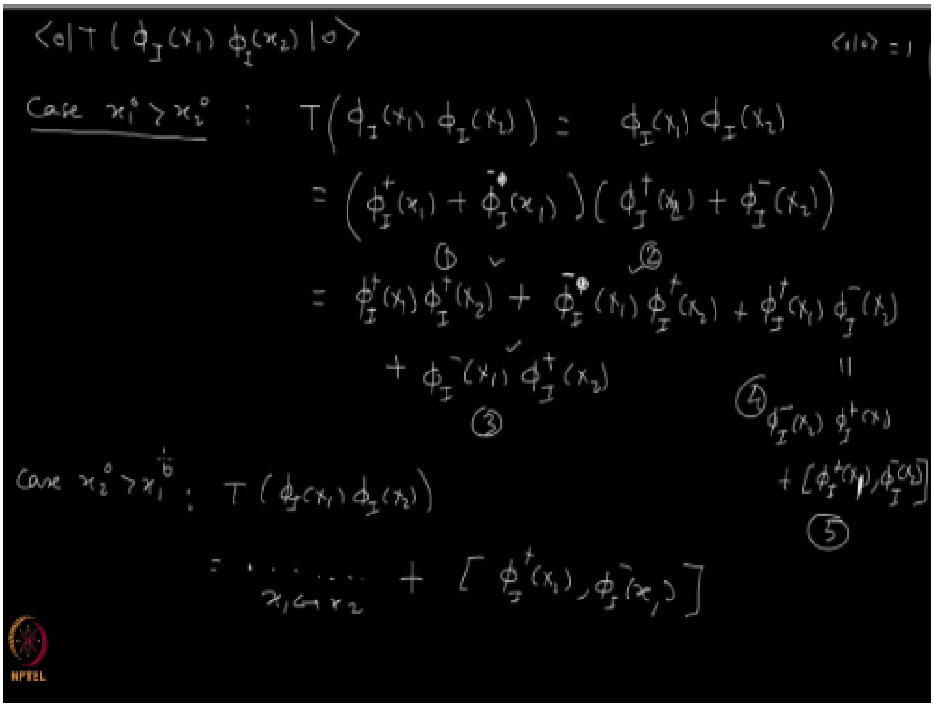


Figure 5: Refer Slide Time: 17:58

four terms that you got they were in a particular order and now you can identify that they were in normal order.

All the ones, all the pluses, if plus contains what? Plus contains a, minus contains a dagger, right. So here all the four terms had a's to the right and a daggers to the left, right? So that is what we have here. We are saying that the time order product of these two fields is a sum of terms in which all the operators are normal ordered plus a term which we got which was this piece, okay.

And this I will denote by, let me write it here. So I had the following. So I will introduce another notation. So I will say phi I x 1, phi I x 2 the symbol is read as phi I x 1, phi I x 2 and this is the contraction between these two, okay. So this is the contraction between phi I x 1 and phi I x 2 and what is the definition of the symbol? It is exactly this piece okay, what we got there, okay.

That is the definition of the symbol. So here I will write phi I x 1, phi I x 2 and I should take the contraction of these two, right? So that is the result that I have proved. Now let us identify what this contraction is okay, and it is something we have already studied and I will just quickly show you what that object is.

$$
\begin{aligned}
& : a(\vec{k}_1)\cdots a(\vec{k}_n)\cdots a^\dagger(\vec{p}_1)\cdots a^\dagger(\vec{p}_m)a(\vec{q}_1)\cdots a(\vec{q}_s) : && (31)\\
= \quad & a^\dagger(\vec{p}_1)\cdots a^\dagger(\vec{p}_m)a(\vec{k}_1)\cdots a(\vec{k}_n)a(\vec{q}_1)\cdots a(\vec{q}_s) && (32)
\end{aligned}
$$

$$
T\bigg(\phi_I(x_1)\phi_I(x_2)\bigg) =: \phi_I(x_1)\phi_I(x_2) : + \overline{\phi_I(x_1)\phi_I(x_2)} \qquad (33)
$$

