# Solid State Physics
# Lecture 24
Thermal Conductivity

Hello. We were discussing Drude model for free electrons and we have already discussed the transport properties of electrons in the Drude model and the electrical conductivity. Now we are going to consider and discuss the Thermal Conductivity the thermal transport of electrons from the Drude model. (Refer Slide Time: 00:51)

So, we are going to discuss the thermal conductivity of metals and if we consider metals or for that matter any other kind of material, we will see that when the lattice is at an elevated temperature the lattice itself starts vibrating; that means, every atom starts vibrating in that system in that crystal. And when that entire lattice starts vibrating a wave propagates throughout the lattice, this kind of wave the quantum of this kind of wave is called phonon and this phonon takes a significant part of the thermal conductivity in a material and metals are no exception for that. However, in case of metals you can see that there is also; an also a thermal conductivity associated with the motion of the electrons. You have possibly noticed that good metals good conductors of electricity are also good conductors of heat and here the contribution comes from the electrons how? Well, if we consider Drude model we have assumed following Drude that the electron after collision it emerges with a velocity that is proportional to the temperature at that point in space. However, the direction of that velocity is random. The magnitude of that velocity that is the speed that is proportional to the temperature. So, in real life also its somewhat similar the temperature plays a significant role in dictating exactly what would be the velocity of that electron after it emerges from a collision. So, if we assume this kind of situation with Drude model, we can calculate the thermal conductivity and the we have to define a thermal current in this case. Let us define the thermal current and let us try to calculate it applying some analytical mathematics. So, let us define thermal current density $\overrightarrow{j}^q$; q is for thermal current density. And how do we define that? As we can clearly understand that the heat conduction is along the opposite direction to temperature gradient. There must be a temperature gradient in order to have some heat conduction and if the temperature gradient is given as the gradient of temperature just like this $\overrightarrow{\nabla}T$. Then this heat conductivity sorry the thermal current density can be expressed as $\overrightarrow{j}^q = -\kappa\overrightarrow{\nabla}T$, it would be along the opposite direction of this temperature gradient and there must be a constant that we call kappa. This kappa is the thermal conductivity which is nothing, but a proportionality constant in this context. Now if we consider for simplicity the situation in one dimension, then we can write $j^q$ the thermal current density which is no longer a vector in one dimension, we do not need to mention its vectors properties $j^q = -\kappa\frac{dT}{dx}$, capital T is the temperature and x is that one dimension. Now, if we write the thermal energy per electron as $\epsilon(T)$, its of course, a function of T the temperature, then an electron whose last collision was at a space point x prime, it will have the thermal energy of $\epsilon(T[x'])$ which is a function of temperature at the spatial point x prime, it had its last collision at x'. According to our assumption within Drude model, the velocity is proportional to the temperature and the thermal energy is dictated by the last collision that took place some time ago and the electrons. So, if we consider a point in space, here in case of one dimension say this is a line here the point is x and there are electrons that are coming from the hotter side say this left hand side is hotter and say the right hand side is colder. So, you will have the electrons coming from the hotter side here and some of the electrons would come from the colder side here. Let us use a different color for colder electrons. So, this kind of an arrangement will be there. If there are n number of electrons here we can assume that half of them $\frac{n}{2}$ is coming from the hotter side and another $\frac{n}{2}$ is coming from the colder side. Now, the thermal energy per electron that is coming from the hotter side that can be given as $\epsilon(T[x - v\tau])$ what is this? x is the coordinate of this point, v is the speed of this electron and tau is the relaxation time the mean free time. So, we are saying that it was here at minus $[x - v\tau]$ this kind of a location when it had the last collision on an average this would be the situation. So, the energy of this electron the thermal energy of this electron on an average would be $\epsilon(T[x - v\tau])$. And if we consider the other kind of electron that is coming from the colder side, then we will have their energy as $\epsilon(T[x + v\tau])$ this kind of an energy would be there. Now, we are having $\frac{n}{2}$ number of electrons coming from the colder side and $\frac{n}{2}$ number of electrons coming from the hotter side. So, this is the total energy of the electrons coming from the hotter side, this is the total energy of the electrons coming from the colder side under this kind of a situation if we add these two we will get the thermal current density. (Refer Slide Time: 10:25)

So, the thermal current density can be given as $j^q = \frac{1}{2}nv[\epsilon(T[x - v\tau]) - \epsilon(T[x + v\tau])]$. Now if we have the variation in temperature over a mean free path that is l, l is the mean free path which can be given as $l = v\tau$. And if this quantity is very small; that means, if there are enough number of electrons in the system to have frequent collisions and the mean free path is not too long then according to the definition of differentiation according to the definition of derivatives we can expand it about the point x and we can write that the thermal conductivity $j^q = nv^2\tau\frac{d\epsilon}{dT}(-\frac{dT}{dx})$, $\frac{dT}{dx}$ is the temperature gradient. So, this difference in energy sorry this is not small t this is temperature that we have written as capital T. So, this happens to be the current the thermal current density if we expand this quantity for small values of mean free path that is $v\tau$. So, for $v\tau$ to be small we can do this and this difference is actually written by the product of $\frac{d\epsilon}{dT}(-\frac{dT}{dx})$. Now, if we extend this from one dimension to three dimension, then we can write v as $v_x$ and if we average over all directions, then $\langle {v_x}^2\rangle = \langle {v_y}^2\rangle = \langle {v_z}^2\rangle = \frac{1}{3}\langle v^2\rangle$ where v is the total velocity. So, $n\frac{d\epsilon}{dT}$ this can be expressed as the total number of electrons over the volume of the system $\frac{N}{V}\frac{d\epsilon}{dT}$ which is nothing, but if we multiply $d\epsilon$ by N we will get $\frac{\frac{dE}{dT}}{V}$ total energy of the system dT divided by the volume which is $C_v$ the heat capacity of the system for a constant volume. That means, given from this equation we can write the thermal conductivity in three dimension the thermal current density in three dimension not the thermal conductivity as $j^q = \frac{1}{3}v^2\tau C_v(-\overrightarrow{\bigtriangledown}T)$. So,if this is the expression for the thermal current density, then the thermal conductivity kappa this quantity can be given as this $(-\overrightarrow{\bigtriangledown}T)$ would be removed the only thing that will remove remain here is$\kappa = \frac{1}{3}v^2\tau C_v$ and this is the thermal conductivity due to the conduction electrons that we obtained from the Drude model and $v^2$ is the mean squared electron speed. So, this is what we obtained from Drude model for the electronic contribution to the thermal conductivity.