Solid State Physics
Lecture 66

Thermodynamics of Superconducting Transition, London Equation

Now, let us discuss the Thermodynamics of the Superconducting Transition. (Refer Slide Time: 00:31)

The transition between the normal and superconducting states is thermodynamically reversible. We treat type 1 superconductor with a complete Meissner effect; that means, the magnetic field equals 0 inside the superconductor. And in the magnetic method the stabilization free energy is found from the value of the applied magnetic field that will destroy the superconducting state at a constant temperature. Now, let us consider the work done on a superconductor which is brought reversibly at constant temperature from a position at infinity to a position r in the field of a permanent magnet. If we have a field of a permanent magnet like this, we bring a superconducting material from infinity to here at constant temperature, this is our superconducting material. Then the work done on this superconductor can be written as $W = -\int_0^{B_a}$, where B a is the applied magnetic field external field;$W = -\int_0^{B_a} \overrightarrow{M} \cdot d\overrightarrow{B_a}$ because the applied field is changing infinitesimally when we bring it from infinity to our place of interest which has a position vector $\overrightarrow{r}$. Now this much work done is per unit volume of the specimen. This work appears in the energy of the magnetic field, the thermodynamic identity of the process the free energy dF in differential free energy can be written as $dF = -\overrightarrow{M} \cdot d\overrightarrow{B_a}$. If we write it this way for a superconductor, then we can we also know that $\frac{M}{B_a}$. Now we write in CGS unit which can be written as $\frac{M}{B_a} = -\frac{1}{4\pi}$. So, we can write for a superconducting state dF_s, s for superconductor is $dF_s = \frac{1}{4\pi} B_a \, dB_a$ in CGS units. The increase in the free energy density for superconductors as we bring into a magnetic field can be given as $F_s(B_a) - F_s(0) = \frac{B_a^2}{8\pi}$. This is the amount of change in the free energy as we bring the specimen into the influence of external magnetic field from infinity. Now, consider a normal non magnetic metal if we neglect the small susceptibility of a metal in the normal state, then magnetization is 0 and the energy of the normal metal is independent of magnetic field. (Refer Slide Time: 05:23)

So, at the critical field we call it B_{ac} critical applied field, $F_N(B_{ac}) = F_N(0)$ this is the normal metallic state. Now, these results are all we need to determine the stabilization energy of superconducting state at absolute 0. At the critical value B_{ac} of the applied magnetic field, the energies are equal in the normal and superconducting states because if we are cooling the material through the superconducting state at through the sorry its not cooling, if we are reducing the magnetic field through high to low magnetic field then we reach that critical field where the free energy is equal. So, we can write as $F_N(B_{ac}) = F_s(B_{ac}) = F_s(0) + \frac{B_{ac}^2}{8\pi}$ as we have obtained earlier here in this equation. Now this is in CGS units. The specimen is stable in either state when the applied field is equal to the critical field it could be normal metal, it could be a superconductor with the help of previous equation we can now write $\Delta F = F_N(0) - F_s(0) = \frac{B_{ac}^2}{8\pi}$. This much is the energy difference in CGS units for no applied field where; that means, ΔF is the stabilization energy density for the superconducting state and this result is an excellent agreement with experiments now we discuss something called London equation. This is an important equation in the context of superconductivity, We saw that the Meissner effect implies a magnetic susceptibility $\chi = -\frac{1}{4\pi}$ in CGS units, in SI units its $\chi = -1$ and this is the characteristic of a Meissner effect perfect diamagnetic state. Electric conduction in the normal state of a metal is described by Ohms law. Ohms law is $\overrightarrow{j} = \sigma \overrightarrow{E}$ which we have written earlier in a different form in the form of resistivity now, here σ is the conductivity. We need to modify this equation drastically to describe conduction and the Meissner effect in a superconducting state. So, we describe conduction as well as Meissner effect not only conduction how do we do that? We make a postulate we postulate that in the superconducting state the current density is directly proportional to the vector potential $\overrightarrow{A}$. (Refer Slide Time: 09:56)

