Statistical Mechanics
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Lecture - 51
Degenerate Fermi Gas

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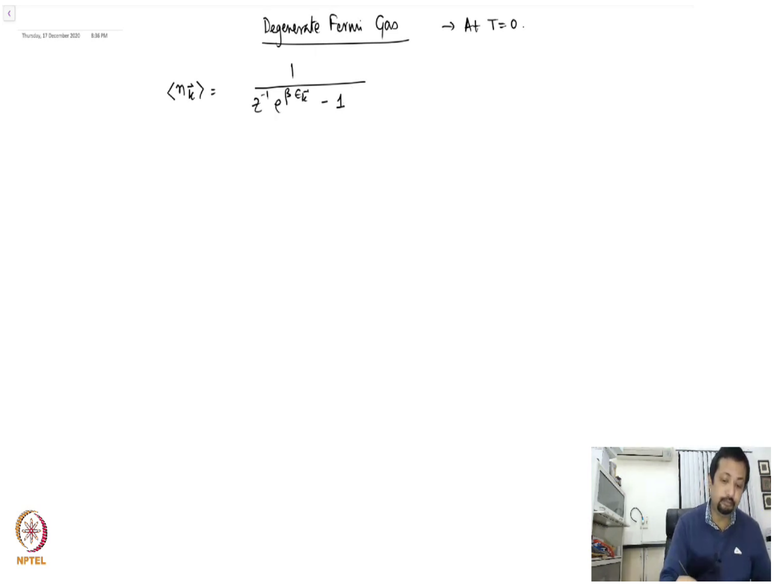

So, we now specifically look at the Degenerate Fermi Gas and that is an ideal fermi gas at the temperature T is equal to 0. Now, the average occupation number n k is given by 1 over Z inverse e to the power beta epsa k minus 1 right.

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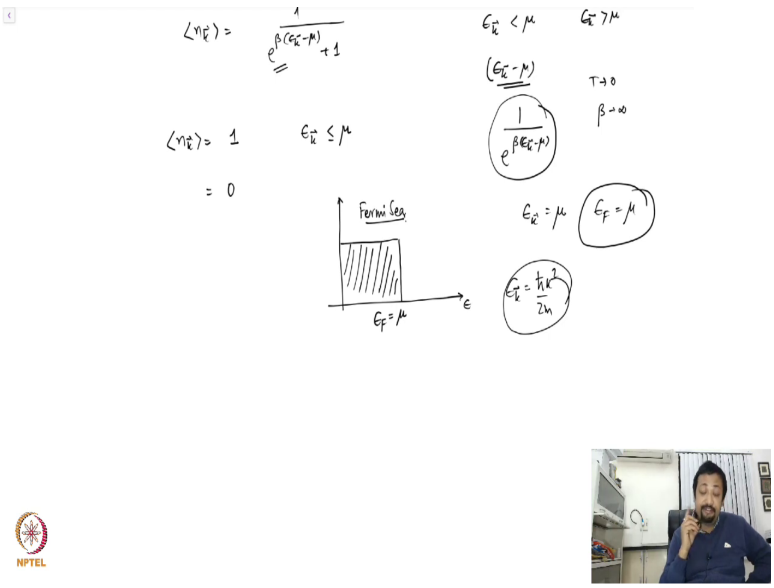

The average occupation number is given by n k is equal to 1 over e to the power beta epsa k minus mu plus 1. So, now, we have to figure this out at T equal to 0, but one has to be careful with this. Because you see that if epsa k is less than mu, then this term I have epsa k minus mu is negative.

So, that this term is actually 1 over e to the power beta epsa k minus mu right. And in the limit of T to 0 or exactly at T equal to 0, beta is infinity. So, that this term vanishes and therefore, you have average occupation number is equal to 1 for all energy levels epsa k which are less than equal to mu.

So, similar argument when epsa k is greater than mu, then you see that this term is positive and the denominator itself it becomes infinitely large and therefore, you get a 0. So, if you want to plot the occupation number as a function of the energy level, then you will see that all the levels up to this chemical potential mu are completely filled at T equal to 0 right and this completely filled levels define my Fermi Sea.

So, this defines my Fermi Sea. All energy levels up to epsa k is equal to mu are completely filled at T equal to 0 and therefore, at T equal to 0, the fermi energy equals to the chemical potential of the system. Now, since I am looking at a ideal gas, where I have h square epsa k is equal to h square k square over twice m, I can define a fermi wave vector.

