Solid State Physics
Lecture 68

Flux Quantization in a Superconducting Ring

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Now, let us discuss Flux Quantization in a Superconducting Ring. We prove that the total magnetic flux that passes through a superconducting ring may assume only quantized values; that means, discrete values. Integer multiples of the flux quantum, the flux quantum is given as $\frac{2\pi\hbar c}{q}$, this is in CGS unit the quantum of flux in a superconducting ring where q has been determined from experiment to be $q = -2e$ that is 2 of the electronic charge. The charge of an electron pair, the cooper pair. Flux quantization is a beautiful manifestation of a long range quantum effect in which the coherence of the superconducting state extends over a ring or a solenoid. Let us first consider the electromagnetic field as an example of a similar boson field. The electric field intensity, let us write it as $E(\overrightarrow{r})$ acts qualitatively as a probability field amplitude. When the total number of photons is large, the energy density may be written as, for large number of photons $\frac{1}{4\pi}E^*(\overrightarrow{r})E(\overrightarrow{r}) \simeq n(\overrightarrow{r})\hbar\omega$, where n is the number density of photons with frequency omega. So, we are just taking this example of photons in electromagnetic field to understand what could be the situation with BCS state the flux of a magnetic field. (Refer Slide Time: 03:31)

Now, we may write the electric field in a semi classical approximation in the following way; $E(\overrightarrow{r}) \simeq (4\pi\hbar\omega)^{1/2}n(\overrightarrow{r})^{1/2}e^{i\theta(\overrightarrow{r})}$. And similarly, $E^*(\overrightarrow{r}) \simeq (4\pi\hbar\omega)^{1/2}n(\overrightarrow{r})^{1/2}e^{-i\theta(\overrightarrow{r})}$, where $\theta(\overrightarrow{r})$ is the phase of the field. Theta a function of r, this is the phase of the field. A similar probability amplitude describes the cooper pair. The arguments that follow apply to a boson gas with a large number of bosons in the same orbital. We then can treat the boson probability amplitude as a classical quantity, just as the electromagnetic field is used for photons. Both amplitude and phase are then meaningful and observable. The arguments do not apply to a metal in the normal state because an electron in the normal state acts as a single unpaired Fermi on that cannot be treated classically. We first show that a charged boson gas obeys London equation. We consider $\psi(\overrightarrow{r})$ to be the particle probability amplitude, has similar meanings like a wave function. We suppose that the pair concentration is given as n, which is written as $\psi^*\psi$. We assume this to be a constant. At absolute 0, n would be $\frac{1}{2}$ of the concentration of electrons in the conduction band for n refers to pairs. Then, we may write $\psi = n^{1/2}e^{i\theta(\overrightarrow{r})}$ and $\psi^* = n^{1/2}e^{-i\theta(\overrightarrow{r})}$ are the phase vector. The phase $\theta(\overrightarrow{r})$ is important for what we will do next. The velocity of a particle as we can write from Maxwell's sorry, Hamilton's equations, we can write the velocity as $\overrightarrow{v} = \frac{1}{m}(\overrightarrow{p} - \frac{q}{c}\overrightarrow{A}) = \frac{1}{m}(-i\hbar\overrightarrow{\bigtriangledown} - \frac{q}{c}\overrightarrow{A})$. This is in CGS units. (Refer Slide Time: 08:16)

