Introduction to Quantum Field Theory

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Lecture 41 : Phi-4 Theory: Interaction Picture Continued -2

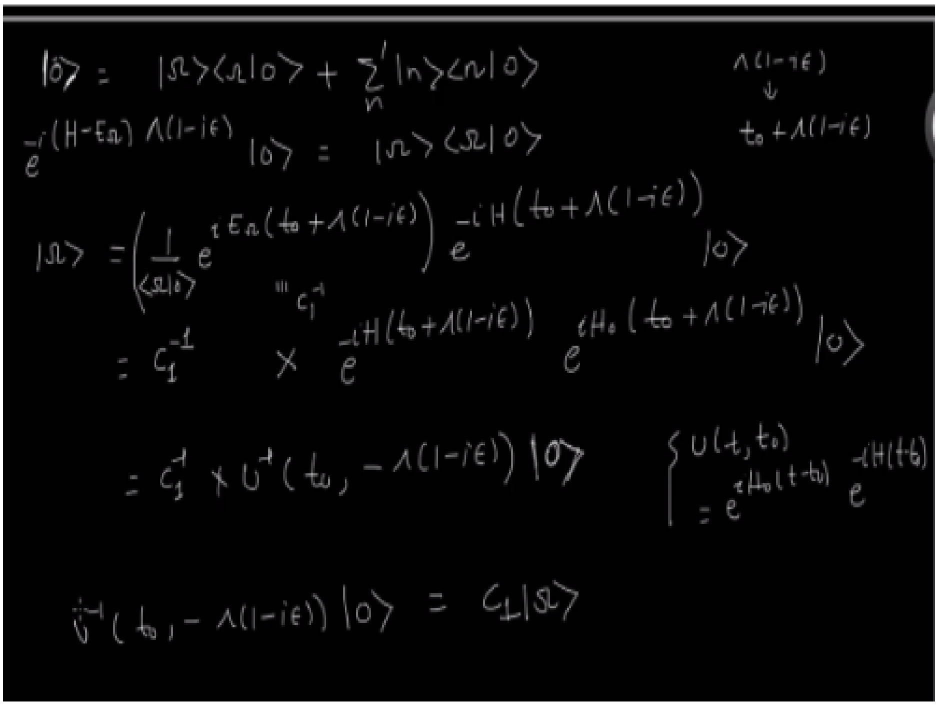

Figure 1: Refer Slide Time: 00:29

Okay, so last time we were looking at the fields and let us finish what we started for the ground state in the interacting theory. Okay, so we had done roughly the following. So if you look at the ground state of the free part of the Hamiltonian okay we had expressed as, there is some lag, I do not know why; like this okay. And then we had acted upon on both the sides with this operator. Okay and this gave us the following because all these states were killed when we took lambda going to infinity limit, keeping epsilon fixed okay, because they were exponentially suppressed.

So this is how we isolated. Now what I will do is I will shift lambda. So this piece is lambda 1 minus epsilon i epsilon. Now lambda is going to infinity, so even if I add or subtract a finite piece from it, it does not matter. So I can make the replacement and replace it but t naught plus lambda 1 minus i epsilon, okay because I am taking lambda to be infinity. Okay, so adding a finite piece does not matter. Okay, so if I do so then you can see that I can express ket omega here as the following. This algebra you can check. It will be e to the i e omega t naught plus lambda 1 minus i epsilon times e to the -i H t naught plus lambda 1 -i epsilon. And if you do not like what I am saying here that I can shift by t naught and if you think this is not nice, you can start with a change expression here.

So instead of starting with this, you can always have a t naught here itself, okay. No one can stop you from putting a t naught there. And then you can proceed and you will get the same things, okay. So whichever way you prefer you can do that. Okay, so this constant there is no operator here. These are just complex numbers. So I will call this c 1 inverse.

Okay, so it is some complex number. I call it c 1 inverse because I am going to eventually take it to this side. So that will become c 1 times ket omega. So here I call it c 1 inverse this thing. This vector is c 1 inverse. And then you have this piece okay. So this is c 1 inverse and let me write here times this factor. Now I will insert a factor here. I do not think I did that the time we were discussing this.

So let us do it now. So I will insert a factor e to the i H naught t naught plus lambda 1 minus i epsilon ket 0. And this I can do because this factor when acts on the free vacuum the Hamiltonian H naught has an eigenvalue 0, right. So this will become e to the 0 and e to the 0 is 1. So that is why I am allowed to do this, okay. So there is no problem in inserting this operator here, okay.

