Solid State Physics
Lecture 38

Transfer Matrix for a Rectangular Barrier

(Refer Slide Time: 00:24)

Now, let us consider a simple situation. We will calculate the Transfer Matrix for a Rectangular Barrier and a Piecewise Potential. (Refer Slide Time: 00:36)

Why do we consider this kind of a potential a rectangular barrier? Because we know how to solve for a rectangular barrier. So, let us first have some more insight into the transfer matrix and the tunneling problem, using a known potential, then we will go for the periodic potential that we have in a solid. So, we will discuss and apply the concepts that we have discussed so far, in case of a rectangular barrier. What kind of a rectangular barrier do we consider? The potential is 0 at the left it is V_0 within this barrier region and 0 at the right again. We have A_L amplitude coming from the left, A_R moving to the right amplitude and B_L amplitude moving to the left, in the left side B_R moving to the left, in the right side. V_0 is the potential here and if we consider this to be 0, this is the V(x) axis the potential axis this is 0 here, this coordinate is 'b'. And, if we name this part as left part, this is the intermediate part I and this is the right part R. If, we name it this way, then in the whole x-axis we can distinguish 3 different regions L, I and R. The general solution of the Schrodinger equation, that can be given in the left side region $\psi_L(x) = A_L e^{iqx} + B_L e^{-iqx}$. This is an oscillatory solution, if we consider that the electron energy $E < V_0$, even then this is the left side. So, here the potential is 0. If, it is if the energy is positive and $E < V_0$ here it would be an oscillatory solution for the region $x < b$. So, this is 'L' region. If, we now consider the intermediate region $\psi_I(x) = A_I e^{\beta x}$, which is not an oscillatory solution provided $V_0 > E$ the energy of the electron $\psi_I(x) = A_I e^{\beta x} + B_I e^{-\beta x}$, this is for the region $0 < x < b$, sorry the first region was $x < 0$. So, this is the intermediate 'I' region and the last region, the right side region,$\psi_R(x) = A_R e^{iqx} + B_R e^{-iqx}$, once again an oscillatory solution for the region $x > b$ which is 'R' region. We have this function q which is the function of the electron energy is given as $q(E) = \sqrt{\frac{2mE}{\hbar^2}}$. And, we have the other quantity β again a function of the electron energy is given as $\beta(E) = \sqrt{\frac{2m(V_0-E)}{\hbar^2}}$. The standard boundary conditions of continuity of the wave function and it is derivative. We will have to have the wave function and it is first order derivative continuous at x = 0. If we consider that we will also need it to be continuous at x = b. (Refer Slide Time: 06:48)

So, we will have from the boundary conditions; $A_L + B_L = A_I + B_I$ at $x = 0$. From the first order derivative at x = 0 we can write $A_L iq - B_L iq = A_I \beta - B_I \beta$, this is also from x = 0. We can express A_I and B_I in terms of A_L and B_L, how do we do that? We can write a matrix equation $\begin{pmatrix} A_I \\ B_I \end{pmatrix} = \frac{1}{2\beta} \begin{pmatrix} (iq+\beta) & (-iq+\beta) \\ (-iq+\beta) & (iq+\beta) \end{pmatrix} \begin{pmatrix} A_L \\ B_L \end{pmatrix}$. Similarly, if we consider the boundary condition at x = b, we can obtain that $\begin{pmatrix} A_R \\ B_R \end{pmatrix} = \frac{1}{2iq} \begin{pmatrix} (iq+\beta)e^{(-iq+\beta)b} & (-iq+\beta)e^{(-iq-\beta)b} \\ (-iq+\beta)e^{(iq+\beta)b} & (iq+\beta)e^{(-iq-\beta)b} \end{pmatrix} \begin{pmatrix} A_I \\ B_I \end{pmatrix}$. This is what the boundary condition gives us and once we have this the direct multiplication of the transfer matrix matrices, gives the transfer matrix for the rectangular barrier in the question. (Refer Slide Time: 10:51)

So, if we do that we get $s_{11} = e^{-iqb}[\cosh \beta b + i\frac{q^2-\beta^2}{2q\beta} \sinh \beta b]$; $s_{12} = e^{-iqb}(-i)\frac{q^2+\beta^2}{2q\beta} \sinh \beta b$. And, we have $s_{22} = s_{11}^*$. So, we can find s_{22} this way and $s_{21} = s_{12}^*$. So, we can obtain s_{21} also this way. So, what did we do? We have multiplied these two transfer matrices here; one is this, the other one is this, to get these matrix elements for the full transfer matrix corresponding to this rectangular potential barrier, this is what we have done. Now, if we consider the barriers shifted by 'd' appropriate phase factor has to be introduced in the transfer matrix. The transfer matrix of an arbitrary piecewise potential can be obtained by multiplying the component matrices in appropriate order. Now, if we consider $|s_{11}|^2 = 1 + \frac{q^2+\beta^2}{24q^2\beta^2 \sinh^2 \beta b]}$. And, the transmission coefficient of the barrier, that is given as capital $T(E) = 1|s_1 1|^2$, which is nothing but $T(E) = \frac{1}{1+\frac{V_0}{24E(V_0-E)\sinh^2\sqrt{\frac{2m(V_0-E)b^2}{\hbar^2}}}}$. By putting the value of every term that we have for $0 \leq E \leq V_0$, in this range of energy, this is the transmission coefficient. And, now if we have the energy of the electron that is $E > V_0$; that means, the energy of the electron is more than the barrier that we have considered here. (Refer Slide Time: 15:24)

In that kind of a situation for $E > V_0$ we can similarly calculate the transmission coefficient, we will get it to be $T(E) = \frac{1}{1+\frac{V_0}{24E(E-V_0)\sin^2\sqrt{\frac{2m(E-V_0)b^2}{\hbar^2}}}}$, this is for $E > V_0$. If, we have $E = V_0$. In this kind of a situation, we will have the transmission coefficient which is the function of energy, which is now $T(V_0) = \frac{1}{1+\frac{1}{4}\beta_0^2 b^2}$. Let us put the write $\frac{\hbar^2\beta_0^2}{2m} = V_0$. This is the expression for the potential barrier when $E = V_0$ and this would be the transmission coefficient why?. Because, $E - V_0$ this quantity here is going to 0 and $\sin^2 0 = 0$. Similarly, from this one we can find that this quantity is going to 0. So, this term is not going to contribute and we will get just this part.