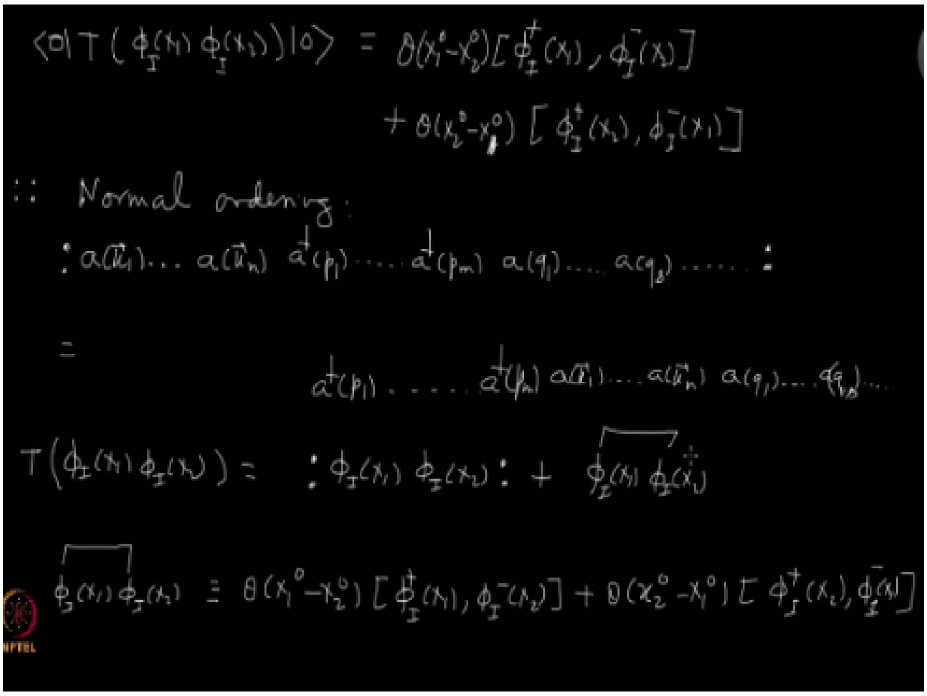
So for that recall that we had defined the Feynman propagator x prime minus x as theta of t prime minus t and then vacuum expectation value of phi x prime phi (x, 0). Now I will put I here because these are the ones which are evolving with free Hamiltonian, okay. And phi I have reserved for interacting case. So I should write phi I and t prime sorry t minus t prime I should write, okay.

So that was the definition of the Feynman propagator. You can go back and see. Now let us look at these quantities. See this quantity is what? This is a phi plus and phi minus. That is a phi plus and a phi minus. Most of it will kill the vacuum. Only one piece will survive, okay and that you can easily check that. Maybe I will leave it as an exercise. It is easy so I do not want to do it.

So you can check that if you take this okay you just get phi I plus x prime commutator phi I x, minus x okay. That is what you will get, okay. And with this you can show that our contraction of phi I x, what was it x 1, x 1 phi I x 2 this contraction is exactly D F x 1 minus x 2, okay. So this is, these are easy exercises to do. And with this we conclude that our time order product of two fields is just the normal product of these two fields normal ordered product of these two fields and a contraction, right.

So let me write it. So we get, we have proved that this is just normal ordered phi I x 1, phi I x 2 plus the Feynman propagator D F x and –x 2, okay. Or maybe I will leave it like this okay, where we know that this contraction is a propagator, okay. So that is a, that is going to be very useful for us. So let us put a box around here, okay.

I will give you another exercise which, in which you have to just follow the same steps, which I have used for this two point function and it is not difficult.



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$$\overline{\phi_I(x_1)\phi_I}(x_2) = \theta(x_1^0 - x_2^0)[\phi_I^+(x_1), \phi_I^-(x_2)] + \theta(x_2^0 - x_1^0)[\phi_I^+(x_2), \phi_I^-(x_1)] \qquad (34)$$

Recall that

So exercise, before I give the exercise, I will introduce a shorthand notation, okay. So I will introduce a notation first. So instead of writing phi I x 1, I will just write phi 1. Instead of writing phi I x 2 I will just write phi 2 two okay, just for ease of notation. Otherwise, I will end up spending too much time.