And if I do this I can write it as c 1 inverse and let us see what these two operators form. This is forming the following. It is U inverse t naught epsilon. Okay, let us check. So you have i H naught times something here and minus i H. Let us recall what U was, here. U is e to the i H naught t minus t naught times e to the –i H t minus t naught. I think I should write it down on the next sheet, on the present sheet. So you see when you take the inverse of it, the orders will get interchange and i will become –i. Note that I am not taking a dagger anymore, because instead of working with real t's see earlier H was Hermitian and t's were real; t and t naught not so real. And then taking dagger and inverse they were same thing, these were unitary.

$$
\begin{aligned}
|0\rangle &= |\Omega\rangle\langle\Omega|0\rangle + \sum_n{}' |n\rangle\langle n|0\rangle && (1)\\
e^{-i(H-E_\Omega)\Lambda(1-i\epsilon)}|0\rangle &= |\Omega\rangle\langle\Omega|0\rangle && (2)\\
|\Omega\rangle &= \left(\frac{1}{\langle\Omega|0\rangle} e^{iE_\Omega(t_0+\Lambda(1-i\epsilon))}\right) e^{-iH(t_0+\Lambda(1-i\epsilon))}|0\rangle && (3)\\
&= c_1^{-1} \times e^{-iH(t_0+\Lambda(1-i\epsilon))} e^{-iH_0(t_0+\Lambda(1-i\epsilon))}|0\rangle && (4)\\
&= c_1^{-1} \times U^{-1}(t_0, -\Lambda(1-i\epsilon))|0\rangle && (5)
\end{aligned}
$$

But now because I am taking time to be complex, we are taking lambda 1 minus epsilon, okay. So I have to be careful. Dagger does not give you inverse, okay. Because just because e to the i z if you were to take a dagger of it, you will get e to the –i z star, right? I am right now just thinking z of as a z not as an operator, but a complex number, okay. And this times this will not be 1, okay.

This times this is 1 means z is real. Or this times, so the dagger is not the inverse and what you should do is take inverse which will just be e to the –i z okay? So that is what we have to take care of. So when you take inverse the orders will change and the signs will change, okay. And that is what we have there, okay. So you see the signs of change and the order has changed.

Maybe I should write down somewhere here. U (t naught, t) is e to the i H naught t minus t naught into the -i H t minus t naught, okay. And with that we see yeah, that this will be the inverse of this, okay. And then of course, your ground state or the free ground state. So that we have and with this I can write the following that U inverse t naught minus lambda 1 minus epsilon acting on free vacuum is this constant c 1 times ket omega, okay.

So that is one relation we have found, okay. And that is useful because U is completely expressed in terms of phi I's. Okay we have already seen that we can do that. So our goal of expressing omega in terms of free vacuum and phi I's is achieved here. And we were interested in phi I's because phi I's are the things which evolve according to free theory, right? So that is why, this is what we wanted to do.

And similarly, we can look at the bra version of the vacuum and get the following. Let me show that part also explicitly. So what I want to do is so you take the vacuum in the bra form, and again, write it in the basis of eigenstates of the full Hamiltonian, okay. And now you act with this operator and when you do so on both the sides, you get the following. Okay, same logic as before. So when this operator is acting on ket omega okay, bra omega the Hamiltonian here will get the eigenvalue e omega, remember H is Hermitian.

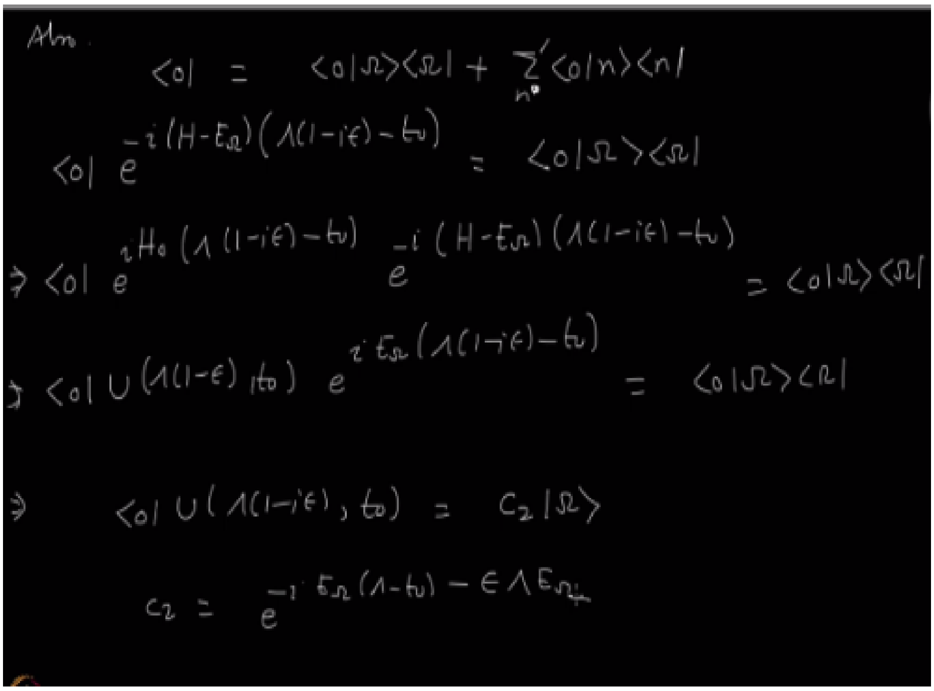

Figure 2: Refer Slide Time: 09:46

So it acts on the bra and gets the same eigenvalue as H acting on the ket. So this will give you e omega minus e omega. That gives you 1, e to the 0 is 1. So that is why all the factors have disappeared and you are left with only this piece, okay. And these ones will give you factors which will vanish when you take lambda to infinity, okay. So exactly the same argument as before, keep epsilon to be fixed, take lambda to infinity.

