Solid State Physics
Lecture 58
The Quantum Theory of Paramagnetism

(Refer Slide Time: 00:24)

(Refer Slide Time: 00:30)

Now, let us move on to Paramagnetism, paramagnetism is as opposed to diamagnetism where you apply a magnetic field along a direction and it develops a magnetic moment small magnetic moment in the opposite direction. The material itself develops a small magnetic moment opposite to the direction of the external magnetic field. In paramagnetism it develops a magnetic moment that is small in magnitude, but that is along the direction of the magnetic field and we will only consider the quantum theory of paramagnetism. Because there are flaws in the classical theory the magnetic moment of an atom or ion in free space that can be given by if μ is the magnetic moment that is given as $\mu = \gamma\hbar\overrightarrow{J} = -g\mu_B\overrightarrow{J}$. Now, there are new terms being introduced I will have to explain what these terms are the total angular momentum that is given as $\hbar\overrightarrow{J}$ which is nothing but the $\hbar\overrightarrow{J} = (\hbar L + \hbar S)$ the orbital and spin angular momentum parts. It is just the sum of these 2 the constant γ here is the ratio of the magnetic moment to the angular momentum it is called the gyromagnetic ratio. For electronic systems we can quantify this other constant g here which is called the g factor or the spectroscopic splitting factor as $g\mu_B = -\gamma\hbar$ from this definition here this equation here you can get this definition of g. And for electron spin g = 2.0023 which is taken usually as 2 for electron spins and that is what we are interested in. For a free atom the g factor can be given by Lande equation, where g is expressed as $g = 1 + \frac{J(J+1)+S(S+1)-L(L+1)}{2J(J+1)}$ this is the definition of the g factor. And we have a quantity μ_B this is the Bohr magneton which equals $\mu_B = \frac{e\hbar}{2mc}$ in CGS unit. And if we express it in SI unit that is $\mu_B = \frac{e\hbar}{2m}$ this is the Bohr magneton which quantifies the spin magnetic moment of one free electron. (Refer Slide Time: 06:07)

Now, we can consider the energy levels of the system in a magnetic field, the energy $U = -\overrightarrow{\mu}\cdot\overrightarrow{B} = m_J g\mu_B B$, where m_J is the azimuthal quantum number and it can take values from J, J-1, ..., -J. For a single spin and if it does not have any orbital moment, then m_J as we know is only $m_J = \pm\frac{1}{2}$ and we know that g = 2. Then $U = \pm\mu_B B$. That means, when we apply a magnetic field the earlier degenerate states would now get splitted into energy now we will have 2 levels like this, this one with $-\mu$ and this one with μ magnet magnetic moments with the separation in energy as $2\mu_B B$ this much is the separation in energy. If a system has only 2 levels then the equilibrium population ratio $\frac{N_1}{N}$, where N_1 is the population of the lower level and N_2 would we would consider as the population of the higher level. And $N = N_1 + N_2$ this ratio $\frac{N_1}{N} = \frac{\exp\mu B/\tau}{\exp\mu B/\tau+\exp-\mu B/\tau}$. Similarly, $\frac{N_2}{N} = \frac{\exp-\mu B/\tau}{\exp\mu B/\tau+\exp-\mu B/\tau}$. As we mentioned earlier $N_1 + N_2 = N$ the total number of electrons, N_1 is the population of the lower level and this is the population of the higher level; N is the total population. With this the projection of the magnetic moment of the upper state along the field direction is $-\mu$ and the lower state is μ, the resultant magnetization for N atoms per unit volume. (Refer Slide Time: 11:14)

That can be expressed if we assume $x = \frac{\mu B}{k_B T}$, then the resultant magnetization for the total N atoms that can be given as M equals the difference of the population becomes useful $M = (N_1 - N_2)\mu = N\mu\frac{e^x-e^{-x}}{e^x+e^{-x}}$, this expression comes from these ratios that we obtained here. And this expression is the expression for $\tanh x$; that means, the magnetic moment the magnetization would be $= N\mu\tanh x$. Now, if we consider low field high temperature limit; that means, B is small, T is large, $x \ll 1$, in this limit we have $\tanh x \approx x$ then the magnetization would be $M \approx N\mu\frac{\mu B}{k_B T}$. Now, if we consider a magnetic field and an atom is in that magnetic field with angular momentum quantum number J, it will have 2(J + 1) equally spaced energy levels. And in that kind of a situation the magnetization would be given by $M = N_g J\mu_B B_J(x)$, this whatever we did up to here was for 2 level system and this one is for total angular momentum J; that means, 2(J + 1) levels system here we have $B_J(x)$. Now, this expression is known as the Curie Brillouin law, we have introduced a new function here and a new quantity x, $x = \frac{gJ\mu_B B}{k_B T}$ and B_J this quantity this function is called the Brillouin function. (Refer Slide Time: 16:06)

Now, this Brillouin function is defined in the following way if we have $B_J(x)$ the expression for this function is $B_J(x) = \frac{2J+1}{2J}\coth\left(\frac{(2J+1)x}{2J}\right) - \frac{1}{2J}\coth\left(\frac{x}{2J}\right)$. Now, if we consider $J = \frac{1}{2}$ then x as we expressed x here $J = \frac{1}{2}$, g 2 2 that will cancel $\frac{\mu_B B}{k_B T}$ that will be left,$x = \frac{\mu_B B}{k_B T} \ll 1$ for high temperature and low magnetic field. If we have this kind of a situation then $\coth x$ can be expressed as $\coth x = \frac{1}{x} + \frac{x}{3} - \frac{x^3}{45} + ...$ Now, the susceptibility which is expressed as χ is nothing but the magnetization over the external magnetic field this quantity can be expressed approximately as $\chi = \frac{N_J(J+1)g^2\mu_B^2}{3k_B T}$. Which is $\frac{N_p^2\mu_B^2}{3k_B T} = \frac{C}{T}$.