Solid State Physics
Lecture 23
AC Electrical Conductivity

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After having this insight on the hall into the hall coefficient, let us move on to the AC Electrical Conductivity of a metal. (Refer Slide Time: 00:36)

When we are talking about AC electrical conductivity; that means, alternating current, we are going to apply a time dependent electric field on the metal. So, to calculate the current induced in a metal by time dependent electric field, we need to assume a form of the electric field. So, the electric field is assumed to have this kind of a form, it is in complex form electric field is $\overrightarrow{E}(t) = Re_{(}\overrightarrow{E}(\omega)e^{-i\omega t})$ and we take the real part of this quantity that is our electric field. Now, the equation of motion of the electrons, for the momentum per electron, that is $\frac{d\overrightarrow{p}}{dt} = -\frac{\overrightarrow{p}}{\tau} - e\overrightarrow{E}$. Now, we want a steady state solution for this problem. So, let us consider a trial solution, for this differential equation, for this differential equation, we can consider an exponential trial solution $\overrightarrow{p}(t) = Re_{(}\overrightarrow{p}(\omega)e^{-i\omega t})$ and we have to take the real part of this quantity, because the momentum is real. And, substituting this complex p not the real part is this entire complex quantity. And, the complex e that is this entire quantity not just the real part into the differential equation, we can obtain something useful and then take the real part to get the physical quantity. That is the standard approach that you have seen elsewhere, maybe including other properties of electromagnetic waves. So, if we do that we would find that we must satisfy a condition $-i\omega\overrightarrow{p}(\omega) = -\frac{\overrightarrow{p}(\omega)}{\tau} - e\overrightarrow{E}(\omega)$. This you will find by putting this expression of p complex expression of the momentum and complex expression for the electric field into this differential equation here ok. Now, our volume current density $\overrightarrow{j} = -\frac{ne\overrightarrow{p}}{m}$, this is the current density. And, in complex term it can be written as in complex form it would be written as $\overrightarrow{j}(t) = Re(\overrightarrow{j}(\omega)e^{-i\omega t})$ and we will take it is real part for the real physical quantity. So, now putting this into appropriate place from this equation here, we can write $\overrightarrow{j}(\omega) = -\frac{ne\overrightarrow{p}(\omega)}{m}$, which is taking the value of $\overrightarrow{p}(\omega)$ from this expression here, we can write $\overrightarrow{j}(\omega) = -\frac{\frac{ne^2}{m}\overrightarrow{E}(\omega)}{\frac{1}{\tau} - i\omega}$.This comes from here ok. (Refer Slide Time: 06:33)

