Solid State Physics
Lecture 62

Paramagnetic Susceptibility of Conduction Electrons

Hello. We have earlier discussed about Diamagnetism and Paramagnetism. Now, we shall discuss about Paramagnetic Susceptibility in case of a Conductor. So, if we have conduction electrons in the system, the lesson that we have learnt about paramagnetism is not valid the way we have learnt it. There should be some modification. (Refer Slide Time: 00:51)

Therefore, we shall here develop our understanding about paramagnetic susceptibility of conduction electrons. Let us discuss the problem first. If you consider classical free electron theory, then we get an unsatisfactory account of the paramagnetic susceptibility of conduction electrons. An electron has associated with it a magnetic moment of 1 Bohr magnetron if we consider 1 electron, the magnetic moment is 1 μ_B. So, one might expect that the conduction electrons would make a curie type paramagnetic contribution to the magnetization of the metal. Then, the magnetization of the metal would be given as $M = \frac{N\mu^2}{k_B T}B$. This would be the magnetization if we have N number of electrons in the system, each giving rise to μ_B amount of magnetic moment. Instead, we observe that the magnetization of most normal ferromagnetic metals is independent of temperature, while according to this expression it is inversely proportional to temperature. Experimentally, it is found to be independent of temperature for most normal metals. Pauli showed that if we apply Fermi-Dirac distribution, then this problem is corrected, it is resolved. We first try to understand it qualitatively. If we consider the probability of an atom lined up parallel to external magnetic field, and the probability of antiparallel orientation to external magnetic field, the difference of parallel orientation and anti-parallel orientation, the difference in probability can be given as $\frac{\mu B}{k_B T}$. If we have N number of atoms per unit volume, then the net magnetization becomes $\frac{N\mu B}{k_B T}$, obviously. Most conduction electrons in a metal have no possibility of turning over when a field is applied. Why? Because most orbitals in the Fermi c with parallel spins are already occupied. Only the electrons which are in the range of $k_B T$ of the top of the Fermi distribution have a chance to turn over in the field. Therefore, only the fraction $\frac{T}{T_F}$, T_F is the Fermi temperature, this fraction of the total number of electrons they contribute to the susceptibility. If we consider that, then the magnetization can be written approximately as $M = \frac{N\mu^2 B}{k_B T}\frac{T}{T_F}$ this fraction. So, it is not N number of electrons that contribute, but NT/T_F that number of electrons contribute which we cancel T - T here, we arrive at $M = \frac{N\mu^2 B}{k_B T_F}$. Now, this expression as you can see here is independent of temperature, T_F is the Fermi temperature which is a constant, therefore, this is independent of temperature. And this is also of the order of magnitude that is observed from experiment. Now, we calculate the expression for the paramagnetic susceptibility for a free electron gas at $T \ll T_F$. Here we need to understand the density of states first. (Refer Slide Time: 07:00)

If we have an external magnetic field applied, then the density of states for parallel and antiparallel electrons, they are not the same. Here, this is the energy axis, on this side we plot the density of states of the majority electrons, and this side we plot the density of states of the minority electrons. Say the Fermi energy is somewhere here, if we have this kind of a situation, then you can clearly see that there are these many majority electrons, while these few minority electrons. You can clearly see this from this picture. That means, if we take the difference then we would get something like this. The majority electron is larger in number, so the difference of majority and minority electrons would populate somewhere in this region. If we just draw that, it would look something like this. The concentration of electrons with magnetic moment parallel to the magnetic field, that can be given as $N_+ = \frac{1}{2}\int_{-\mu_B}^{\epsilon_F} d\epsilon D(\epsilon + \mu B) \approx \frac{1}{2}\int_0^{\epsilon_F} d\epsilon D(\epsilon) + \frac{1}{2}\mu B D(\epsilon_F)$. We arrive at this conclusion from these pictures that we have drawn here, and this is written for absolute 0 temperature. Now, the approximation is written for, this approximation is valid for $k_B T \ll \epsilon_F$, which we expressed in other way as temperature much much less than Fermi temperature. Now, the concentration of electrons with magnetic moment antiparallel to the magnetic field, that we write as $N_- = \frac{1}{2}\int_{\mu_B}^{\epsilon_F} d\epsilon D(\epsilon - \mu B) \approx$

$\frac{1}{2}\int_0^{\epsilon_F} d\epsilon D(\epsilon) - \frac{1}{2}\mu B D(\epsilon_F)$. This we can write just similar to the previous expression. (Refer Slide Time: 11:40)

If we have this then the magnetization would be given as $M = \mu(N_+ - N_-)$, obviously. So, if we calculate this, it will turn out to be $M = \mu^2 D(\epsilon_F)B = \frac{3N\mu^2}{2k_BT_F}B$. This would be the expression for magnetization putting the value of density of states for free electron model. Now, this expression gives us the Fermi sorry, the gives us the Pauli spin magnetization of the conduction electron for $k_BT \ll \epsilon_F$. And the density of states at the Fermi energy for free electron, that is given as $D(\epsilon_F) = \frac{3N}{3\epsilon_F} = \frac{3N}{2k_BT_F}$, putting this, we get this. Now, in deriving the paramagnetic susceptibility we have supposed that the spatial motion of the electron is not affected by the magnetic field, but the wave functions are modified by the magnetic field. That is what we have assumed. Landau has shown that for free electron this cause a diamagnetic moment equal to minus one-third of the paramagnetic moment. If we consider that the wave functions are altered, but the spatial motion of is not altered. Thus, the total magnetization of the free electron gas in that case after subtracting the diamagnetic part that would be $M = \frac{N\mu_B^2}{2k_BT_F}B$. If we subtract one-third from here this is what we get. One-third from this expression that we obtained here we get this to be the value of the magnetization. Now, before comparing this expression with experiment, we must take into account of the diamagnetism of ionic cores, the band effect, and of electron-electron interaction. If we take an example of sodium the interaction effect increases the spin susceptibility by about 75%. The interactions like band interaction, electron-electron interaction, and ionic core effect, that increases the sub spin susceptibility by 75%. Quite large number. The magnetic susceptibility is considerably higher for most of the transition metals that is the metals where some inner shell is not completely filled. So, these are more for transition metals for than that of alkali metals. The higher values suggest that the density of states is unusually high for the transition metals. If we have high density of states at the Fermi level, then this would be the situation. And this argument that the density of states must be higher in case of transition metal, that argument is in agreement with the measurement of the electron electronic part of the heat capacity that has to do with the density of states. So, this is how we explain the anomaly that we discussed to begin with that the magnetization is not dependent on temperature, and it after take taking into consideration the Fermi-Dirac distribution, and working out the magnetization, and the susceptibility this way, it almost agrees with the experiment.