You will see you get an exponential suppression. So here you can see minus minus plus, okay. So minus, something is not looking good. Let me check. No sorry. Yeah, sorry I was thinking wrongly. So for any other state, higher state this H minus e omega will give you a positive number right because the state will have higher energy than e omega. So that will be positive number. And then you have -i times -i okay. That becomes a -1. So minus epsilon lambda times a positive number and when you take lambda to infinity that will give you exponential suppression. So all is good. Now what I will do is as before I will insert a factor here, which will contain H naught. So what I do is I write the above equation as follows.

So this is what I insert here and because this H naught has an eigenvalue 0 on this bra 0, this exponential gives you really 1 when it acts on the vacuum. So this is allowed and I should keep my this vector this one. So this is a complex number here. And then you have bra omega, okay. And you can check that what you have on the left hand side is the following.

So it is again U operator where time runs from t naught to lambda 1 minus epsilon, okay. If you look at the definition of U that we have, this is what you will get, okay. And then of course this piece. Not this piece but a piece coming from e omega. So I can write it down. Okay, so that is almost the same as before.

And with this I can finally write an equivalent expression to this one, the equivalent of this guy, okay which relates omega to free vacuum and the expression is this. So we have ket sorry bra 0 U of lambda 1 minus i epsilon t naught, okay. And this is equal to some constant c 2 times omega and where c 2 is following, okay. So this is good. So let me quickly summarize where we stand right now.

Where,

$$\Lambda(1-i\epsilon) \to t_0 + \Lambda(1-i\epsilon) \tag{6}$$
$$U(t,t_0) = e^{iH_0(t-t_0)}e^{iH(t-t_0)} \tag{7}$$

$$U^{-1}(t_0, -\Lambda(1-i\epsilon))\,|0\rangle = c_1\,|\Omega\rangle \tag{8}$$

Also,

$$\begin{aligned} \langle 0| &= \langle 0|\Omega\rangle\,\langle\Omega| + \sum_n{}' \langle 0|n\rangle\,\langle n| \qquad (9) \\ \langle 0|\, e^{-i(H-E_\Omega)(\Lambda(1-i\epsilon)-t_0)} &= \langle 0|\Omega\rangle\,\langle\Omega| \qquad (10) \end{aligned}$$

Maybe here, okay. So I have phi t x as phi i t x and you have U (t 0, t) and U inverse t 0, t okay. Now I am not writing dagger but writing inverse because of the reason I explained a little back, little time back, okay. And we have also shown that this operator U t 0, t we can express this time ordered exponential. So we have seen that this is -i t naught to t d tau H i tau, okay.

And then we have further shown just now that U inverse, let me give some space U inverse lambda 1 minus i epsilon, okay minus lambda 1 minus epsilon. This is minus to t naught this operator is acting okay so this operator acting on ket 0 is c 1 ket omega and the same thing for bra vector is the following, okay. So this is what we have found till now. Okay, so now we are almost there.

A little bit of work is left and we can then have a nice expression at the end. So we said earlier that all that you can ask in quantum field theory can be expressed in following terms. So you can look at these quantities, okay. And that they will answer your most general questions that you can ask if you have knowledge of all these objects, okay.

But now in this course which is anyway about to finish soon, we will be only interested in quantities that appear when you are studying scatterings, okay. So when you study scattering you start from some initial state and look at what is the amplitude that it will evolve into some final state and when you are looking at those objects, okay you do not require to know this entire set of inner products that you can construct.

Rather you require a much more restricted set and they are of this form, okay. So require these and what is this? This time ordering operator just takes this entire product of fields here and orders it according to time keeping the earliest time to the right most and as you go from right to left, the time increases, okay. So that is what this time ordering operator is. Now what I will do is I will look at look at this object, okay.

