Statistical Mechanics

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Lecture - 56
Pressure of an Ideal Bose Gas

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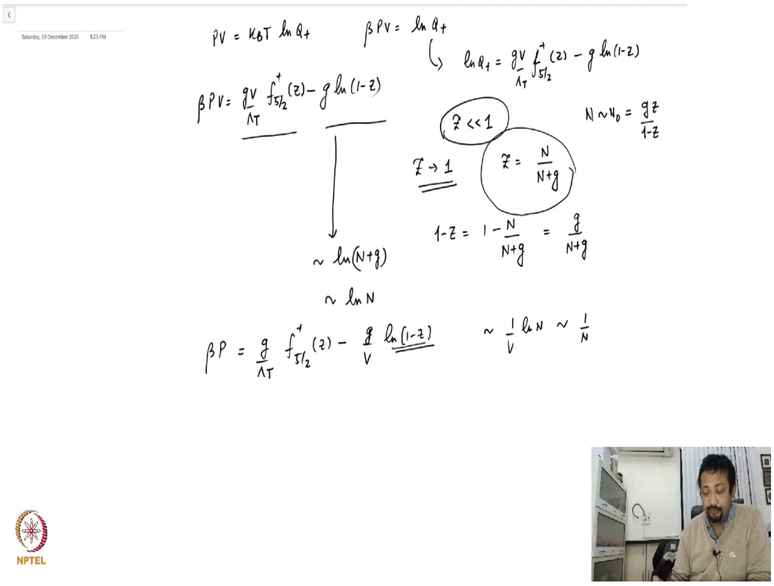

Now, we know that P V in the grand canonical ensemble was K B T ln Q plus. So, that beta P V is going to be ln of Q plus. So, we write down beta P V if you recall this expression of ln Q plus that we have ln of Q plus was g V over lambda T f of 5 by 2 plus Z minus g of ln 1 minus Z. So, this expression of beta P V takes the form lambda g V over lambda T f plus 5 by 2 of Z minus g of ln 1 minus Z.

Now, if you compare the first term in this expression which is this one, and the second term. You see that the second term can always be ignored in the thermodynamic limit, when Z is far away from 1 which means Z is less than very very less than 1 the high temperature limit.

Then I know that this quantity is 0 in contrast, when Z approaches 1 then I know that Z is going to be N over N plus g. We did that in our earlier lectures, when we said that the total particle number is compared to N naught which is equal to g Z over 1 minus Z.

And this gaves this gives us this expression for Z. So, therefore, 1 minus Z is going to be 1 minus N divided by N plus g which is going to be g over N plus g. So, that this term then as Z approaches 1 becomes the order of ln N plus g and since g is very very small compared to N, then this term is ln N. In contrast the first term since there is a volume factor, I can divide throughout the volume fact from the left hand side and the right hand side.

And I am going to have beta P is equal to g over lambda T f plus 5 by 2 minus Z minus 1 by V g times; ln of 1 minus Z. And you see when Z tends to 1; then this goes as 1 by V ln N which is of the order of 1 over N. So, this in the thermodynamic limit vanishes, as well as when Z is much much less than 1, then when Z is much much less than 1 than this is just value then this 1 also vanishes is just 1 by V right.

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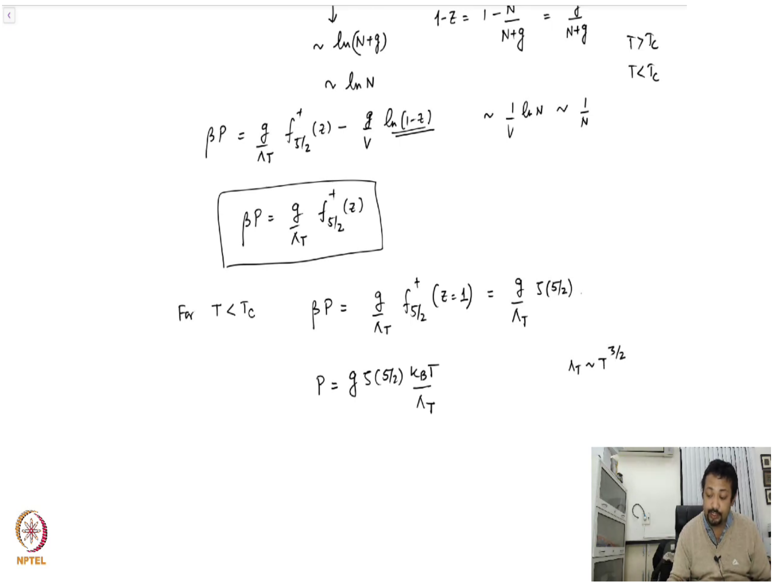

So, therefore, in both the limiting cases when T is greater than T C as well as when T is less than T c, I have beta P is equal to g over lambda T f plus 5 by 2 Z. Now, for T less than T C I have beta P is equal to g over lambda T f 5 by 2 plus Z is equal to 1 right. And this quantity is g over lambda T zeta 5 by 2.