So use this and show the following that if you take the time ordered product of four fields okay where I have used this shorthand, you should show that this is equal to the following. Normal order product, so there will be normal ordering involved. So you have phi 1, phi 2, phi 3, phi 4. Okay that is equivalent of what you got here. So you got just these two. In fact, this result I can write like this.

I can write it as phi I x 1, phi I x 2 plus this contraction and put the normal ordering outside, right? I can do so because this is just a complex number, right. So there is no meaning of ordering between these. This is just the propagator. There is no operator left in here. It is proportional to identity operator. So I can pull out this thing and put it here, okay the same thing.

So in the case of with four fields, you will be able to show that you have this term plus phi 1, phi 2, phi 3, phi 4 and a contraction between phi 1 and phi 2, okay plus now instead of 1 getting contracting with 2, 1 will get contracted with 3. Now 1 will get contracted with 4. Then you have possibility of 2 getting contracted with 4 and then 2 could be contracted with 4, sorry and this one 2 was contracted with 3 and this one is 2 with 4. Then you can have 3 contracted with 4, correct? Plus 1 with 2 and 3 with 4. 1 with 3 and 2 with 4. 1 with 4 and 2 with 3. And this entire thing is normal ordered. So you see you get one term which has all the four operators.

Then you get 6 terms 1, 2, 3, 4, 5, 6 which have only two operators, right? This one has only 5 3 and 5 4 as operators. And this contraction is a complex number, right? That is a propagator. Similarly, this one has only two operators 5 2 and 5 4. And this contraction gives you a propagator between x 1 and x 3, okay. So these six contain two operators each and these three they are fully contracted.

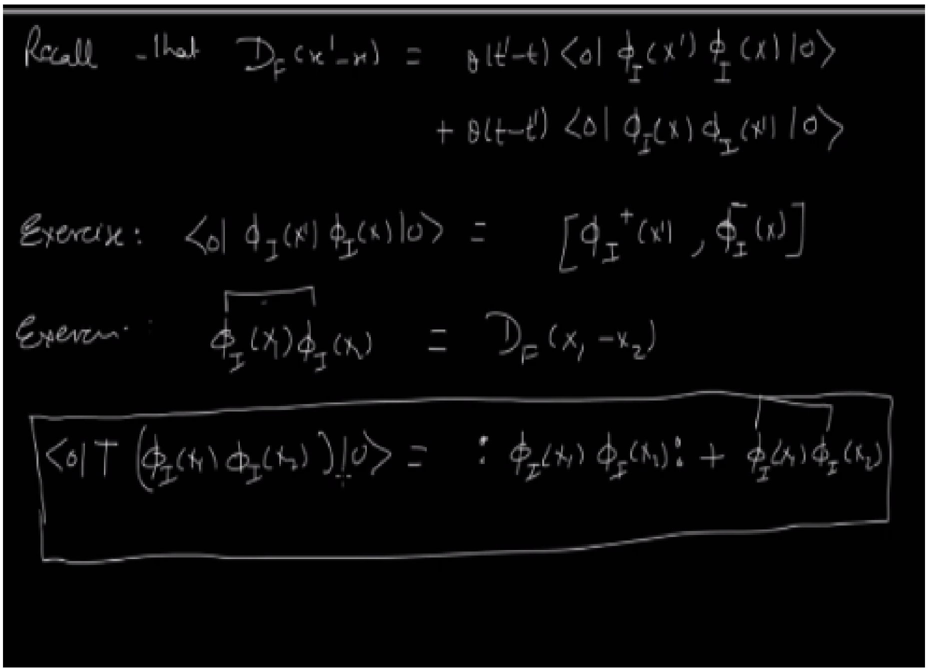

Figure 7: Refer Slide Time: 31:54

So there is no operator left at all here. Okay, I think I have written wrongly. I should omit this. It is just the time ordered product, okay. This is the result. Now you see that if you were to sandwich this time order product of these 4 fields between vacuum, this will go away. This will contribute to 0 because they will kill the vacuum because of normal ordering. So what I am saying is, what I am using is the following.

So if you have a normal order of a sum of operators, okay. So let us say O i's are some operators. Then this is just sum of the O i's in normal order okay, that is what I am using. So I can take the these normal orderings and put on the individual terms, okay. So then when you take this in the normal order, then this will kill the vacuum on if you sandwich the vacuum on both the sides. Similarly, this one this one and all the six.

