Solid State Physics
Lecture 30

The Sommerfeld Theory for Conduction in Metals

Hello. We were discussing the Sommerfeld theory for free electrons the transport, the electrical conductivity, the thermal conductivity in metals. So, the difference between Drude model and Sommerfeld model is that in Drude model the electrons were treated as classical particles Maxwell Boltzmann statistics was used for it and Sommerfeld changed that by using Fermi-Dirac statistics for describing the electrons. So, now we will discuss the Sommerfeld theory of conduction in metals. (Refer Slide Time: 00:59)

So, for developing the theory for conduction we need a velocity distribution for the electrons in the metals and how do we find that? Let us consider a small volume element in the reciprocal space k space something like this a small volume element in the k-space. So, its let us say relative to some origin it has a position vector $\overrightarrow{k}$ and it has the differential volume d^3k. Now, if we allow for the 2-fold spin degeneracy; that means, one state is occupied by can be occupied by electrons of two different spins. Then the number of electronic levels in the volume element would be given as $\frac{V}{4\pi^3}d^3k$. This is the number of electronic levels. So, the probability of each level being occupied this is given by the Fermi function $f(\epsilon(\overrightarrow{k}))$ which is a function of the energy. So, then we can write that the total number of electrons in the k-space volume element in d^3k this much k-space volume is given as $\frac{V}{4\pi^3}f(\epsilon(\overrightarrow{k}))d^3k$. and what is energy here? Energy for free electrons is nothing, but $\epsilon(\overrightarrow{k}) = \frac{\hbar^2k^2}{2m}$, where m is the mass of the electron. Velocity of a free electron having wave vector $\overrightarrow{k}$ would be given as $\overrightarrow{v} = \frac{\hbar k}{m}$. So, the total number of electrons per unit volume of real space in a velocity space element if we consider a velocity space element d^3v, then its about the point v the total number of electrons per unit volume that can be given as $f(\overrightarrow{v})d^3v$, f is the Fermi function again in velocity space. This is the total number of electrons per unit volume about v about v being the velocity. So, we have written the Fermi function for velocity distribution. (Refer Slide Time: 06:17)

So, what is the velocity distribution function? $f(\overrightarrow{v})$ this quantity can be obtained to be $f(\overrightarrow{v}) = \frac{(m/\hbar)^3}{4\pi^3}\frac{1}{\exp\left[(1/2mv^2-\mu)/K_BT\right]+1}$. This is the velocity distribution function. So, Sommerfeld then using this distribution function reexamined the Drude model and then replaced the classical Maxwell–Boltzmann velocity distribution by this Fermi–Dirac velocity distribution function and after doing that a typical electron in a metal that has momentum of the order of $\hbar k_F$. Considering this Sommerfeld got an expression for the mean free path of the electrons. So, if the Fermi velocity v_F is considered as a measure of the typical electronic speed then the mean free path 'l' that can be calculated as $l = v_F\tau$. Earlier in Drude's case we had the typical electronic velocity as the average velocity of all free electrons in the system and that instead of that here we have found v_F which is the Fermi velocity. And, in considering Sommerfeld model we can see that there are many electrons most of the electrons that have much lower velocity, but these are the electrons that participate in the conduction much more than anything else. So, their contribution is more in the conduction and therefore, their velocity matters these are the mobile electrons mainly that transport the charge. And, hence when we are calculating the mean free path we must multiply this Fermi velocity and the relaxation time that will give us the mean free path which would; obviously, be larger than the Drude's estimate. So, this mean free path l can be given as $l = \frac{(r_s/a_0)^2}{\rho_\mu} \times 92$ Å.