Solid State Physics
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Lecture - 70
AC Josephson Effect and Microscopic Quantum Interference

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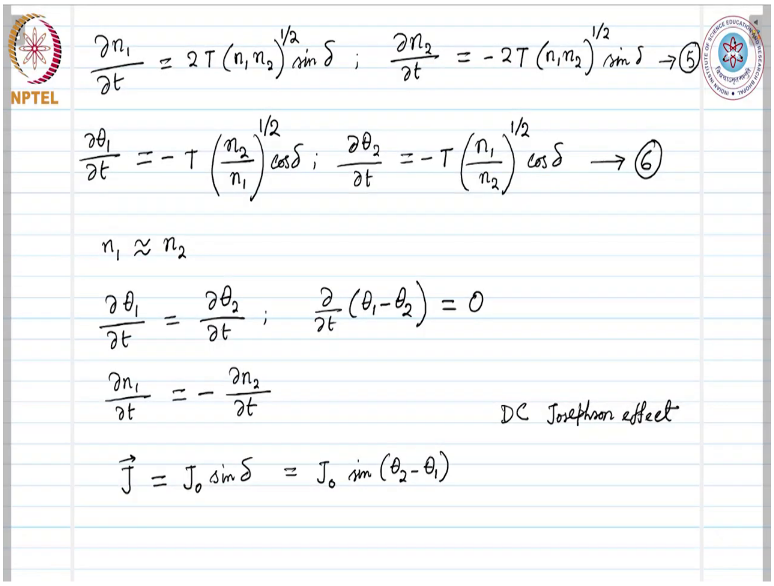

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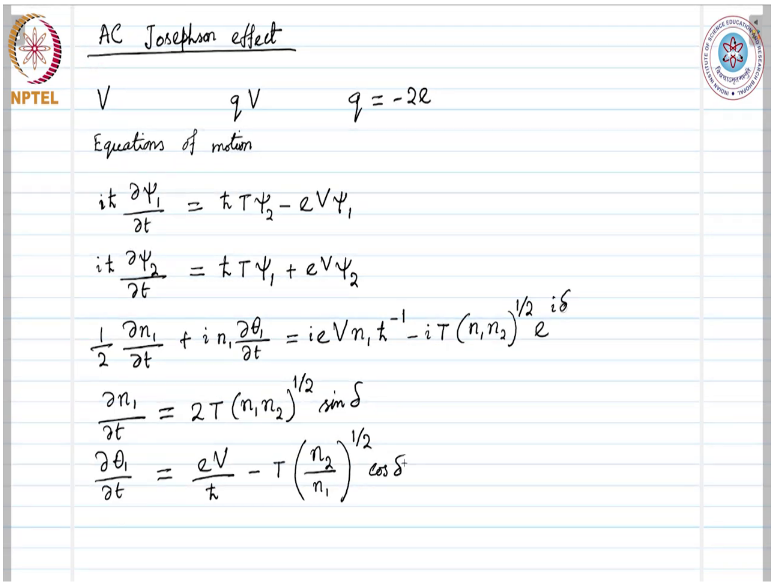

After this let us move on to AC Josephson Effect. In developing the mathematical treatment of AC Josephson effect, we will heavily rely on whatever we have done for the DC Josephson effect. So, AC Josephson effect is where we apply a DC voltage and get an oscillatory current. Let a DC voltage V be applied across the junction. We can do this because the junction is an insulator across that is that insulator we can apply a voltage there is no problem.

An electron pair experiences a potential energy difference; the energy difference would be q V where q is minus twice e, where e is the charge of a proton. We can say that a pair of one side is at potential energy minus e V and the other side is at plus e V for this one.

