Solid State Physics
Lecture 52
Quantization of Elastic Waves

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Now, let us consider quantization of the lattice vibration Quantization of Elastic Waves. The energy of a lattice vibration is quantized, the quantum of the energyas we have already discussed is called phonon. This is a boson that can be created and annihilated that means, if there is excitation, the there will be phonons if there is no excitation at all, there will be no phonon, and this is similar to the energy of elastic mode of an angular frequency ω like quantum harmonic oscillator. So, the energy $\epsilon = (n + \frac{1}{2})\hbar\omega$ the energy of an angular harmonic oscillator, quantum harmonic oscillator where n is the number of phonons. If there is no phonon at all that means, n = 0, we will still have some $\epsilon = \frac{1}{2}\hbar\omega$ that is the zero-point energy. We can quantize the mean square phonon amplitude, consider the standing wave mode of amplitude that is $u = u_0 \cos Kx \cos \omega t$. Here u is the displacement of a volume element from its equilibrium position. So, if we assume the equilibrium position to be x, it is the displacement from x so, it is the displacement x from the equilibrium position in the crystal and the energy in this mode as in any harmonic oscillator is half kinetic energy and half potential energy on an average, average over time I mean. So, the kinetic energy density can be expressed as $\frac{1}{2}\rho(\frac{du}{dt})^2$, ρ is the mass density. In a crystal of volume v, the volume integral of kinetic energy can be given as $\frac{1}{2}\rho V \omega^2 u_0^2 \sin \omega t^2$, V is the volume. The time average kinetic energycan be obtained as $\frac{1}{8}\rho V \omega^2 u_0^2$. How do we get this? Because the time averaged of $\langle sin^2(\omega t)\rangle_t = \frac{1}{2}$ so, we get this quantity which is $\frac{1}{8}\rho V \omega^2 u_0^2 = \frac{1}{2}(n + \frac{1}{2})\hbar$. (Refer Slide Time: 05:58)

So, the square of the amplitude can be written as $u_0^2 = \frac{4(n+\frac{1}{2})\hbar}{\rho V \omega}$. This relates the displacement in a given mode to the phonon occupancy n of the mode. So, after understanding this, let us move on to understanding the phonon momentum. A phonon of wave vector capital K will interact with particles such asphotons or neutrons and electrons as if it had a momentum of $\hbar K$. In crystals, there exist wave effect wave vector selection rule in the context of x-ray dispersion x-ray diffraction, we have found that $\overrightarrow{k'} = \overrightarrow{k} + \overrightarrow{G}$, this was the selection rule in the context of wave diffraction, and these were the wave vectors $\overrightarrow{k}$ and $\overrightarrow{G}$, these are wave vectors of electrons, this is what we have found. Now, the total wave vector of the interacting waves is conserved in a periodic lattice with the possible addition of a reciprocal lattice vector capital $\overrightarrow{G}$ that is allowed. So, the true momentum of the whole system is always rigorously conserved that means, if the scattering of the of a photon is inelastic that creates a phonon of wave vector capital K. So, if there is a photon that scatters in a in elastically creating a phonon of wave vector capital K, remember we are talking about phonons with wave vectors capital K and electrons with wave vector small k, then the selection rule will become selection rule for creation of phonon that is $\overrightarrow{k'} + \overrightarrow{K} = \overrightarrow{k} + \overrightarrow{G}$. On the other hand, if a phonon is absorbed in the process, then the selection rule is $\overrightarrow{k'} = \overrightarrow{k} + \overrightarrow{K} + \overrightarrow{G}$. So, these are the selection rules for creation and absorption of phonons.