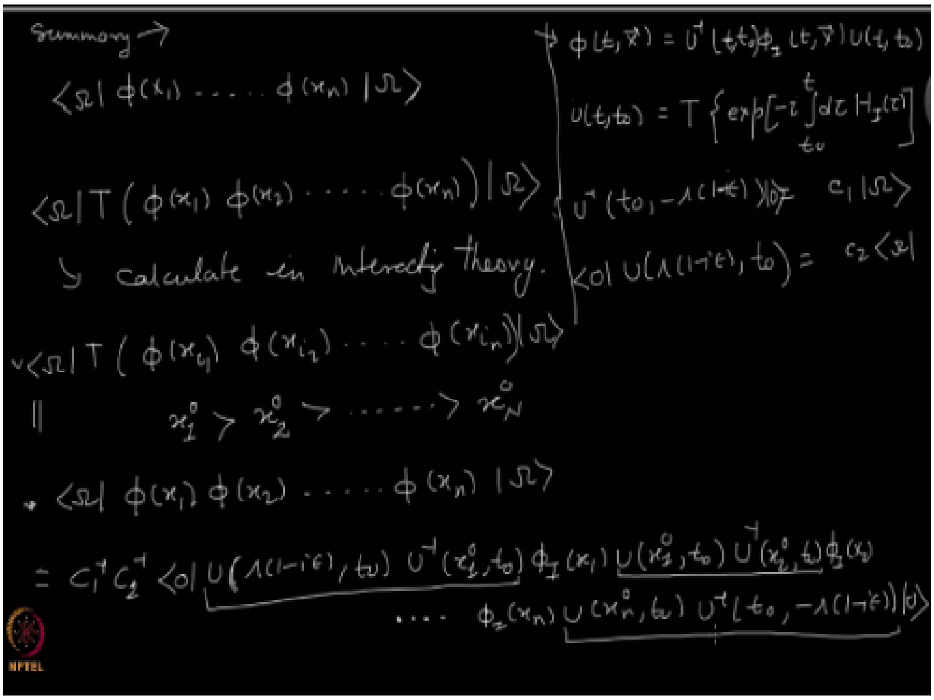

Figure 3: Refer Slide Time: 15:09

So for us, this is the object that we want to calculate in the interacting theory, okay. So now let us look at this. So I start with this one, okay. So I look at time ordered product of n number of fields, which are having their space time argument as x i 1, x i 2 and so forth, okay. And suppose that the times of these space time points are having this relation. So suppose if you choose x 1 naught.

So out of these x i 1, x i 2 and so forth, someone will have the maximum time largest time argument and that one I am calling the one that guy is called as x 1 and the time component is x 1 0 for that one okay. So that is the largest one and then one of them is x 2 and time corresponding to that is the second largest and so forth, okay.

So we have picked up we have picked out I mean, we have found out which one which time arguments are largest and which and how they are ordered, okay. So now if this is the case, then the above expression I can write with this knowledge as following, right? So now these are in the right order. So this is the time ordered. So this and this are same because I have taken care of time ordering now, okay.

Now what I do is I use these results which I have here okay and for omega in the bra form and omega in the ket form I will use this. And for the phi's I will use this, okay. So I, what do I get? So this guy is equal to so it will have a c 1 inverse, c 2 inverse, that those constants will be multiplied here. And then you have free vacuum, this piece. Then you have U of lambda 1 minus i epsilon, t naught.

$$
\begin{aligned}
\langle 0|\, e^{iH_0(\Lambda(1-i\epsilon)-t_0)} e^{-i(H-E_\Omega)(\Lambda(1-i\epsilon)-t_0)} &= \langle 0|\Omega\rangle \langle\Omega| && (11)\\
\langle 0|\, U(\Lambda(1-i\epsilon),t_0) e^{iE_\Omega(\Lambda(1-i\epsilon)-t_0)} &= \langle 0|\Omega\rangle \langle\Omega| && (12)\\
\langle 0|\, U(\Lambda(1-i\epsilon),t_0) &= c_2 \langle\Omega| && (13)\\
c_2 &= e^{-iE_\Omega(\Lambda-t_0)-\epsilon\Lambda E_\Omega} && (14)
\end{aligned}
$$

Summary

So that is one operator you have here. And then U (x 1) I will write in this form. Sorry phi (x 1) I will write using this expression. And what is that? It has a factor of U inverse, so U inverse x 1. So x 1 means I will use x 1 naught. So here t will be replaced by x 1 naught and the reference time t naught okay and then you have phi i at x 1 okay. Then you have U x 1 naught t naught, okay. Let us continue further.

So for phi x 2, I will again make this replacement. So I will have U inverse x 2 naught t naught phi i x 2. And then you will have again a factor of U here, I am not writing. And then you go further and then you will have at the end phi i x n. There will be a factor of U here, which I am or rather U inverse here, which I am not writing right now.

But I will write the one on the right and this will be U x n 0 t 0 and then this last piece, the ket omega, okay. This will give you U inverse t 0 minus lambda 1 minus epsilon ket 0, okay. So that is what we are going to get. Now you see that this combination is going to appear frequently, U, U inverse, this one. Here again U, U inverse. There U, U inverse and in between also.

All these places will have phi's and then U inverse in between, okay. And also note that when you have U, U inverse, the first argument is always t naught here, the first argument of both U and U universe is t naught and same. Okay, this one is opposite, so we will worry about it. Okay, good then. So what I will do is I will define an object v t prime t to be exactly what we are getting there. So we are getting U inverse some t prime. And the first argument is t naught. And then we had a factor of U with again the first argument as t naught and the second argument was something else, so let us call it t. So that is the definition of v, okay. And if you check substituting what U inverse is and what U is, you will get the following. That is one line algebra that you can do.

Okay, so check this and also check that if you were to take a derivative of v with respect to t, you are going to get at H I t v (t, t prime), okay. So that you can do. You already know how things act on U. So this will work out. And of course, from the differential here itself, it is clear that if you take v t prime t prime that is 1 okay. Because if t prime is same as t, this is 0, this is 0.