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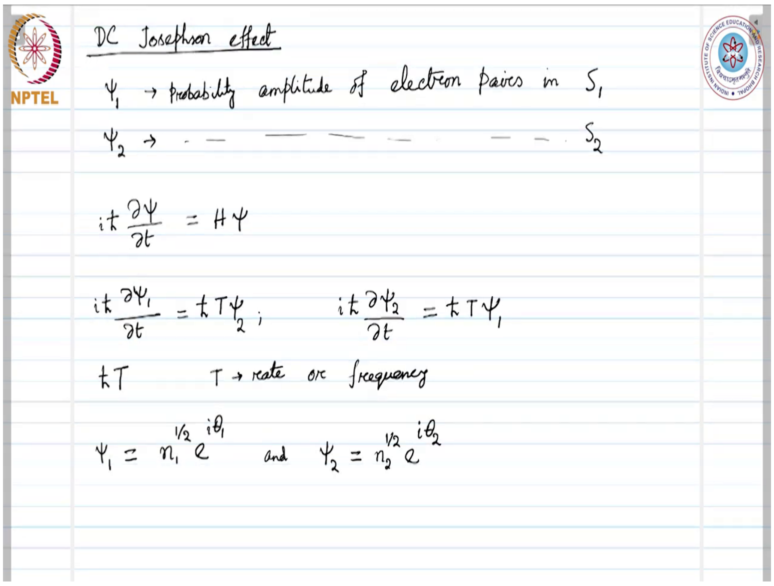

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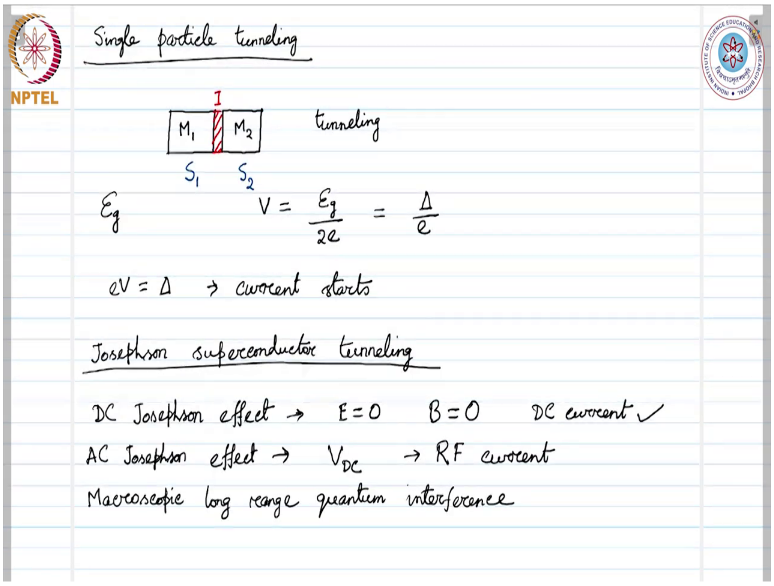

So, the left side we imagine in this picture as the potential minus eV and the right side at plus e V potential. Then the equations of motion would become i h cross del psi 1 del t equals h cross t psi 2 minus e V psi 1.

And similarly i h cross del psi 2 del t this quantity would become h cross t psi 1 plus e V psi 2. Now we proceed just like earlier and to find in place of equation 3 that we had earlier here equation 3. Here we would find just similarly for this applied voltage half del n 1 del t plus i times n 1 del theta 1 del t equals i e this e is the charge of proton V n 1 h cross power minus 1 minus i capital T n 1 n 2 power half e power i delta.

This equation if we now compare the real parts and the imaginary parts. From the real parts we obtain equating from both sides the real parts del n 1 del t would be equal to twice T n 1 n 2 power half sin delta. And from the imaginary part we would obtain del theta 1 del t that equals to e V over h cross minus T n 2 over n 1 power half cosine of delta.

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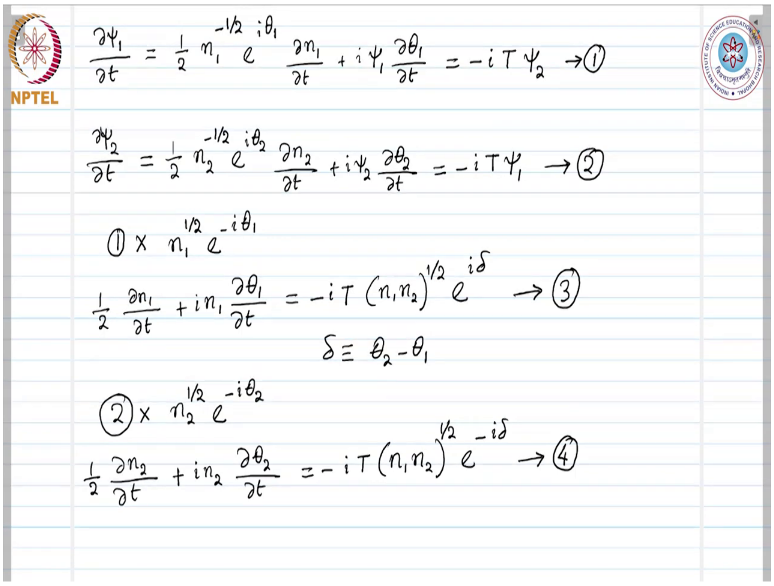

