Solid State Physics
Lecture 36
Tunneling of Electrons

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Hello. Now, we shall discuss Tunneling of Electrons. As we have discussed, in the context of covalent bonds, and in the context of linear chain of hydrogen atoms, that electrons interact, that is hopped from one atom to another, and that hopping is a tunneling method. It cannot, it does not really have enough energy to overcome that potential barrier. Nevertheless, in quantum mechanics it can tunnel from one atom to another. And that kind of tunneling happens in almost every solid. This is not transport. This is just hopping, that electron may come back, but this process is not very fast. And not many electrons tunnel at the same time, it is very few that does it. So, it is not transport, it is something else. So, transmission and reflection of electron has to be considered that to through an arbitrary potential barrier. And in this context, we will use the transfer matrix method to understand exactly what goes on in the tunneling process. So, here we consider the propagation of electron in one-dimensional crystal from one point to another. So, we can find out, we can determine the reflected and transmitted components of a wave that is electronic wave that hits a given potential. Let us consider an arbitrary profile of a potential barrier. Let us consider that we have potential axis here $V(x)$, and this is the x axis in one-dimension. And let us say this is the potential profile, on the left side it is x_L, on the right side it is x_R, this is an arbitrary potential profile. And we consider that there is an amplitude A_L of the wave approaching the barrier from the left. So, this is this electron is moving right, A_R, that is also moving right, but this is on the right side of the potential, and B_L wave moving towards the left, at the left side of the barrier, and B_R wave moving towards the left at the right side of the barrier. Let us say we have this kind of an arrangement. And outside this region x_L to x_R that is this part and so right part and left part, for the simplicity we consider that the potential is 0 in these regions. If we consider this, then the general solution to the Schrodinger equation for a positive energy E, let us say the energy e is somewhere here, anywhere, but that is less than the barrier. If we have that kind of a situation, then we can write the left side left hand side part of the wave function $\psi_L(x) = A_L e^{iqx} + B_L e^{-iqx}$ moving along the positive x direction $+B_L e^{-iqx}$ moving along the negative x direction that is left towards the left. This is for $x \leq x_L$. And for the right hand side, $\psi_R(x) = A_R e^{iqx} + B_R e^{-iqx}$that is moving towards the right $+B_R e^{-iqx}$ that is moving towards the left. And this is for the region $x \geq x_R$. Now, we have this q here in the exponential part. The q as we have learned from the potential barriers, the understanding of potential barriers, this is a function of the energy of the electrons, and this is given as $q(E) = \frac{2mE}{\hbar^2}$. This is in the left and in the right side of the barrier. Now, let us consider the Schrodinger equation. Since, the Schrodinger equation is a linear second order differential equation, the amplitudes A_R and B_R on the right hand side of the right hand lead if we consider this part to be a lead, depends linearly from the amplitudes A_L and B_L that is also left hand side. (Refer Slide Time: 07:24)

So, we can represent the case of the barrier using a transfer matrix like $\begin{pmatrix} A_R \\ B_R \end{pmatrix}$ = S(E)$\begin{pmatrix} A_L \\ B_L \end{pmatrix}$ So, the transfer matrix will be a 2 by 2 matrix, which can be written as $\begin{pmatrix} s_{11}(E) & s_{12}(E) \\ s_{21}(E) & s_{22}(E) \end{pmatrix} \begin{pmatrix} A_L \\ B_L \end{pmatrix}$. So, this is our transfer matrix. And this is the same as S(E) here. So, if we now perform this matrix multiplication, we can write in terms of these matrix elements the linear equations $A_R = s_{11}A_L + s_{12}B_L$. Similarly, $B_R = s_{21}A_L + s_{22}B_L$. Just the matrix multiplication gives us this linear, set of linear equations, and the transfer matrix S(E) corresponds to a given potential V(x), that we have drawn here, and of an arbitrary shape and the chosen energy E of the electron. So, this can be obtained by integrating the Schrodinger equation. Notice that the complex conjugate of any solution of Schrodinger equation is on its own write a solution for the same energy. Therefore, we can write

$s_{11} = s^*_{22}$ that has to be true and $s_{12} = s^*_{21}$ that has to be correct, in order to be us, in order to have a solution to the Schrodinger equation. Now, we can also show that the determinant of this matrix transfer matrix is 1, which means the matrix is unimodular. (Refer Slide Time: 11:22)

So, if we consider the physical meaning of the amplitudes of the left hand side and the right hand side waves, and if we consider the electronic current density. Then, we can write, considering the electronic current density and the and its correspondence with the amplitudes, we can write $|A_L|^2 - |B_L|^2 = |A_R|^2 - |B_R|^2$. So, now by considering this form of the transfer matrix, and these constraints on the transfer matrix elements, we can write $|A_L|^2 - |B_L|^2 = |s_{11}A_L + s_{12}B_L|^2 - |s_{21}A_L + s_{22}B_L|^2$, which is equal to $[|s_{11}|^2 - |s_{21}|^2][|A_L|^2 - |B_L|^2]$. So, now we can write from this that $|s_{11}|^2 - |s_{21}|^2$, which if we look at the form of this transfer matrix, we can find that this is nothing but the determinant of the transfer matrix, subject to these constraints and this must be det(S) = 1. So, we have found that the transfer matrix between two leads that is the left hand side lead and the right hand side lead, connecting the barrier is unimodular because this quantity, according to this calculation here s 11 square, $|s_{11}|^2 + |s_{21}|^2 = 1$ which is same as the determinant of this transfer matrix. We have proved S to be unimodular.