This is, so there is something wrong. This is t naught, okay. So, sorry so what I was saying was not correct. So if you put t equal to t prime, this will go away, it will be zero. So this will become 1 this factor and then you see that these two will just kill it, kill each other, these two exponential and then you get v t prime t prime as well. So that will be your, that will serve as your boundary condition.

And you already know how to solve this equation, right? This is the one we solved earlier where v was written as U earlier. So it will be the same solution because the differential equation is same, the boundary condition is same, so you get the same result. So you are going to get the following, okay. So now what I can do is I can take this result and substitute in the previous expression.

$$\phi(\vec{x},t) = U^{\dagger}(t,t_0)\phi_I(\vec{x},t)U(t,t_0) \quad (15)$$

$$U(t,t_0) = T\Big\{exp\Big[-i\int_{t_0}^{t} d\tau H_i(\tau)\Big]\Big\} \quad (16)$$

$$U^{-1}(t_0,-\Lambda(1-i\epsilon))\,|0\rangle = c_1\,|\Omega\rangle \quad (17)$$

$$\langle 0|\,U(\Lambda(1-i\epsilon),t_0) = c_2\,\langle\Omega| \quad (18)$$

So you when you go from here to there, you are still going towards higher time. You continue like this and the order is never broken, the time order is never broken as you can see here. When

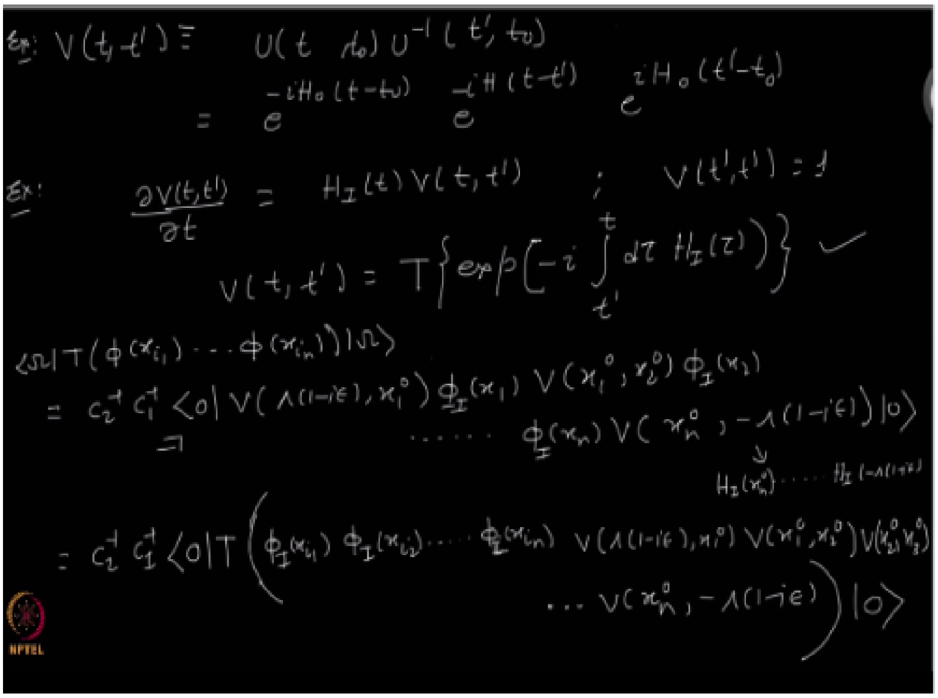

Figure 4: Refer Slide Time: 25:29

you reach here for example, the x 2, so this field is at time x 2 naught and then all the fields that appear in here, all the H I pieces, H I factors, they are ordered from in the right time order starting with x 2 naught and going towards higher time x 1 naught, right?

And then you again have x 1 naught. So this is also in right time order. And then from x 1 naught to infinity, plus infinity. So you see everything is correctly ordered in time within this expression. So what I can do is I can instead of having just t's here okay, I can just use one time ordering operator okay, and write this, okay. Phi I, now I will write x 1 sorry x i 1 0, okay. Because now it does not matter, right? Once I put t outside, what does it say? It says put everything inside in the right time order. So now I do not have to worry whether this is really the one which is of having the highest time argument, because even if it is not, this operator will reshuffle the fields which appear in here and bring phi i x 1 here, okay. So I can, in fact I should write it even more neatly. I should write it as phi i x i 1, phi i x i 2, phi i x i n.

And then we have all these v's which I should keep. So lambda 1 minus epsilon x 1 0. Then we have v (x 1 0, x 2 0) okay and so forth. Maybe I can write one more. X 3 0, x 2 0. Then you reach x n 0 minus lambda 1 minus epsilon, okay. I hope this is not difficult. Maybe let me try to say again in case it is not clear.

So if you agree with me that in this expression all the fields whether they are in v or whether they are explicitly written as phi I's, they all appear in correct time order, right. That is what we argued. Now when I come here, all I am saying is no matter which way you write the v's and phi I's in what order. This time ordering operator outside will put everything in the right order, right?

