Solid State Physics
Lecture 28
Fermi-Dirac Distribution

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Now, let us define a very useful quantity ${f_i}^N$ subscript i superscript N, how do we define this? This is the probability of there being an electron in the particular single electron level i when N electron system is in thermal equilibrium that means, this probability is some of the independent weights that we have found earlier $P_N(E)$ that is the probability of finding N electron system in any of those states. So, this quantity is ${f_i}^N = \Sigma P_N({E_\alpha}^N)$ and this is the probability of there being an electron in the particular single electron level i in the N electron system under thermal equilibrium. So, we are summing over in this sum, this sum runs over all such states alpha where there is an electron in the level i. Now, we make certain observations to be able to evaluate this quantity ${f_i}^N$. The 1st observation is that the probability of an electron being in the level i that can be given as ${f_i}^N$ which we write now as $1-\Sigma P_N({E_\gamma}^N)$, what does it mean? The summation is over all N electron states in which there is no electron at i, i^{th} state, i^{th} level that is what we are saying so, in order to be consistent. So, these are the states, these are the probabilities for which there is no electron at i^{th} level and 1 minus that sum over that probability gives us the probability of finding one electron at i^{th} level that is the idea, that is how we are going to approach this. (Refer Slide Time: 03:44)

The 2nd observation that we make is if we consider (N + 1) electron state, we increase the number of electrons by 1 and (N + 1) electron state in such a way that there is one electron at i^{th} level. If we have this and for this, if we get the energy equals ${E_\gamma}^N$, then we can write ${f_i}^N = 1 - \Sigma P_N({E_\alpha}^{(N+1)} - \epsilon_i)$. So, what are these quantities? The sum runs over all states alpha that is (N + 1) in count and this one is the energy of an electron at i^{th} level that means what are we doing? We are following this expression here and in this expression, we are changing the argument of P_N by putting this energy of (N + 1) particles and subtracting the energy of the electron at i^{th} level. So, we are going back to the same energy that was here for the N electron system. So, this is fine no problem here so far. So, the part that we have summed here, this part sorry not the sum itself, this part, this part may be written as $P_N({E_\alpha}^{(N+1)} - \epsilon_i)$, let us find what this is. According to the definition of P_N, this may be written as $e^{(\epsilon_i-\mu)/K_BT} P_{N+1}({E_\alpha}^{N+1})$, we have introduced something new that is μ here, this quantity is the chemical potential. So, what is the chemical potential? Chemical potential is at a given temperature say T, it is the energy cost for one particle, the energy difference that one particle makes in the system. Free energy of (N + 1) electron system - the free energy of N electron system, $\mu = F_{N+1} - F_N$ that is how we define chemical potential. (Refer Slide Time: 08:56)

Now, we have this quantity defined now, this equation obtained now and if we now substitute this into here back into this one, what do we obtain? The expression for ${f_i}^N$ that we will obtain would look like by making this substitution, it will be ${f_i}^N = 1 - e^{(\epsilon_i-\mu)/K_BT}\Sigma P_{N+1}({E_\alpha}^{N+1})$, this is what we get and now, if we compare this with the definition of ${f_i}^N$, we would see that this part here makes something similar to ${f_i}^N$. In more precise language, we can write ${f_i}^N = 1-e^{(\epsilon_i-\mu)/K_BT} {f_i}^{N+1}$. Just by looking at the definition of ${f_i}^N$ here, we obtain, we understand that this circled quantity is ${f_i}^{N+1}$. Now, we make another observation, the last observation that is the above equation gives an exact relation between the probability of one electron level i, being occupied at a temperature T in an N electron system and in an (N + 1) electron system. Now, when the number of electrons in the system that is N is very large of the order of Avogadro number let us say that is the kind of system we are interested in, we are not interested in three-four electrons roaming around somewhere, we are interested in a real metal or conductor so, the number of electrons that we are interested in is of the order of Avogadro number. And when that is the situation, adding a single electron would not really change the probability, it will make completely insignificant change in the probability so, we can ignore that and if we ignore that in place of ${f_i}^{N+1}$, we would write ${f_i}^N$ and there is no significant difference between these two quantities provided N is of the order of Avogadro number that is clearly understandable. And if we do that from this expression, what we will obtain is ${f_i}^N = \frac{1}{e^{(\epsilon_i - \mu)/K_B T} + 1}$, this is what we will obtain and since we are only interested in very large values of N where N being a little different by one or two electrons does not matter so, this index n is also irrelevant that means, we can easily drop our explicit reference to N and write $f_i = \frac{1}{e^{(\epsilon_i - \mu)/K_B T} + 1}$, this is the Fermi-Dirac distribution function. And then, the total number of electrons after finding this Fermi-Dirac distribution function would be simply sum over i that is the single particle states $N = \Sigma_i f_i = \Sigma_i \frac{1}{e^{(\epsilon_i - \mu)/K_B T} + 1}$, this will give us the total number of electrons N. So, we have learnt the Fermi-Dirac distribution, we have derived the Fermi-Dirac distribution by considering the probability, many of you have already gone through this derivation, those who are going through this derivation for the first time, take some time to think about it because this derivation is not something trivial, you need to think about the probability very accurately and in peace, then only you will arrive at this kind of a; this kind of distribution function.