Once, we have obtained this, then it is the $\overrightarrow{j}(\omega)$ the current density can be written as customary form of the Ohms law, $\sigma(\omega)\overrightarrow{E}(\omega)$ the conductivity as a function of omega times the electric field, which is also a function of omega now. Here this quantity $\sigma(\omega)$ is called the frequency dependent conductivity or the AC conductivity. And, what it is; what is it is value? $\sigma(\omega)$ can be written as $\frac{\sigma_0}{1-i\omega\tau}$. And, σ_0 as we have defined earlier is nothing but $\frac{ne^2\tau}{m}$. Now, if we consider 0 frequency, then $\sigma(\omega)$ becomes σ_0, $\omega \rightarrow 0$ here and if $\omega \rightarrow 0$ it is $\frac{\sigma_0}{1}$, which is σ_0. So, it correctly reduces to the drude result for 0 frequency. The most important application of this result is the propagation of electromagnetic radiation in a metal. So, how does an electromagnetic radiation propagate in a metal? Maybe you have already learned that electromagnetic radiation does not propagate in an ideal conductor. There is no electric field inside an ideal conductor and if there is no electric field inside an ideal conductor, there is no question of propagation of the electromagnetic wave that is perfect. But, there is nothing called an ideal conductor. Every conductor that we are surrounded with are real conductors and at very high frequency, you cannot expect that the electric field would instantaneously be nullified by the conductor. At low if the electric field is of very low frequency then that is valid no problem with that, but if you have very high frequency, then it does not work. So, how does the electric field get nullified, they are charges free, charges in the conductor in the metal, that moves from one place to another to nullify this electric field. And, that movement of charges that requires certain time, that depends on the properties of the material. And, if enough time is not given for those electrons to move around, then the electric field would not get completely nullified. And, electromagnetic wave to some extent can propagate inside a metal. Provided the electromagnetic wave is of very high frequency. Let us understand that mathematically. So, if we consider current density $\overrightarrow{j}(\overrightarrow{r},\omega)$, then it can be written as $\sigma(\omega)\overrightarrow{E}(\overrightarrow{r},\omega)$. This is valid whenever the wavelength of the electric field that is λ, that is large compare to the electronic mean free path that is 'l'. So; that means, this is valid when λ that is the wavelength of the electric field, when this is greater than l, which is the mean free path of the electrons. Now, this condition normally satisfies is normally satisfied, in case of ordinary metals and visible lights. Now, if we assume that the wavelength is large compare to the mean free path we may proceed like, in the presence of specified current density that is $\overrightarrow{j}$, we can write the Maxwell's equation if the $\overrightarrow{j}$ is given, the Maxwell's equations would take the form in CGS units of course, because we have been working with CGS units. $\overrightarrow{\bigtriangledown}\cdot\overrightarrow{E}=0$ of electric field inside a metal that is a charge free region is going to be 0, $\overrightarrow{\bigtriangledown}\cdot\overrightarrow{H}=0$. $\overrightarrow{\bigtriangledown}\times\overrightarrow{E}=-\frac{1}{c}\frac{\partial\overrightarrow{H}}{\partial t}$ in CGS unit and $\overrightarrow{\bigtriangledown}\times\overrightarrow{H}=\frac{4\pi}{c}\overrightarrow{j}+\frac{1}{c}\frac{\partial\overrightarrow{E}}{\partial t}$, which is the amperes law modified by Maxwell and expressed in CGS units. (Refer Slide Time: 13:22)

If, we have these Maxwell's equations and if we look for a time dependence of the form of $e^{-i\omega t}$. Then, in case of metals, we can express the current density $\overrightarrow{j}$ in terms of the electric field $\overrightarrow{E}$, using the $\overrightarrow{\bigtriangledown}\times(\overrightarrow{\bigtriangledown}\times\overrightarrow{E})=-\overrightarrow{\bigtriangledown}^2\overrightarrow{E}=\frac{i\omega}{c}(\overrightarrow{\bigtriangledown}\times\overrightarrow{H})$. This quantity $=\frac{i\omega}{c}(\frac{4\pi\sigma}{c}\overrightarrow{E}-\frac{i\omega}{c}\overrightarrow{E})$ this implies that $-\overrightarrow{\bigtriangledown}^2\overrightarrow{E}=\frac{\omega^2}{c^2}(1+\frac{4\pi i\sigma}{\omega})\overrightarrow{E}$. This has the form of the usual wave equation. The usual wave equation looks pretty similar to this. And, if this is similar to the usual wave equation, let us write it in the compact form of the wave equation $-\overrightarrow{\bigtriangledown}^2\overrightarrow{E}=\frac{\omega^2}{c^2}\epsilon(\omega)\overrightarrow{E}$, where $\epsilon(\omega)$ is the complex dielectric constant. So, this equation is just like the equation of electromagnetic wave in a dielectric material with this dielectric constant here. And, what is the value of this dielectric constant now? The complex dielectric constant this $\epsilon(\omega)=1+\frac{4\pi i\sigma}{\omega}$, but this expression whatever we have obtained, this is not for a dielectric material, this is for a metal. (Refer Slide Time: 16:29)