But these ones, these are complex numbers and proportional to identity operator, only these three will survive, right. So we have the result that phi 2, phi 3, phi 4 in vacuum is just phi 1, phi 2 contraction and phi 3, phi 4 contraction. This and this okay we I have used the fact that this is equal to 1, okay. So you see that our problem has now reduced to writing the time ordered product of several fields in terms of normal order, okay.

$$D_F(x'-x) = \theta(t'-t)\,\langle 0|\phi_I(x')\phi_I(x)|0\rangle + \theta(t-t')\,\langle 0|\phi_I(x)\phi_I(x')|0\rangle \tag{35}$$

Exercise:

$$\langle 0|\phi_I(x')\,\phi_I(x)|0\rangle = [\phi_I^+(x'), \phi_I^-(x)] \tag{36}$$

Exercise

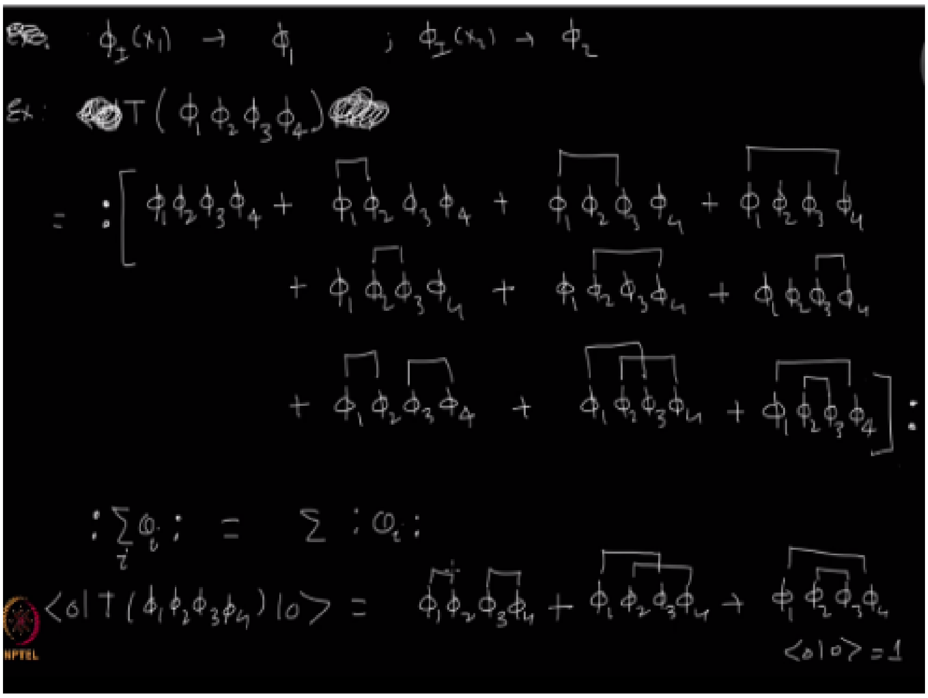

Figure 8: Refer Slide Time: 35:36

$$\overline{\phi_I(x_1)\phi_I}(x_2) = D_F(x_1 - x_2) \tag{37}$$

$$\langle 0|\, T\Big(\phi_I(x_1)\phi_I(x_2)\Big)\,|0\rangle =: \phi_I(x_1)\phi_I(x_2) : + \overline{\phi_I(x_1)\phi_I}(x_2) \tag{38}$$

$$\phi_I(x_1) \rightarrow \phi_I \quad ; \quad \phi_I(x_2) \rightarrow \phi_I \tag{39}$$

Because once we do that, all we are left with is these contractions which are nothing but the propagators, right? So if you wish I can write it further as D (x 1- x 2) D F (x 3 – x 4) + D F (x 1 – x 3) D F (x 2 – x 4) + D F(x 1 – x 4) and D F (x 2 – x 3) right, it is the same thing as before. Now let me state the most general case. So it goes by the name Wick's theorem. And it states that if you take time ordered product of n number of fields okay, then that can be written as a normal order.