The particle flux that can be given from this expression for velocity coming from Hamilton's equation as, $\psi^*\overrightarrow{v}\psi = \frac{n}{m}(\hbar\overrightarrow{\bigtriangledown}\theta - \frac{q}{c}\overrightarrow{A})$. So, the electric current density can be given as $\overrightarrow{j} = q\psi^*\overrightarrow{v}\psi = \frac{nq}{m}(\hbar\overrightarrow{\bigtriangledown}\theta - \frac{q}{c}\overrightarrow{A})$. Everything so far is in CGS unit. We take the curl of both sides of this equation and that gives us $\overrightarrow{\bigtriangledown} \times \overrightarrow{j} = \frac{nq^2}{mc}\overrightarrow{B}$ equals curl of a gradient which goes to 0. And what we are left with is $\overrightarrow{\bigtriangledown} \times \overrightarrow{A}$. This is in CGS units. Here, we use the fact that the curl of a gradient of a scalar that always goes to 0, to get rid of the contribution from this term. We recall that the Meissner effect is a consequence of the London equation. Quantization of the magnetic flux through a ring is a dramatic consequence of this equation; the equation of the current density here. Now, let us consider let us discuss the picture of a ring, superconducting ring. Let us consider the superconducting ring like this. Between these 2 lines that we have drawn here, the material the superconducting material exists and the flux through the ring is something that we are interested in. The flux lines go like this, somewhat like this. Now, we take a closed path through the interior of the superconducting material. We mark it in red here. This is through the interior of the superconducting material; that means, inside the superconducting material, does not go outside. We take a closed path that we call C. The Meissner effect tells us that the magnetic field and the current would be 0 in the interior. Now the current is 0, if we put current 0 in this equation here, equation of the j, then we will obtain $\hbar c \overrightarrow{\bigtriangledown} \theta q \overrightarrow{A}$. Because, only this part going to 0 can make the current 0. Then, we can write down that a $\oint_C \overrightarrow{\bigtriangledown} \theta \cdot d\overrightarrow{l} = \theta_2 - \theta_1$ for the change of phase on going once around the ring. The probability amplitude ψ that we have considered earlier is measurable in the classical approximation. So, that ψ must be single valued and $\theta_2 - \theta_1$, that must be integer multiple of $\theta_2 - \theta_1 = 2\pi s$, where s is an integer. (Refer Slide Time: 14:07)

Now, if we apply stokes theorem, we can write $\oint_C \overrightarrow{A} \cdot d\overrightarrow{l} = \int_C (\overrightarrow{\bigtriangledown} \times \overrightarrow{A}) \cdot d\overrightarrow{a}$, where $d\overrightarrow{a}$ is an area element. $\overrightarrow{\bigtriangledown} \times \overrightarrow{A} = \overrightarrow{B}$ the magnetic field, $\overrightarrow{B} \cdot d\overrightarrow{a} = \phi$, which is this is the flux. $d\overrightarrow{a}$ is an area element on the surface bounded by the curve C and ϕ is the magnetic flux through the curve C. Comparing previous equations that we have obtained earlier, we can now write, $2\pi \hbar c s = q\phi$; that means, $\phi = \frac{2\pi\hbar c}{q} s$, where s is an integer. Therefore, we find that the flux through the ring is quantized in integer multiples of $\frac{2\pi\hbar c}{q}$. By experiment we mentioned earlier that q = - 2e, that is as appropriate for electron pairs. So the quantum flux is in a superconductor can be written as phi naught, that is the quantum of flux. An integer multiple will give the actual flux. So, this quantum of flux can be written as $\phi_0 = \frac{2\pi\hbar c}{2e} \simeq$ the charge of a proton that is 2.0678 $\times 10^{-7}$ Gauss cm^2. And, if we write in SI unit phi naught can be written as $\phi_0 = \frac{2\pi\hbar}{2e} \simeq$ 2.0678 $\times 10^{-15}$ Tesla m^2. The flux quantum is called fluxoid or fluxon. The flux through the ring is the sum of the external flux that we can write as ϕ_{ext} and the superconducting flux the flux coming from the superconductor i.e. ϕ_{sc}. Normally there is no quantization condition on the flux from external sources. Therefore, the superconducting flux has to adjust itself appropriately; that means, the current through the superconductor has to adjust itself appropriately. So, that this sum is always quantized in terms of ϕ_0. It should be an integer multiple of ϕ_0, that is all for now.