Solid State Physics
Lecture 63
Ferromagnetism

Hello, after discussing diamagnetism and paramagnetism, we will now move onto Ferromagnetism. (Refer Slide Time: 00:33)

Ferromagnets have spontaneous magnetic polarization that means they have a magnetization in the material even without an external magnetic field. So, there is some inter important concepts that we must learn, so that we can understand ferromagnetism. The concepts are called Curie point and exchange integral. If we consider a paramagnet with a concentration of capital N number of ions with spin S, and given an internal interaction tending to line these magnetic moments these spins along a certain direction parallel to each other if we have that kind of a situation, then we have ferromagnet. That means what? Let us consider N = 2, normally they are randomly aligned in any direction if there exist an internal field in the system that would try to make them line up parallel to each other. If that kind of an internal field exists, then all the spins will align along the same direction and then we will call it a ferromagnet. Of course this is an over simplified description. Ferromagnet is not that simple that always all the spins are aligned along the same direction. But to begin with this would be the first, first order description of ferromagnet ok. If we have that kind of a field, we can call this internal field as exchange field. If we treat the exchange field as equivalent to an external magnetic field because its exchange field we call it B_E, E for exchange. And applied magnetic field we call $\overrightarrow{B_a}$, so that is not relevant in this situation yet. And the magnitude of this exchange field this could be as high as $B_E \sim 10^3$ Tesla, which is extremely high magnetic field, a 1000 Tesla is extremely high magnetic field. This internal exchange field could be that high in magnitude. And we assume this exchange field to be proportional to the magnetization of the material magnetization is given as M. So, $B_E \propto M$ the magnetization. If we define the magnetization as magnetic moment per unit volume which is the conventional definition of magnetization, then in the mean field approximation, that means, when we consider that there is a mean field around an atom and the atom is under the influence of that mean field here it is a spin of that atom which is under the influence of that mean field, we can assume that each magnetic atom experiences a field proportional to the magnetization. That means, we can write from this expression the proportionality constant as lambda, so $\overrightarrow{B_E} = \lambda\overrightarrow{M}$ this is what we can write for the expression of the exchange field. Where λ is a constant, and it is supposed to be independent of temperature, and each spin sees the average magnetization of all other spins that is the mean field description of the problem. In reality, it may see only nearest neighbors. One spin does not see every other spin in its surrounding rather the nearest neighbors, few nearest neighbors that is all it can see, it can experience. But our simplification is good for first look at the problem. Now, we introduce Curie temperature. It is called it symbolized as T_c. This is the temperature above which the spontaneous magnetization vanishes it separates the disordered paramagnetic phase from the ordered ferromagnetic phase. Consider now the paramagnetic phase an applied field $\overrightarrow{B}_a$ will cause a finite magnetization and this in turn will cause a finite exchange field. So, $\overrightarrow{B_a}$ will cause the magnetization and that will cause $\overrightarrow{B}$, because $\overrightarrow{B_E}$ and $\overrightarrow{B_E}$ are related this way. So, then we can write $\mu_0 M = \chi_p(\overrightarrow{B_a} + \overrightarrow{B_E})$. This is in SI unit. And if we write in CGS unit that simple just μ_0 goes away that would be $\mu_0 M = \chi_p(\overrightarrow{B_a} + \overrightarrow{B_E})$ ok. If we have this kind of an expression which is the obvious expression according to the definition of susceptibility, then the magnetization is equal to a constant susceptibility times a field only if the fractional alignment is small. If we have all the spins aligned along one direction, then this expression does not hold good. (Refer Slide Time: 07:59)

The paramagnetic susceptibility can be given by the Curie law which is $\chi = \frac{C}{T}$. C is a constant. This is called the Curie constant. So, now, we can find that $MT = C(B_a + \lambda M)$. This expression we can find by combining all these expressions that we have. This gives us $M = \chi B_a + B_E$. And

$B_E = \lambda M$. So, putting all that together, we get this. And once we get this, that means, that leads to the expression for susceptibility $\chi = \frac{M}{B_a}$. And if we do that, we would find we can write it as equal to $\chi = \frac{C}{T - C\lambda}$. This is of course in CGS units. This is an important expression. The susceptibility has a singularity at $T = C\lambda$ as you can see from here. If $T = C\lambda$, then susceptibility would diverge. At this temperature and below, there exists a spontaneous magnetization because if $\chi = \infty$ we can have a finite magnetization for no applied magnetic field, zero magnetic field externally applied. So, this expression leads to another expression for χ which is C, $\chi = \frac{C}{T - T_C}$. As we argued that $T_C = C\lambda$. Therefore, we can write it this way, of course, in CGS units. This expression describes fairly well the observed susceptibility variation in paramagnetic region above the Curie point. We may determine the value of the mean field constant if by looking at whatever expression we have so far $\lambda = \frac{T_C}{T} = \frac{3k_B T_C}{Ng^2 S(S+1)\mu_B^2}$ in CGS units. The exchange field gives an approximate representation of the quantum mechanical exchange interaction. On certain assumptions, it is shown in quantum theory that the energy of interaction of atoms. (Refer Slide Time: 11:53)

For example, if we consider an i^{th} atom here and a j^{th} atom here, then i^{th} atom has a spin $\overrightarrow{S_i}$ of course, a spin vector according to classical theory is j. Spin vector here where j is $\overrightarrow{S_i}$ and $\overrightarrow{S_j}$ are the spins of these two atoms, then they have the exchange energy $U = -2J\overrightarrow{S_i} \cdot \overrightarrow{S_j}$. This can be derived from quantum mechanics, some simple model where J is called the exchange integral. This is related to the overlap of the charge distribution of atom i and atom j. And this model, this expression for the exchange interaction is called the Heisenberg model, Heisenberg spin model. Now, we can establish an approximate connection between the exchange integral j and the Curie temperature. Suppose that the atom under consideration has z number of nearest neighbors and each connected with the central atom by the same interaction J. For more distant neighbors, we take J = 0; we only consider interaction with the nearest neighbors. We are no longer within mean field theory when we are taking this kind of a model this is called a lattice model. So, J in that kind of a situation can be given as $J = \frac{3k_B T_C}{2zS(S+1)}$. So, this is about our discussion related to Ferromagnetism.