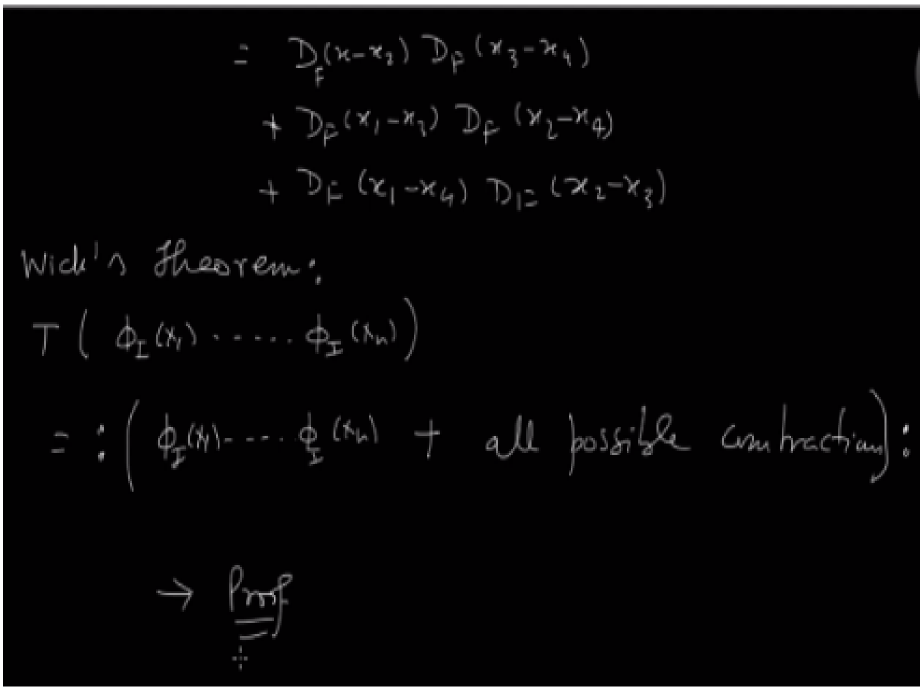

Figure 9: Refer Slide Time: 42:44

Example:

$$\mathrm{T}\Big(\phi_1\phi_3\phi_3\phi_4\Big) = \ : \Big[\phi_1\phi_3\phi_3\phi_4 + \phi_1\phi_2\phi_3\phi_4 + \phi_1\phi_2\phi_3\phi_4 + \phi_1\phi_2\phi_3\phi_4 \tag{40}$$

$$+\phi_1\phi_2\phi_3\phi_4 + \phi_1\phi_2\phi_3\phi_4 + \phi_1\phi_2\phi_3\phi_4 \tag{41}$$

$$+\phi_1\phi_2\phi_3\phi_4 + \phi_1\phi_2\phi_3\phi_4 + \phi_1\phi_2\phi_3\phi_4\Big] \tag{42}$$

$$:\sum \mathcal{O}_i = \sum : \mathcal{O}_i : \tag{43}$$

$$\langle 0|\mathrm{T}\Big(\phi_1\phi_3\phi_3\phi_4\Big)|0\rangle = \phi_1\phi_2\phi_3\phi_4 + \phi_1\phi_2\phi_3\phi_4 + \phi_1\phi_2\phi_3\phi_4 \tag{44}$$

$$= D_F(x_1 - x_2)D_F(x_3 - x_4) + D_F(x_1 - x_3)D_F(x_2 - x_4) \tag{45}$$

$$+D_F(x_1 - x_4)D_F(x_2 - x_3) \tag{46}$$

Wick's theorem

$$T\left(\phi_I(x_1)\cdots\phi_I(x_n)\right) =: \left(\phi_I(x_1)\cdots\phi_I(x_n) + \text{all possible contraction}\right): \tag{47}$$

Okay, so this most general case can be written as the following. So you take all the fields phi I x 1 up to phi I x n just like in your case of four fields plus you should do all possible contractions okay, just as we did here. So you have to construct all possible contractions starting with only one contraction, then going to two contractions, then three, four and so forth, okay.

And we will prove this result in the next video, okay. I will provide the proof in the next video. Okay, so let us meet in the next video then.