It will start from the lowest, field at the lowest values or lowest time values and go towards field at the highest time values. So the moment t acts on this, it gives you this expression, okay. So that is why I can just write it in this way, okay. If you agree with this then we are in very good shape because then I can do something. I can look at such products, this one and the next two v's and so forth.

And let us see, did I make a mistake? No, I did not. Okay. So I am going to utilize some property of v which is easy to check. Oops what happened? I know what happened. So here is another exercise that you can check. So show that if you have v (t 2, t 3) and v (t 2, t 1), okay. So this argument the second argument of this factor of this v and the first argument of this v if they are same, then you get v (t 1, t 3), okay. So this will not be difficult to check. And if you use that, then here what will happen? This x 1 naught argument will disappear, right? So let us read this. So what it does is this argument disappears these t 2.

And you get one v with this argument, first argument and this argument as the second argument. Okay, I am sorry. Sometimes I am saying this is the first argument. That is the second argument. But you see, you understand what I am saying. So what do we get? These two v's will combine to give v of lambda 1 minus epsilon, x 2 0 okay.

But then you have, so this becomes 1 v whose this the rightmost argument will be x 1 0. But then you have another v here whose first argument is x 2 naught. So that can combine with these two and then they together will give you, these three will together give you v of lambda 1 minus i epsilon x 1 naught x naught 1 naught they disappear x 2 naught, disappear and you get x 3 naught okay.

So if you just run this thing in the chain, you will end up with eventually one v because all the v's are going to combine with, so you get one v where the leftmost argument will be lambda 1 minus epsilon and the rightmost argument will be minus lambda 1 minus epsilon, okay. So that also you can check. Maybe I can write it as an exercise. So you have v of lambda 1 minus i epsilon x 1 0.

$$\langle\Omega|\phi(x_1)\phi(x_2)\cdots\phi(x_n)|\Omega\rangle \tag{19}$$

$$\langle\Omega|T\bigg(\phi(x_1)\phi(x_2)\cdots\phi(x_n)\bigg)|\Omega\rangle \tag{20}$$

Calculate the ground state in interacting theory.

$$\langle\Omega|T\bigg(\phi(x_{i_1})\phi(x_{i_2})\cdots\phi(x_{i_n})\bigg)|\Omega\rangle \tag{21}$$

$$x_1^0 > x_2^0\cdots > x_n^0 \tag{22}$$

$$\langle\Omega|\phi(x_1)\phi(x_2)\cdots\phi(x_n)|\Omega\rangle \tag{23}$$

Then you have v (x 1 0, x 2 0) and then you have v (x 2 0, x 3 0) and so forth. And you have eventually x n 0 minus lambda 1 minus i epsilon. And show that using this above result this is just here that is kind of obvious. But you can always check. Now we can substitute this result in the previous one, okay. So what do you get. You get, in fact I can jump one more step. So now I will use this result.

See what is v t, t prime. That is just t prime and t appear here. So t prime is the lower limit and the t is the upper limit and you have this time ordered exponential, okay. So I can take this and use it for this entire chain of products and substitute in this result, okay. So what do I get eventually? I get the following. Omega, now this no gain in writing x i 1, x i 2.

See here now whatever arguments are appearing here it is the same arguments appearing here. So I can just stop worrying about those and just write x 1 phi (x 2) phi (x n) okay and what this is equal to free vacuum one big time ordering operator. It is not big, it is just the bracket is big. And phi x 1, phi x 2, phi x n and then we had our exponential factor. What was that?

Exponential of minus i minus lambda 1 minus i epsilon to lambda 1 minus i epsilon d tau H I (tau), okay. Okay that is a nice result except for the fact that I have forgotten c 1 inverse c two inverse. And also another thing this should be I. Now everything looks okay, let me check. Okay, it does look okay. Okay, very good.

So this is very nice, because all these correlators okay, these are called by the way I forgot to tell you the names. So these are called correlators or Green's functions okay. So these objects are called Green functions. And if you have n number of fields, you call n point Green function or n point n point correlator. So this is the name given to this object.