Now, let us consider different ranges of frequency. If, we are at a frequency that is high enough to satisfy this condition $\omega\tau\gg 1$, if this condition is satisfied. Then to a first approximation we can write the $\epsilon(\omega)=1-\frac{{\omega_p}^2}{\omega^2}$. So, we have introduced a new quantity ω_p here, ω_p is known as the plasma frequency. This quantity is given as ${\omega_p}^2=\frac{4\pi ne^2}{m}$. Now, when ϵ that is the dielectric constant is real and negative; that means, when $\omega<\omega_p$ in this kind of a situation, the solution would be an exponential decay in space, there would not be any propagation. That means, if this electromagnetic wave enters a metal, it is intensity will decay very rapidly as it gets inside the metal, as it progresses through the metal, but that electromagnetic wave would not propagate into the metal. So, there would be a skin region of that metal, in which that electromagnetic wave will enter and sharply decay the intensity will sharply decay for this kind of a situation, that is no propagation. When the dielectric constant ϵ is positive; that means, $\omega>\omega_p$ in this situation the solution will become oscillatory and the electromagnetic wave can propagate inside the metal. In this condition the metal becomes transparent, EM wave propagates, transparent metal. This is something interesting metal should not in general be transparent metals usually reflect electromagnetic wave. But, when this condition is attained that is the frequency of the incident wave is pretty high, higher than the characteristic plasma frequency for that metal, that electromagnetic wave propagates; that means, the electrons cannot move so, fast that it can nullify the electric field, inside the metal. Now, if we express τ that is the relaxation time in terms of resistivity, then we can write $\omega_p\tau=1.6\times 10^2(\frac{r_s}{a^0})^{\frac{3}{2}}(\frac{1}{\rho_\mu})$. The alkali in case of the alkali metals, it has been observed that for ultraviolet rays the alkali metals become transparent. So, the plasma frequency the ω was the angular frequency, ν is the actual frequency that is $\nu_p=\frac{\omega_p}{2\pi}=11.4(\frac{r_s}{a^0})^{-\frac{3}{2}}\times 10^{15}$ Hertz. Or the wavelength λ_p the plasma wavelength that $=\frac{c}{\nu}=0.26(\frac{r_s}{a^0})^{\frac{3}{2}}\times 10^3$ Å. So, this is valid for the alkali metals that is what we find this should be minus ok. So, this is a very important consequence that is at very high frequency the metal becomes transparent. Let us consider another important consequence the second important consequence; the electric charge density has an oscillatory time dependence. So, the electric charge density is it goes as $e^{-i\omega t}$. (Refer Slide Time: 23:10)

If, we write down the equation of continuity, that we have learnt in electromagnetism, then we can see that the $\overrightarrow{\nabla} \cdot \overrightarrow{j} = -\frac{\partial \rho}{\partial t}$, which is nothing, but the conservation of charge. So, ρ is the volume charge density in this context and this means $\overrightarrow{\nabla} \cdot \overrightarrow{j}(\omega) = i\omega\rho(\omega)$. And, now if we consider the Gauss law, we can write $\overrightarrow{\nabla} \cdot \overrightarrow{E}(\omega) = 4\pi\rho(\omega)$. Now, with this comparing these 2 equations we find that $i\omega\rho(\omega) = 4\pi\sigma(\omega)\rho(\omega)$. And, this equation will have a solution provided we have $1 + \frac{4\pi i\sigma(\omega)}{\omega} = 0$. Only then this equation will have a solution. And, this is the same as the condition for propagation. Now, what does this mean? This means a charge density wave is created for the propagation of the electromagnetic wave. So, there is a charge density of the electrons in the system and that charge density itself oscillates, when the electromagnetic wave propagates through the metal. And, this kind of oscillation is called the plasma oscillation. So, we can see a wave in the charge density itself as a function of as a consequence of the electromagnetic wave passing through the metal. And, this happens only when the frequency of the incoming electromagnetic wave is pretty high. Otherwise, that electromagnetic wave gets reflected only it can only penetrate the skin of the metal nothing beyond that, but with a very high frequency it can go through the metal, pass through the metal, and it makes it creates an oscillation in the charge that is there in the metal.