Solid State Physics
Lecture 47
Carrier Concentration

Hello, after introducing the idea of semiconductors, its electronic structure, motion of electrons and holes in the semiconductor subject to an external electric field and the concept of effective mass; now, we will discuss about Carrier Concentration in semiconductors. (Refer Slide Time: 00:51)

We want the concentration of intrinsic carriers as a function of temperature in terms of the band gap; that is something we want to evaluate. We have already denoted the chemical potential as μ. This is the chemical potential for excitement of electrons to the conduction band at a given temperature T. In semiconductor physics, μ may be called the Fermi energy. At the temperature of interest, we may suppose for the conduction band of the semiconductor, if we have small temperature $E - \mu$, if this quantity is much greater than k_BT, also large E large energy ϵ and small temperature. For this kind of a situation, we will have this satisfied. At in semiconductor physics, μ is the Fermi energy because at the gap, the Fermi energy sits and in that kind of a situation, that will be the chemical potential. Unlike metal across the Fermi energy, we do not have states available on both sides; only one side we have states available, on the other side there is no state. Therefore, the energy associated with exchanging particles cannot be different from the Fermi energy. It has to be the same as the Fermi energy. Now, in this kind of a condition, if this condition holds good then we can write the Fermi Dirac distribution function $f_e \simeq \exp{(\frac{\mu - \epsilon}{k_B T})}$. How can we do that? This quantity the numerator becomes very large, the denominator becomes small, if this holds and in that kind of a situation; sorry, this is not the numerator. The distribution actually is $\frac{1}{\exp{(\frac{\mu-\epsilon}{k_B T})}+1}$. This was the distribution and this quantity becomes large, this exponential quantity compared to 1. So, we can ignore 1 here and this when taken in the numerator becomes this minus sign comes in the power. So, that is the reason, this expression is valid, when this approximation is valid. So, the Fermi Dirac distribution reduces to this and the probability that a conduction electron orbital is occupied that in this approximation would be $f_e \ll 1$ of and of a conduction band to be occupied; but still there is finite probability of some carriers being present in the conduction band. So, the energy of an electron in the conduction band would be given as $\epsilon_k = \epsilon_c + \frac{\hbar^2 k^2}{2m_e}$. This is assuming parabolic bands. What is ϵ_c? This is the energy of the conduction band edge or the conduction band minimum. Now, if we replace m_e that is mass of the electron with the effective mass of the electron, then the above equation is valid even for non-free electrons; otherwise, this equation looks pretty much like a free electron equation. This is the energy of a free electron. But if we put the effective mass here in place of the mass of the electron, then this equation will hold good even for electrons that are not free electrons. (Refer Slide Time: 07:00)

The corresponding density of states can be written as $D(\epsilon) = \frac{1}{2\pi^2}(\frac{2m_e}{\hbar^2})^{3/2}(\epsilon - \epsilon_c)^{1/2}$. The concentration of electrons in the conduction band would be given as $n = \int_{\epsilon_c}^{\infty} D_e(\epsilon) f_e(\epsilon) d\epsilon$. The density of electrons electronic states times the Fermi function times the differential energy. This quantity calculates to be $n = \frac{1}{2\pi^2}(\frac{2m_e}{\hbar^2})^{3/2} \exp{(\frac{\mu}{k_B T})} \int_{\epsilon_c}^{\infty} (\epsilon - \epsilon_c)^{1/2} \exp -(\frac{\epsilon}{k_B T}) d\epsilon$. Putting this expression of $D(\epsilon)$ here; putting this expression of $D(\epsilon)$ in this, we get this. Now, if we perform the integral, we would get n the concentration of electrons in the conduction band $n = 2(\frac{m_e k_B T}{2\pi\hbar^2})^{3/2} \exp{(\frac{\mu - \epsilon_c}{k_B T})}$. The problem is solved for n, if we know the value of μ; otherwise, we cannot calculate the value of n. Now, it is useful to calculate the equilibrium concentration of holes. Let us call the concentration of holes p. (Refer Slide Time: 10:39)

The distribution function Fermi Dirac like distribution function for holes would be given, it would be related with the distribution function for electrons and it would be given as $f_h = 1 - f_e$. Holes are basically missing electrons; therefore, this kind of a distribution function is justified. And then, we can write f_h knowing the Fermi Dirac distribution function $1 - \frac{1}{\exp{(\frac{\epsilon - \mu}{k_B T})}+1}$. This can be rewritten as $\frac{1}{\exp\left(\frac{\mu-\epsilon}{k_BT}\right)+1}$ which is $\simeq \exp\left(\frac{\epsilon-\mu}{k_BT}\right)$, provided $(\mu - \epsilon) \gg k_BT$. If the holes are near the top of the valence band that is the region where we consider holes because electrons are missing from there. So, the lack of electrons, they work as they behave as carriers and they are the holes. So, they behave like a particle with effective mass m_h. (Refer Slide Time: 13:35)

With this kind of an arrangement, the density of states for the holes can be given by $D_h(\epsilon) = \frac{1}{2\pi^2}\left(\frac{2m_h}{\hbar^2}\right)^{3/2}(\epsilon_v - \epsilon)^{1/2}$. What is ϵ_v? This quantity is the valence energy of the valence band edge. So, if we have a band structure of a semiconductor somewhat like this, this is the valence band edge energy ϵ_v, this is the conduction band end edge energy ϵ_c. Now, if we proceed just like the earlier, we can calculate the concentration of holes $p = \int_{-\infty}^{\epsilon_c} D_h(\epsilon) f_h(\epsilon) d\epsilon$. Let us put the same $\epsilon_c D_h(\epsilon) f_h(\epsilon) d\epsilon$. This quantity equals $p = 2\left(\frac{m_h k_BT}{2\pi\hbar^2}\right)^{3/2} \exp\left(\frac{\epsilon_c-\mu}{k_BT}\right)$. For the concentration of holes in the valence band, this is the expression we get and the concentration of electrons times the concentration of holes, this quantity is always a constant. You can verify it. So, in the mathematical steps, we have never considered any intrinsic carrier. So, it holds for also extrinsic that is doped semiconductors. We did not anywhere assume intrinsic semiconductor. Therefore, the same treatment carries for the doped semiconductor as well.