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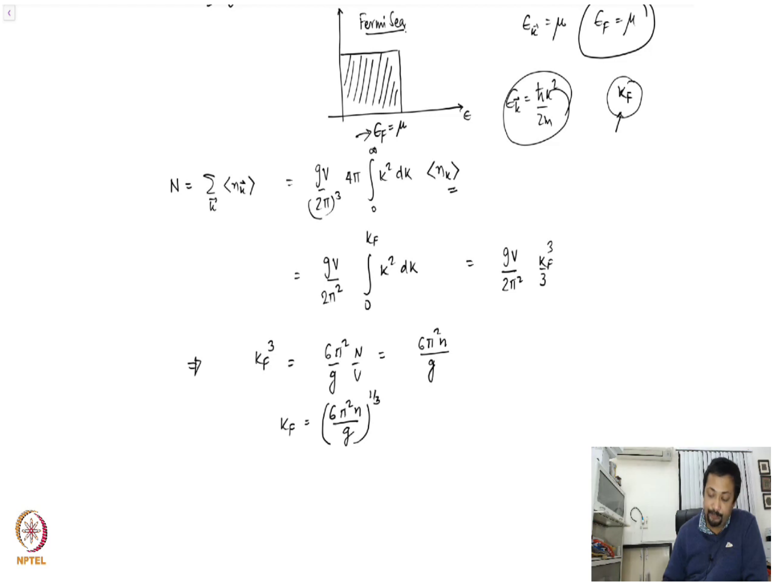

Now, to define the fermi wave vector, I note that total number of particles is sum over k average of n k right and the sum over k, I know is g V over twice pi whole cube times 4 pi times k square d k right and this goes from 0 to infinity and then, I have n k right but n k.

So, the idea is to determine define a fermi vector, k F corresponding to this fermi energy epsilon f and therefore, n k is 1, only up to this fermi vector k F. So, one has g V as 2 pi. This is going to be 2 pi square 0 to k F k square d k because beyond that beyond this value of k F, the average occupation number is 0.

So, this quantity becomes g V over 2 pi square k F cube over 3. You immediately see that this implies that k F cube is going to be 6 pi square over g times N over V which is 6 pi square the number density n divided by the g. So, that k F is 6 pi square n over g raised to the power one-third.

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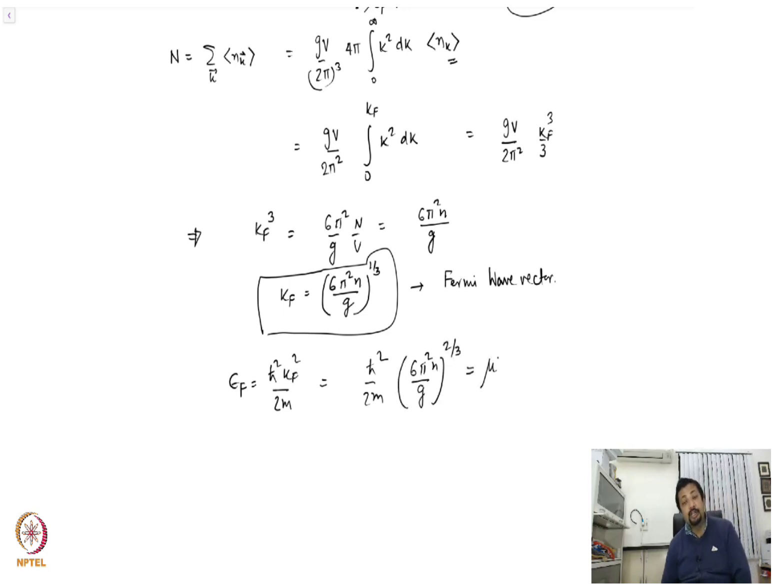

So, this defines my Fermi Wave vector. And the fermi energy which is epsa F is h square k F square over twice m is given by h square twice m 6 pi square n over g raised to the power two-third and this is the chemical potential at 0 temperature.

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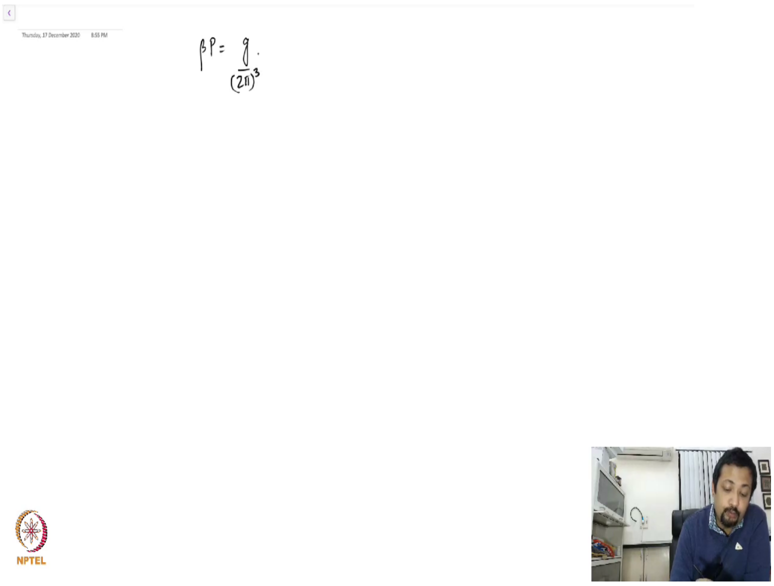

So, let us look at the thermodynamic pressure. The thermodynamic pressure, we have been looking at this expression. So, many times this was g over 2 pi whole cube or rather if I just take care of things, it is going to be eta is plus 1.

