Solid State Physics
Lecture 67
BCS Theory of Superconductivity

Now let us move on to BCS Theory the quantum theory that explains that describes the properties of superconductors quite nicely over a wide range. Of course, it does not cover every superconducting state, but this is a ground breaking theory paradigm shift in the field of Superconductivity. (Refer Slide Time: 00:47)

Many discoveries and inventions in the field of superconductivity fetched to Nobel Prize. The observation of superconductivity was certainly one and BCS theory was certainly one of those that fetched Nobel Prize. The basis of a quantum theory of superconductivity was laid by classic papers in the year 1957 by Bardeen, Cooper and Schrieffer. And hence the name acronym BCS there is a BCS theory of superconductivity with a very wide range of applicability from helium atoms in their condensed phase to type 1 and type 2 metallic superconductors. And also to high temperature superconductors based on planes of cuprite ions. Further there is a BCS wave function corresponding to particle pairs $\overrightarrow{k}\uparrow$ and $-\overrightarrow{k}\downarrow$ with wave vector $\overrightarrow{k}$ and spin up and wave vector $-\overrightarrow{k}$ and spin down. There is a wave function that correlates these two states; which means when treated by BCS theory gives the family a superconductivity observed in metals and exhibit the energy gap that we discussed earlier, this pairing is also known as s wave pairing. There are other forms of particle pairing possible within BCS theory. The BCS theory is quite an accomplished theory the main accomplishments of BCS theory we are going to discuss. Now, we will not derive the entire BCS theory because that is quite cumbersome and beyond the scope of this course we will rather discuss the outcome of BCS theory and the accomplishments whatever it achieved. So, this is an attract there is an attractive interaction between electrons that is an interesting part of this BCS theory an attractive interaction in the superconducting state between electrons, that can lead to a ground state separated from excited state by an energy gap this super conduct this is the superconducting gap very important. The critical field the thermal properties and most of the electromagnetic properties are consequences of the energy gap that we observe in superconductors every other property follows that. The electron lattice electron interaction leads to an energy gap of the observed magnitude. The indirect interaction proceeds when one electron interacts with the lattice and deforms it a second electron sees the deformed lattice and adjusts itself to take advantage of the deformation to lower its energy. Thus the second electron interacts with the first electron via the lattice deformation. The penetration depth and the coherence length emerge as natural consequences of the BCS theory. The London equation is obtained for magnetic fields that vary slowly in space. Thus the central phenomenon in superconductivity the Meissner effect is obtained in a natural way. The criterion for the transition temperature of an element or alloy involves the electron density of states which we write as $D(\epsilon_F)$ of one spin at the Fermi level and the electron lattice interaction U which can be estimated from the electrical resistivity; because the resistivity at room temperature is a measure of the electron phonon interaction. If we have U$D(\epsilon_F) \ll 1$, then BCS theory predicts that the T_C transition temperature would be $T_C = 1.140\theta \exp\left[-\frac{1}{UD(\epsilon_F)}\right]$, where θ is the Debye temperature and U is an attractive interaction. The result for T_C is satisfied at least qualitatively by the experimental data. There is an interesting apparent paradox the higher the resistivity at room temperature the higher is U and thus more likely it is that the metal will be a superconductor when cooled. Magnetic flux through a superconducting ring is quantized. And the effective unit of charge is twice that of the electronic charge rather than the electronic charge the BCS ground state involves pairing of electrons making flux quantization in terms of the pair charge twice E its a consequence of this BCS theory. Now let us discuss the BCS ground state. The filled Fermi in the ground state of a Fermi gas of non interacting electrons that is for normal metal the state allows arbitrarily small excitations. So, if we draw that kind of a state here we will see that the x axis is the energy axis and y axis is the probability of a particle being occupied at that energy. If we have a normal metallic state it looks like this where this is the Fermi energy and this is at absolute 0. And this allows for every small excitation everything is possible here increase the temperature little bit or give supply some energy some very tiny energy to an electron it will go to an excited state. The BCS theory shows that with an appropriate attractive interaction between electrons the new ground state is superconducting and is separated by a finite energy e_g the gap from its lowest excited state. What kind of situation would that make?. This is the energy this is the probability of the state being occupied at that energy, it would move like this where this much is the energy gap and this is the Fermi energy. The BCS state that is described in b here that contains admixture of electronic state from above the Fermi energy ϵ_F. That is marked here at first sight we can see that BCS state appears to have a higher energy than the Fermi state here in a, but that is about the kinetic energy that we see here. In BCA state there is an attractive potential which is not visible in this picture that potential energy brings the BCS state lower in energy with respect to the Fermi state for certain temperature and magnetic field within a range of temperature and external magnetic field. And that is the situation where the BCS state is more stable. When the BCS ground state of many electron system is described in terms of occupancy of one particle orbitals, those near the Fermi energy are filled somewhat like Fermi Dirac distribution for some finite temperature. As we are familiar with this kind of a distribution the central feature of the BCS state is that one particle orbital are occupied in pairs. (Refer Slide Time: 11:33)

If an orbital with wave function k and spin up is occupied, then then a state with vector $-\overrightarrow{k}$ and spin down will certainly be occupied. If this is unoccupied $\overrightarrow{k}\uparrow$ is unoccupied then $\overrightarrow{k} - \overrightarrow{k}\downarrow$ will also certainly be unoccupied. This kind of a correlation exists in the BCS ground state. The pairs are called Cooper pairs they have spin 0 and many attributes of bosons.