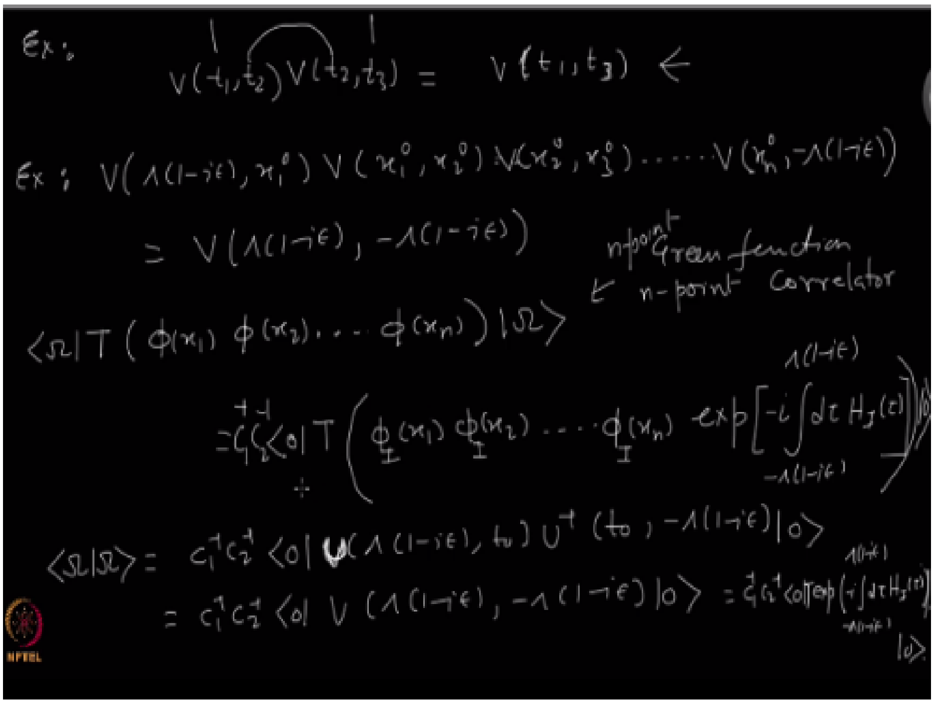

Figure 5: Refer Slide Time: 38:41

$$\begin{aligned} \langle\Omega|\phi(x_1)\phi(x_2)\cdots\phi(x_n)|\Omega\rangle =& c_1^{-1}c_2^{-1}\,\langle 0|\,U(\Lambda(1-i\epsilon),t_0)\,U^{-1}(x_1^0,t_0)\phi_I(x_1)\,U(x_1^0,t_0) \\ &\times U^{-1}(x_2^0,t_0)\phi_I(x_2)\cdots\phi_I(x_n) \\ &\times U(x_n^0,t_0)U^{-1}(t_0,-\Lambda(1-i\epsilon))\,|0\rangle \end{aligned} \tag{24}$$

Ex.

$$V(t,t') = U(t,t_0)U^{-1}(t,t_0) \tag{25}$$
$$= e^{-iH_0(t-t_0)}e^{-iH(t-t')}e^{-iH_0(t'-t_0)} \tag{26}$$

Ex.

$$\frac{\partial V(t,t')}{\partial t} = H_I(t)V(t,t') \quad ; \quad V(t',t') = 1 \tag{27}$$
$$V(t,t') = T\Big\{exp\Big[-i\int_{t_0}^{t} d\tau H_i(\tau)\Big]\Big\} \tag{28}$$

$$
\begin{aligned}
\langle\Omega|\phi(x_1)\phi(x_2)\cdots\phi(x_n)|\Omega\rangle =& c_2^{-1}c_1^{-1}\,\langle 0|\,V(\Lambda(1-i\epsilon),x_1^0)\,\phi_I(x_1) \\
& \times V(x_1^0,x_2^0)\phi_I(x_2)\cdots\phi_I(x_n) \\
& \times V(x_n^0,-\Lambda(1-i\epsilon))\,|0\rangle
\end{aligned}
\tag{29}
$$

$$
\begin{aligned}
\langle\Omega|\phi(x_1)\phi(x_2)\cdots\phi(x_n)|\Omega\rangle = c_2^{-1}c_1^{-1}\,\langle 0|\,T\Bigg(&\phi(x_{i_1})\phi(x_{i_2})\cdots\phi(x_{i_n})V(\Lambda(1-i\epsilon),x_1^0) \\
&\times V(x_1^0,x_2^0)V(x_2^0,x_3^0)\cdots V(x_n^0,-\Lambda(1-i\epsilon))\Bigg)\,|0\rangle
\end{aligned}
\tag{30}
$$