So, that the thermodynamic pressure is K B T g well we will just write down like this g zeta 5 by 2 K B T over lambda T recall that lambda T will go as T to the power 3 half.

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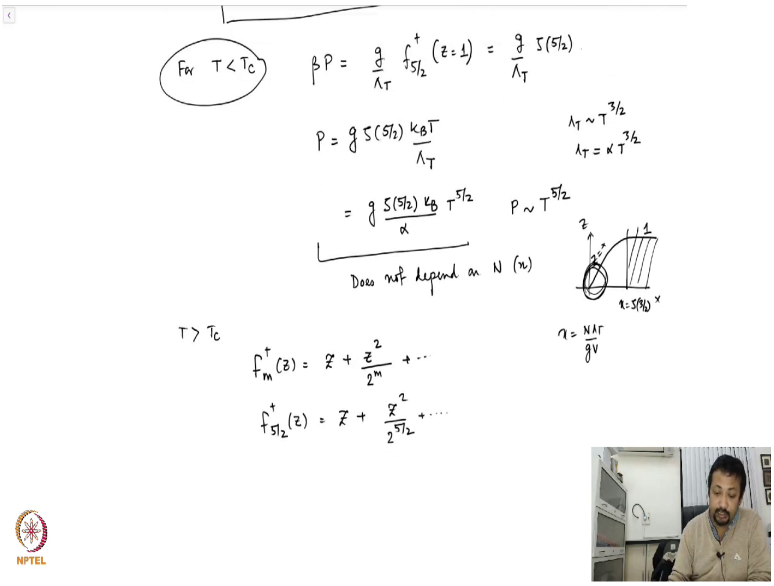

And effectively therefore, you have g zeta 5 by 2 K B some constant. So, I can write down lambda T is equal to alpha T to the power 3 half alpha T to the power 5 half; which means that the thermodynamic pressure as for temperatures. When you are below the convince condensate Bose Einstein condensation searching, you have P going as T to the power 5 by 2. What is interesting to note that this pressure does not depend on the number density N.

So, this does not depend on number density or rather we will say that it does not depend on N or n or 1 by or rho that is also fine the density right. Now, for T greater than T C I know the expansion of this function and that is going to be Z plus Z to the power I have Z square divided by m square sorry, it was k to the power n.

So, this is going to be 2 to the power m plus higher order terms, which means f plus of 5 by 2 Z is going to be Z plus Z square over 2 to the power 5 by 2 plus higher order terms.

And I have seen how the chemical potential depends on x equal to N lambda T over g V, we did this we saw that if I plot Z as a function of x, then for small x this goes linearly Z equal to x and beyond x equal to zeta 3 half this is 1. So, we are not looking in this part of the regime, but rather we are looking in the high temperature regime, where we are close to 0 Z 0 value of Z.

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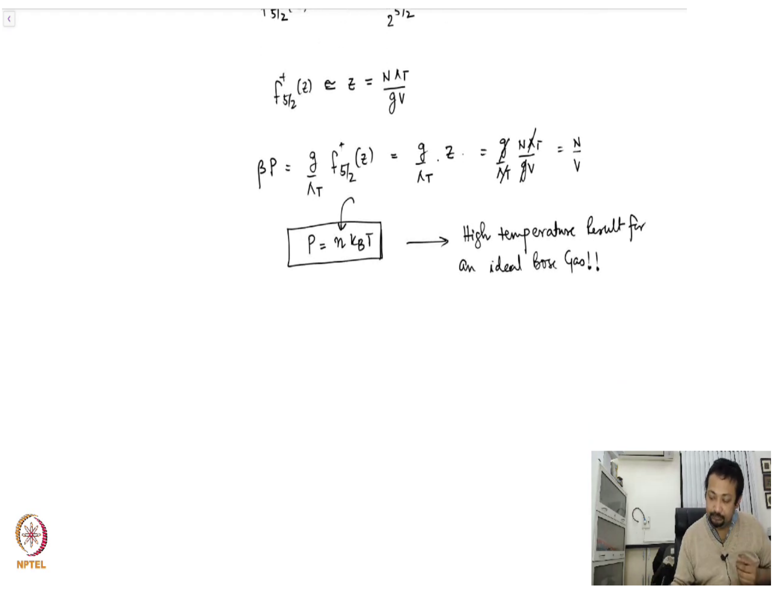

So, therefore, what we will do is we will substitute f plus 5 by 2 Z is equal is approximately equal to Z which is going to be N lambda T over g V. So, that I have pressure beta P which was g over lambda T times f plus 5 by 2 Z becomes g over lambda T times Z, which is equal to g over lambda T N lambda T over g V.

