Solid State Physics
Lecture 26
Introduction to Sommerfeld Theory - II

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This is given as $\frac{V}{8\pi^3}$ because we assume that the electrons are not interacting with each other. We can build up the N electron ground state by placing the electrons into the allowed one electron levels; that means, the k points. Now, the one electron levels are specified by the wave vector k that we have here and the projection of the electron spin about any arbitrary direction. So, we know that electrons spin half particle; that means, its projection are $\pm\frac{\hbar}{2}$. This is the projection of the electron spin. What is the lowest energy lowest possible energy for an electron? We have $E = \frac{\hbar^2 k^2}{2m}$. So, if $\overrightarrow{k} \rightarrow 0$, then the E = 0 and we fill the lowest states with electrons first. So, the energy of the single electron level is directly proportional to k squared. When the number of electrons that is capital N in the system is very large, then it will the electrons occupying the lowest energy states first and then, subsequently, higher energy states, that kind of an arrangement will look like a sphere in the reciprocal space in the k space. So, we will have a sphere with a certain radius. Let us call this radius in the k space as the Fermi-wave vector k_F and k_F is the radius of this sphere inside which all the k points, all the allowed k points are filled with electrons; outside which nothing is filled with electron, no allowed k point is filled with electron at absolute 0 temperature. So, we have this volume, Ω for this sphere which is nothing but $\frac{4}{3}\pi {k_F}^3$ and this volume is filled. So, the number of allowed k values within the sphere that would be given as $\Omega\frac{V}{8\pi^3}$ which is nothing but $\frac{{k_F}^3}{6\pi^2}V$, where V is the volume of the cube that we have considered as the size of the entire crystal. Now, since each allowed k value leads to 2 single electron levels, one for each spin. We have considered 2 spins here. Then, in order to accumulate N electrons, we will have N equals ah. So, this is the number of total number of electrons in the system, it is $N = 2\frac{{k_F}^3}{6\pi^2}V = \frac{{k_F}^3}{3\pi^2}V$. (Refer Slide Time: 06:01)

So, if we have N number of electrons in a volume, then for volume V, the density of electrons would become $n = \frac{N}{V}$. Then, the ground state of this N electron system that would be formed by occupying all single particle states for the region $k < k_F$ and k_F in this case, the Fermi wave vector would be given as so we can write $N = \frac{{k_F}^3}{3\pi^2}$. This expression gives us the value of k_F. The sphere of radius k_F contains the occupied single electron levels and this kind of a sphere is called the Fermi sphere and surface of the sphere is called the Fermi surface. The momentum $\hbar k_F$, this is the this can also be called p_F which is the highest momentum, highest possible momentum at 0 temperature is also called the Fermi momentum. And if we write v_F the velocity of electrons on the Fermi surface that is $\frac{p_F}{m}$, this velocity is called again the Fermi velocity. Finally, the energy of those electrons on the Fermi surface that is the maximum energy $\frac{\hbar^2 {k_F}^2}{2m}$ is called the Fermi energy at absolute 0 temperature. So, the total energy of the ground state system of capital N number of electrons can be obtained by $E = 2\Sigma_{k<k_F}\frac{\hbar^2}{2m}k^2$. So, if we sum a smooth function over all allowed values of k, we may proceed like the following. We have the volume of k-space per allowed k value. (Refer Slide Time: 09:53)

So, the volume of k space per allowed k value, we are calling it $\Delta k = \frac{8\pi^3}{V}$ as we have discussed earlier. Now, if we Σ_k, any function $F(\overrightarrow{k}$ smooth function that would give us $\frac{V}{8\pi^3}\Sigma_k F(\overrightarrow{k})\Delta k$. Now, if we have many number of electrons, a large volume of this cube, we will have $\Delta k \rightarrow 0$ and that means, the $V \rightarrow \infty$, then only we will achieve this situation $\Delta k \rightarrow 0$. Then, the sum, this sum here would approach an integral. So, this sum can be written as $\int d^3k F(\overrightarrow{k})$. So, we can now rearrange and write $\lim_{V\rightarrow\infty}\frac{1}{V}\Sigma_k F(\overrightarrow{k})$. This quantity equals $\int \frac{d^3k}{8\pi^3}F(\overrightarrow{k})$. So, this is converting the summation to integration under certain condition that the volume tends to infinity, then only this thing works and this kind of a formation is very useful in condensed matter physics. So, remember this quite well. Now, when applying this for finite but large system, we assume that our sum differs very little from the integration limit. We are considering from something finite, but we assume that our sum differs almost negligibly from this integral. And now, using this to evaluate the energy, we find the energy density of the electron gas that is $\frac{E}{V}$ energy per unit volume $\frac{E}{V} = \frac{1}{4\pi^3}\int_{k<k_F} d^3k \frac{\hbar^2 k^2}{m} = \frac{1}{\pi^2}\frac{\hbar^2 {k_F}^5}{10m}$. So, d^3k, if we expand this in spherical coordinate system, we will get k^2dk kind of a term and that will bring us k^5 and provided this limit, we will have ${k_F}^5$ this kind of a term here. (Refer Slide Time: 13:58)

So, to find the energy per electron instead of energy per unit volume, we can write $\frac{E}{V} = \frac{3}{10}\frac{\hbar^2 {k_F}^2}{m} = \frac{3}{5}E_F$. So, by simplifying this in terms of the Fermi wave vector, we can write $\frac{N}{V} = \frac{{k_F}^3}{3\pi^2}$. So, this is the number density of electrons. The number density of electrons can be written in terms of the Fermi wave vector in this fashion. So, we have discussed the electronic properties in the context of Sommerfeld model at the absolute 0 temperature. Now, we will see what happens, if we increase the temperature.