Solid State Physics
Lecture 37

Reflection and Transmission Amplitudes and Coefficients

Now, let us consider the Reflection and Transmission Amplitudes and the Coefficients. (Refer Slide Time: 00:30)

The transfer matrix that we have provided, that we have discussed here that provides an intuitive description of the in the electron tunneling through the given potential region. Now, if we consider the stationary solution of the Schrodinger equation, if we consider an impinging wave that comes from the left and gets partially reflected and partially transmitted through the potential region. (Refer Slide Time: 01:31)

A wave is coming from the left here one electron is moving in this direction towards the right, it hits the potential barrier, it gets partially reflected and partially transmitted. If we consider that kind of a situation then comes the question of reflection and transmission, then we can clearly see that this amplitude B_R has to be 0 in that kind of a situation. And, if $B_R = 0$, then the linear equations that we have obtained from the transfer matrix can be written as $A_R = s_{11}A_L + s_{12}B_L$ while $B_R = 0$, its $0 = s_{21}A_L + s_{22}B_L$. So, we can obtain the reflection and transmission coefficients as we will discuss now. The reflection amplitude if we call it r small $r = \frac{B_L}{A_L}$. Referring back to this picture B_L is the reflected part and A_L is the incoming part. So, $\frac{B_L}{A_L}$ gives us the reflection amplitude which is from the transfer matrix equations here $-\frac{s_{21}}{s_{22}}$. Now, the reflection coefficient which is represented as the $R = rr^* == |r|^2 = |\frac{s_{21}}{s_{22}}|^2$. Now, let us discuss the transmission amplitude given as $t = \frac{A_R}{A_L}$ and if we invoke the unimodularity of the transfer matrix that is the determinant equals 1 we can write the transmission coefficient $T = tt^* = |t|^2 = |\frac{1}{s_{22}}|^2$. (Refer Slide Time: 05:43)

And, the unimodularity of the transfer matrix, we can verify that R + T = 1. So, in terms of the reflection and transmission amplitudes, we can write the transfer matrix S = $\begin{pmatrix} \frac{1}{t^*} & \frac{-r^*}{t^*} \\ \frac{-r}{t} & \frac{1}{t} \end{pmatrix}$ This is the form that the transfer matrix takes for our problem of electrons moving from the left to the right hitting the barrier. So, now this transfer matrix has a very clear physical meaning. Next we will connect the transfer matrix corresponding to a given potential. We will consider a potential V(x) and we will have a transfer matrix S(d). Now, we will have a connection between them corresponding to a rigid displacement in the potential if we transform the potential from $V(x) \to V(x-d)$. If we do something like this, then how does the transfer matrix change? The general solution that we have considered earlier from that we can write this kind of a displacement in the potential the corresponding transfer matrix S(d) = $\begin{pmatrix} s_{11} & s_{12}e^{-2iqd} \\ s_{21}e^{2iqd} & s_{22} \end{pmatrix}$ This is the form of the transfer matrix upon this translation of in the potential. This when we shift the potential along the x-axis by displacement d the diagonal matrix elements remain unchanged. Only the off-diagonal matrix elements they acquire a phase factor $e^{\pm iqd}$.