So, that lambda T lambda T and g and g cancels out and you have N over V. And you see you nicely recall recover the ideal gas result pressure is going to be n K B T. This is the high temperature result for an Ideal Bose Gas, interesting in the high temperature limit you see that the pressure depends on the number density right.

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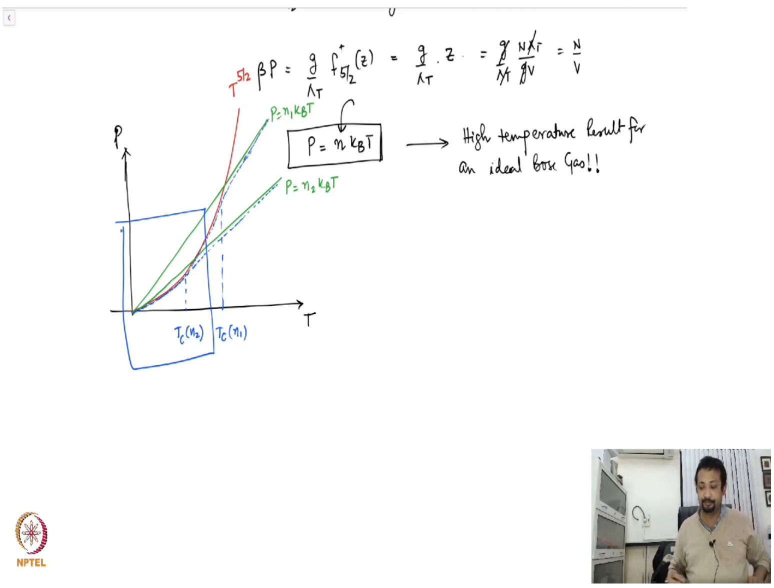

So, now, if I want to plot P as a function of temperature, then the first thing I have to fix C that this is the dependence when T is less than T c. And I have no dependence on number density or density itself. So, if I do it like this, T to the power 5 by 2 at in the high temperature limit I have n K B T. So, I can for 2 densities I can draw it like no. So, let us draw it oops so, let us draw T to the power 5 by 2 like this way I have 2 densities in which case the pressure is this.

And in the other case the pressure P V this is P is equal to n 1 K B T and this is P is equal to n 2 K B T. And, then if I really measure the pressure, you will see that the pressure is going to follow this red line. And, then it is going to crossover to this line in the asymptotic limit of high temperature. And, similarly for the other one how do I do that? Well, let us do it like this it is going to follow this line up to here, when it starts to deviate and then go to the asymptotic limit.

So, these would be the critical temperatures T C of n 1 and these would be the this would be the critical temperature T C of n 2. So, this is how your pressure temperature diagram would look for an ideal Bose gas in the high temperature limit, you are going to recover your ideal gas result.

And, in the low temperature limit below the critical temperature, you are going to recover that this pressure thermodynamic pressure of the Ideal Bose Gas is essentially independent of the density.

Now, this is reflective of the fact that below the critical temperature, if you keep on in decreasing the temperature then more and more number of particles see you have an upper limit on the number of particles you can put in right in the excited levels. So, you reach that limit and if you keep on decreasing the temperature, then more and more number of particles from the excited states come to the ground state.

But, in the ground state epsilon is equal to 0, they do not have any kinetic energy. And therefore, you do not get essentially any contribution to the pressure. And that is essentially reflected; here in this part of the diagram, where your pressure is independent of the density of the gas because, your condenser has started to appear in the system. Now just as. So, what did we do it?

