Solid State Physics
Lecture 46
Effective Mass

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When we look at the energy momentum that is wave vector relationship, we find that $E = \frac{\hbar^2}{2m}k^2$ for free electrons. Now if we differentiate this expression for group velocity, we can find $\frac{dv_g}{dt} = \hbar^{-1}\frac{d^2\epsilon}{dk\,dt}$ = $\hbar^{-1}(\frac{d^2\epsilon}{dk^2}\frac{dk}{dt})$ this is what we obtain. We know that $\frac{dk}{dt} = \frac{F}{\hbar}$ and if that is this quantity then $\frac{dv_g}{dt} = (\frac{1}{\hbar^2}\frac{d^2\epsilon}{dk^2})F$. So, $F = \frac{\hbar^2}{(\frac{d^2\epsilon}{dk^2})}\frac{dv_g}{dt}$. If we look at this equation and compare this with the Newton's law of motion, then we find that this quantity is force, this quantity is the acceleration. Its the rate of change of the group velocity therefore, it is the acceleration and this quantity represents mass in analogy with Newton's law; that means, we can write if we have m* equals effective mass which would be actually a tensor $(\frac{1}{m^*})_{\mu\nu} = \frac{1}{\hbar^2}\frac{d^2\epsilon}{dk_\mu dk_\nu}$. This is because along different directions the effective mass in general could be different therefore, we need to write down it into a tensor form although if we have perfect kind of a crystal perfect kind of a system, the effective mass may be a scalar quantity as well, but in general it is a tensor. So, this is the situation for anisotropic case. For isotropic case we can write uals effective mass which would be actually a tensor $(\frac{1}{m^*}) = \frac{1}{\hbar^2}\frac{d^2\epsilon}{dk^2}$ where the effective mass is a constant.
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Now, what does effective mass physically mean? We found within nearly free electron approximation that the wave function corresponding to the parabolic part of the band can be expressed as plane wave that the plane wave function is with a normalization constant e^{ikx} and the momentum was $\hbar k$. And the wave component that is $e^{i(k-G)x}$ with momentum $\hbar(k-G)$. This is small and increases only slowly with increasing k. So, in this kind of a region where the electron behaves more like a free electron, we have the effective mass m* is nearly equal to the real mass of the electron. Near the Brillouin zone boundary this component $(k-G)$ component is quite large and when this grows large a single electron in an energy band may have positive or negative effective mass negative effective mass occur near the top of the band. So, narrow bands have large effective mass and wide bands have small effective mass this you can find from the expression of the effective mass. Now, let us discuss about the effective mass in semiconductor. In many semiconductors it has been possible to determine the effective mass using the cyclotron resonance how do we do that? The effective masses of carriers in a doped semiconductor say we have p type or n type carriers and they can move very rapidly in the system and if we put them under the influence of an electromagnetic field then there is a cyclotron resonance in the semiconductor. Let us consider that the carriers are accelerated in a helical orbit about the axis of a static magnetic field. So, we have applying a magnetic field with. So, the magnetic field is static and if we make the electrons or holes that is the carriers move along an orbit it will have an angular frequency, cyclotron frequency and we will reach a resonance. So, if the if the resonance cyclotron frequency is given as ω_c we can write it as $\omega_c = \frac{eB}{m^*c}$, c is the velocity of light in CGS unit. And the expression of the cyclotron frequency in SI unit would be $\omega_c = \frac{eB}{m^*}$. So, what are we doing? We are applying a static magnetic field and making the electrons or holes that is the carriers move about that magnetic field m^* is the appropriate cyclotron effective mass and if we reach the resonance by applying a radio frequency electric field perpendicular to the static magnetic field, then we will make the electrons move along the circle. Then the from the magnetic field and the resonance frequency we can find out the effective mass of the system. Now holes and electrons in this kind of an arrangement rotate oppositely. So, here what kind of situation we are considering? We are considering a magnetic field applied in this direction and electric field is radio frequency electric fields that is it moves along both directions changes its side this is applied here. So, the electrons may be made to orbit like this and when it reaches resonance subject to this electric field, then at resonance we can find out the resonance frequency subject to the electric and magnetic field of course, and then we can find out the effective mass from this expression. This is how we can determine the effective mass from experiment.