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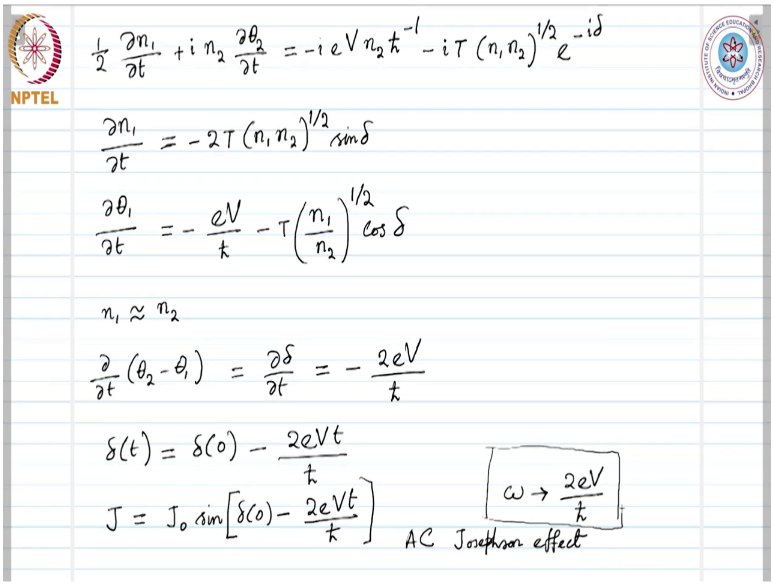

And if we now consider something similar to equation 4 here for the context of applied external voltage, we will find a similar equation half del n 1 del t plus i n 2 del theta 2 del t equals minus i e V n 2 h cross power minus 1 minus i capital T n 1 n 2 power half e power minus i delta.

Here we have to remember that del n 1 del t; this is given as minus twice T n 1 n 2 power half sin delta. And del theta 1 del t equals minus e V over h cross minus T n 1 over n 2 power half cosine of delta this we obtain by equating the real parts and the imaginary parts of the above equation here. From the real parts we obtain the first one and from the imaginary part we obtain the second one.

Now, because we have assumed n 1 nearly equal to n 2 because the superconductors are of identical material and if we use the above equations, we can write del del t of theta 2 minus theta 1 which is del delta del t that would be equals minus twice e V over h cross. If we integrate these equations then we find with a DC voltage across the junction the relative phase of the probability amplitudes as delta t equals delta 0 minus twice e V t over h cross.

The superconducting current for this phase can be given just like earlier that is J equals J naught sin delta 0 minus twice e V t over h cross which is oscillatory in time. And the frequency omega is given as twice e V over h cross; that is the frequency of this current that we see.

This is the AC Josephson effect where by applying a DC voltage we obtained an oscillation of the current with this frequency; twice e V over h cross. A DC voltage of 1 micro volt produces a frequency of 483.6 megahertz. So, by measuring voltage and frequency it is possible to obtain a very precise value of the ratio of fundamental constants e over h cross.

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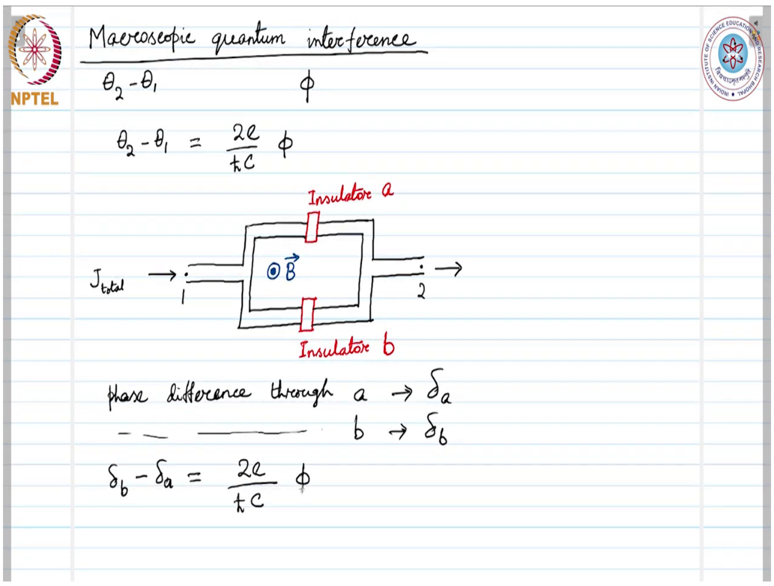

After discussing this we move on to macroscopic quantum interference. The phase difference theta 2 minus theta 1 around a closed circuit which encompasses a total magnetic flux phi that can be given as theta 2 minus theta 1 equals twice e over h cross c phi.