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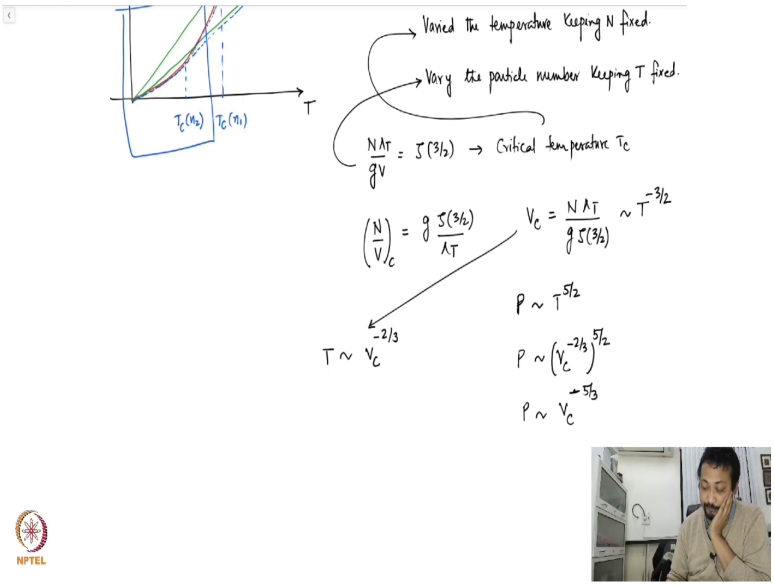

So, you so, essentially we varied the temperature keeping N fixed one can also vary the particle number keeping T fixed right, in which case the criteria that N lambda T over g V which gave us zeta 3 half. This was the criteria which gave us the critical temperature T C for this case. Now I can use the same relation to determine a density critical density in the second case, when I am varying the particle number keeping the temperature fixed.

So, that I will have N by V is equal to g zeta 3 half and we will write this critical over lambda T. So, that V C becomes N lambda T divided by g over zeta 3 half. And this one you see scales as T to the power minus 3 by 2, because lambda T scales as T to the power minus 3 by 2 well. Now, when I have this critical V C scaling as T to the power minus 3 by 2 and I have this expression for pressure, where pressure skilled as T to the power 5 by 2 I can easily eliminate this.

So, that from this expression I can easily see that temperature will scale as V C raised to the power minus two-third so, that the pressure will scale as V C minus 2 3rd raised to the power 5 by 2 and you have pressure scaling as V C raised to the power 5 minus of five-third.

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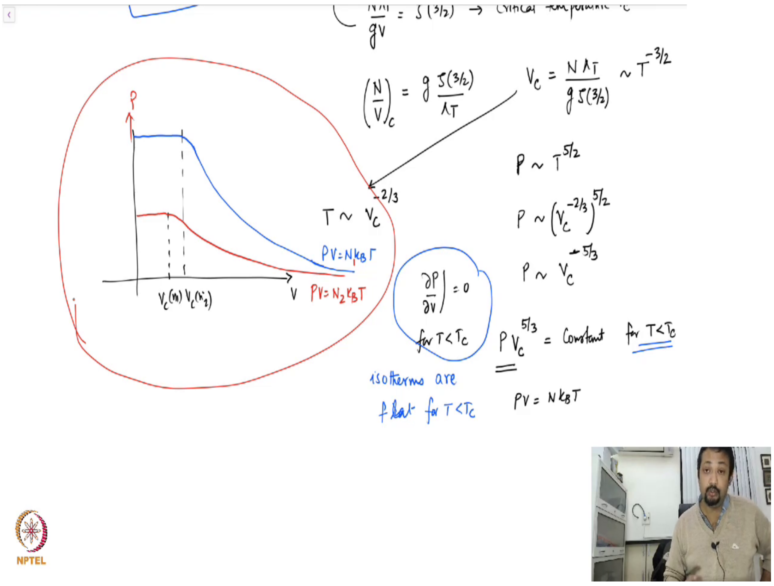

So, that you have P of V C raised to the power five-third is equal to constant for T less than T C the moment. Therefore, the isotherms are very very curious here, why is curious? Because you see pressure does not depend on the volume again.

So, del P del V is going to be 0 for T less than T C on the other hand for high temperatures you still have P V is equal to N K B T. So, that if you plot the isotherms and let us say this is 1 volume V, V C n 1 and the other 1 over here is V C N of 2, then in the high temperature limit you know that this has to follow P V is equal to N K B T. So, let it be so, this goes as P

V is equal to N K B T, but below the critical temperature this is the pressure is nearly constant. So, it goes and saturates to this value.

This essentially means that the isotherms are flat for T less than T C. Similarly here also you will see the sorry, you are going to get a flat region up to this value after which it slowly goes to P V is equal to so, we will call this N 1 and this will be N 2 K B T. So, this is going to be your pressure volume diagram. So, this is how your P V diagram looks for an ideal Bose gas and this is how your P T diagram looks for an ideal Bose gas.