This is the notation for the vector potential of the local magnetic field and this vector potential is related with the magnetic field as $\overrightarrow{B} = \overrightarrow{\nabla} \times \overrightarrow{A}$ because the divergence of a magnetic field is always 0, it can be expressed in terms of a vector potential and as we have discussed as we have learnt earlier in electromagnetism that a vector potential can have some gauge as long as this expression gives us the correct magnetic field we are satisfied with it. The gauge of this vector potential will be specified now. In CGS units we write the constant of proportionality between $\overrightarrow{j}$ the current density and $\overrightarrow{A}$ the vector potential as $-\frac{C}{4\pi\lambda_L^2}$. Now why this kind of a form? We will come to that later that makes the current density $\overrightarrow{j} = -\frac{C}{4\pi\lambda_L^2}\overrightarrow{A}$ in CGS units of course, this equation is called the London equation. Now if we take curl on both sides we get $\overrightarrow{\nabla} \times \overrightarrow{j} = -\frac{C}{4\pi\lambda_L^2}\overrightarrow{B}$ in CGS units. The London equation is understood to be written with the vector potential in the London gauge. London gauge means that applies certain condition on the vector potential, the first condition is $\overrightarrow{\nabla} \cdot \overrightarrow{A} = 0$. And the second condition is the $A_n = 0$ on any external surface through which no external current is fed; n signifies this subscript n signifies the normal component. So, these two conditions make it a London gauge. So, if we have this kind of a gauge we will have the divergence of $\overrightarrow{\nabla} \cdot \overrightarrow{j} = 0$ and $j_n = 0$ the normal component of the current this also goes to 0 and this gives us the actual boundary condition. Because vector potential is not a measurable quantity rather current is a measurable quantity therefore, the expression in terms of the current that gives us the actual practical boundary conditions. First we verify that the London equation leads to Meissner effect we have obtained this equation lets verify it. Now let us write down a Maxwell's equation which is also known as Amperes law in this form in CGS units.$\overrightarrow{\nabla} \times \overrightarrow{B} = \frac{4\pi}{c}\overrightarrow{j}$. (Refer Slide Time: 14:34)

And this is under static condition where there is no displacement current. This is only the Amperes law without Maxwell's correction; that means, there is no displacement current in here now we take the curl on both sides of this equation that will lead us to $\overrightarrow{\nabla} \times (\overrightarrow{\nabla} \times \overrightarrow{B}) = -\overrightarrow{\nabla}^2\overrightarrow{B} = \frac{4\pi}{c}\overrightarrow{\nabla} \times \overrightarrow{j}$ this is what we obtain in again CGS units. Now, this may be combined with London equation to for a given superconductor for a superconductor this gives us $\overrightarrow{\nabla}^2\overrightarrow{B} = \frac{\vec{B}}{\lambda_L^2}$ if we club this with London equation written here. This equation is seen to account for Meissner effect because it does not allow a solution uniform in space so, that a uniform magnetic field cannot exist in a superconductor. If we now consider the magnetic field $\overrightarrow{B(\overrightarrow{r})} = \overrightarrow{B_0}$ = constant. This is not a solution to the above equation unless the trivial situation that is $\overrightarrow{B_0} = 0$. Of course, that kind of a solution would exist, but that is not of any interest further the Maxwell's equation that is the Amperes law ensures that the $\overrightarrow{j} \rightarrow 0$ when the $\overrightarrow{B} \rightarrow 0$. In the pure superconducting state, only field is allowed externally damped as we go in the external go in from the external surface. Let us consider a semi infinite superconductor occupy the space the right part of it is the superconductor the shaded region is a superconductor here. And if we apply a magnetic field here like this uniformly, then it would be decay decayed sharply as we entered the superconductor and go to 0 soon. This is the applied magnetic field and this is the magnetic field inside the superconductor. So, this can be expressed mathematically as $B(x)$, x is increasing as we move right in here $= B_0 \exp\frac{-x}{\lambda_L}$. In this example the magnetic field is assumed parallel to the boundary thus we see λ_L measures the depth of penetration of the magnetic field this is known as London penetration depth. Actual penetration depths are not described precisely by this lambda alone, for the London equation is known to be somewhat over simplified.