The flux is the sum of that due to external field and due to the current in the circuit itself as we have discussed in the context of flux quantization through a superconducting ring. So, here we consider two Josephson junctions in parallel. Here is an insulator, here we have another insulator and we have superconductors connecting them like this and they would be connected in parallel like this.

So, we have a current J total that is coming through this side through this point and this is point 1 and this is exiting through this point which is point 2. So, this red block here is insulator a and the other red block is insulator b. And we consider a magnetic field pointing into the screen. We have this kind of an arrangement of course, the insulators have been drawn thick here in practice it has to be very thin, otherwise the effect would not work we would not have any tunnelling.

Now, this is the situation of a interference for an interference. No voltage is applied and there is a magnetic field. Let the phase difference between point 1 and 2 taken on a path through junction a be delta 1 delta a. Similarly the phase difference through b we call it delta b.

If there is no magnetic field then these phases must be equal, but if we have a magnetic field and a magnetic flux phi passes through the interior of the circuit; phi is the flux that is enough we do not really need to consider what kind of field distribution we have.

Then usually we do it with a straight solenoid normal to the plane of the screen and lying inside the circuit. If we have that kind of a flux then we can write the difference between these two phases delta b minus delta a as twice e over h cross c times phi.

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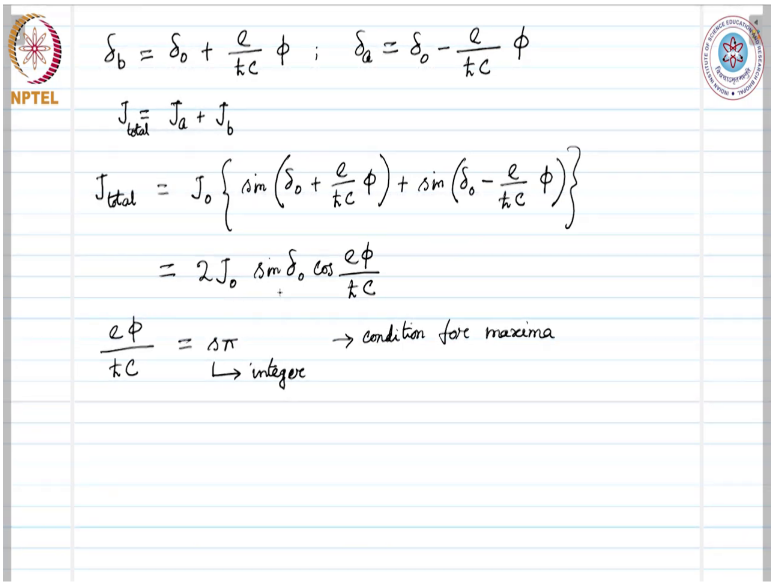

Then in details we can write delta b equals delta naught plus e over h cross c phi and delta a equals delta naught minus e over h cross c phi. The total current is the sum of J a and J b through a and through b. So, we can write it as J total equals J naught times sin of delta naught plus e over h cross c phi plus sin of delta naught minus e over h cross c phi. And that would be equal to twice J naught sine delta naught cosine of e phi over h cross c by applying the trigonometric identities we can obtain this.

So, we have a sine term multiplied with a cosine term in here. The current varies with phi the magnetic flux and it has a max it has maxima many maxima when e phi times h cross c equals s times pi where s is an integer it could be any integer and that gives to maxima.

This has a short period corresponding to this cosine term ah; that means, the short period variation of the current is produced by interference from the two junctions two junctions interfere and produce this cosine term. The longer period variation is a diffraction effect that is this sine term here and that arises from the finite dimension of each junction because the junctions are not really extended.

This interference measurement is so high precision that squid magnetometer uses this principle for measuring a samples magnetic properties. So, this is the end of the course.