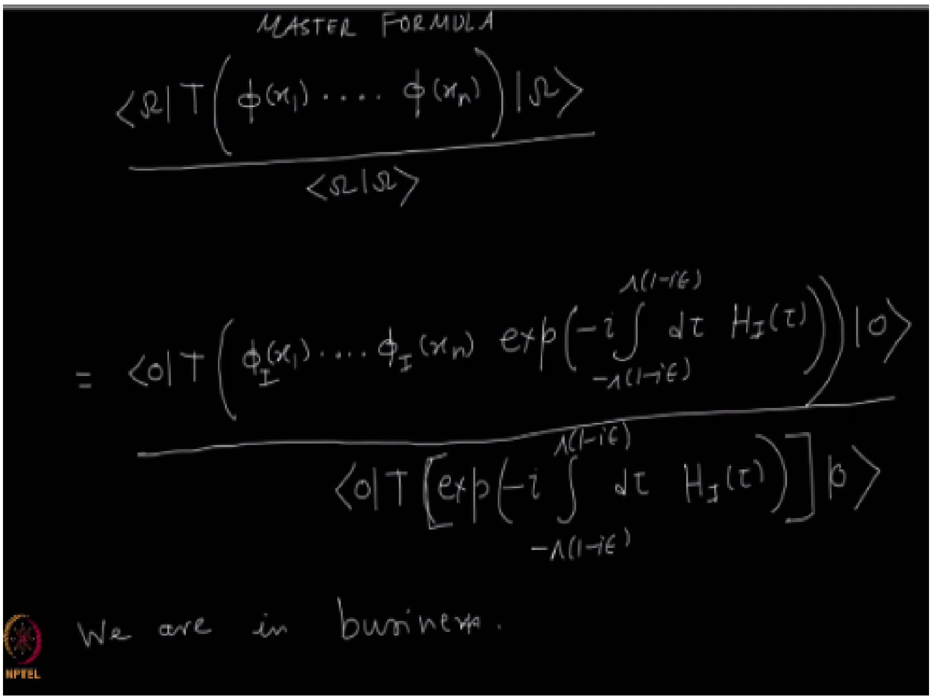

Figure 6: Refer Slide Time: 48:15

Ex.

$$
V(t_1,t_2)V(t_2,t_3) = V(t_1,t_3) \tag{31}
$$

$$
V(\Lambda(1-i\epsilon),x_1^0)\,V(x_1^0,x_2^0)\,V(x_2^0,x_3^0)\cdots V(x_n^0,-\Lambda(1-i\epsilon)) \tag{32}
$$

$$
= V\Big(\Lambda(1-i\epsilon),-\Lambda(1-i\epsilon)\Big) \tag{33}
$$

And that object, in interacting theory I have expressed completely in terms of quantities that exist in free theory, okay. The phi I's evolved with free Hamiltonian. The vacuum is of that of free theory. H I is also are made out of operators which are phi I's or in our specific case it is phi I to the 4, okay. So this is nice except for this c 1 inverse and c 2 inverse everything is good.

So what I can do is I can just construct another quantity which, with which I can get rid of the c 1 inverse, c 2 inverse. So what I will do is I will look at this object which is just c 1 inverse c 2 inverse and you can check I think it is I can write it. It is just U of lambda 1 minus epsilon t naught. And then you have U inverse t naught minus lambda 1 minus epsilon, okay.

And this we have just now seen that these combine nicely into one v, this is v. So that was U, okay. And you can check that this will c 2 inverse. This will combine into as we have shown above, it will combine into one v, okay. And then this is again simply c 2 inverse this exponential. There should be a time ordering here. I can just pull out the time ordering okay just like in this case.

And you have minus i again minus lambda 1 minus i epsilon to lambda 1 minus epsilon d tau H I tau okay. And then you have I am sorry this is now becoming. Okay, so this is what you get. Now this is, with help of this if I divide this Green function with this inner product, the c 1 inverse c 2 inverse will cancel and we will have nothing unknown here, okay.

So let me write down this final result that we have got nicely on this one page. And this will be our master formula to work with in the case of interacting theory, okay. So let me write it. I will call it master formula. Yeah.

n-point Green function / n-point correlator

$$\begin{aligned}\langle\Omega|T\Big(\phi(x_{i_1})\phi(x_{i_2})\cdots\phi(x_{i_n})\Big)|\Omega\rangle =& c_1^{-1}c_2^{-1}\langle 0|\, T\Big(\phi(x_1)\phi(x_2)\cdots\phi(x_n)\\ &\times\exp\Big[-i\int_{-\Lambda(1-i\epsilon)}^{\Lambda(1-i\epsilon)} d\tau H_I(\tau)\Big]\Big)\,|0\rangle \end{aligned} \tag{34}$$

$$\langle\Omega|\Omega\rangle = c_1^{-1}c_2^{-1}\langle 0|\, U(\Lambda(1-i\epsilon),t_0)U^{-1}(t_0,-\Lambda(1-i\epsilon))\,|0\rangle \tag{35}$$

$$\langle\Omega|\Omega\rangle = c_1^{-1}c_2^{-1}\langle 0|\, V(\Lambda(1-i\epsilon),-\Lambda(1-i\epsilon))\,|0\rangle \tag{36}$$

$$\langle\Omega|\Omega\rangle = c_1^{-1}c_2^{-1}\langle 0|\exp\Big[-i\int_{-\Lambda(1-i\epsilon)}^{\Lambda(1-i\epsilon)} d\tau H_I(\tau)\Big]|0\rangle \tag{37}$$

Master formula,

$$\frac{\langle\Omega|T\Big(\phi(x_1)\phi(x_2)\cdots\phi(x_n)\Big)|\Omega\rangle}{\langle\Omega|\Omega\rangle} = \frac{\langle 0|\, T\Big(\phi(x_1)\phi(x_2)\cdots\phi(x_n)\exp\Big[-i\int_{-\Lambda(1-i\epsilon)}^{\Lambda(1-i\epsilon)} d\tau H_I(\tau)\Big]\Big)\,|0\rangle}{\langle 0|\, T\Big(\exp\Big[-i\int_{-\Lambda(1-i\epsilon)}^{\Lambda(1-i\epsilon)} d\tau H_I(\tau)\Big]\Big)\,|0\rangle} \tag{38}$$

We are in business now.