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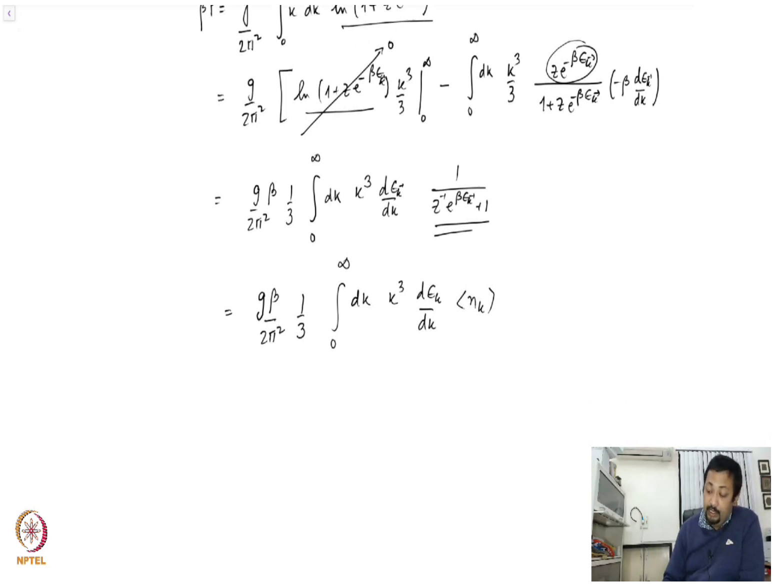

So, this is going to be g over 2 pi square integral k square d k ln 1 plus Z e to the power minus beta epsa. Now, this I can manipulate and write down as integration. So, let us say this is 0 to infinity; this is going to be d k. Choose this as the first function, then you have ln, there is no d k over here in the first term, when you integrate by parts, it is going to be ln 1 plus Z e to the power minus beta epsa.

Of course, it is a function of k. So, we can still keep that over here k cube by 3, 0 to infinity minus this is going to be integration 0 to infinity d k. I will have k cube by 3 and then, derivative of this log with respect to k. So, that takes me 1 plus Z e to the power minus beta epsa k and then, I have Z e to the power minus beta epsa k times minus of beta del epsa k del k.

Well, it is a d epsa k d k; d epsa k d k right. So, this term is going to vanish. So, that you will be left out with g over 2 pi square; the minus and minus is going to give you a plus; the beta comes out and you are left out with 0 to infinity d k 1 by 3 k cube and this is Z 1 by.

So, this is going to be k d epsa k d k times 1 by Z inverse e to the power beta epsa k plus 1. And if you carefully look at this, then this is nothing but the average expectation value; x over sorry, its nothing but the average occupation number. So, that you have g beta over 2 pi square one-third 0 to infinity d k k cube d epsa k d k average of n k.

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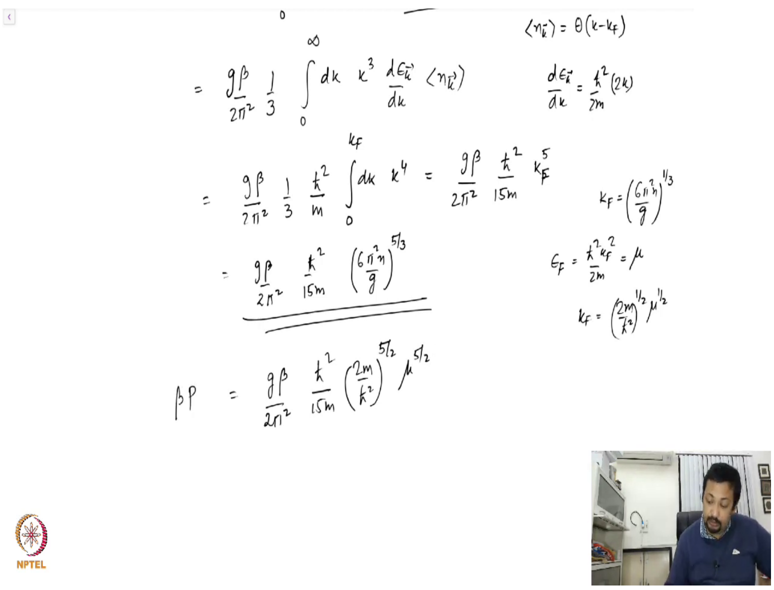

Now, the average of the occupation number at 0 temperature has the form theta of k minus k F; where theta is the heavy side function. So, that this expression is g beta 2 pi square one-third d epsa k d k is going to be h square over twice m times twice k. So, that I have h square over m integral 0 to k F d k, k to the power 4 which is g beta over 2 pi square. I am going to have h square over 15 m times k F to the power 5 and now, k F is 6 pi square n over g raised to the power 1 by 3. So, that I have g beta over twice pi square 1 h bar square over 15 m and then, I have 6 pi square n over g raised to the power of 5 by 3.

Now, I also know that epsa f is going to be h square ks f square over twice m which is going to be mu at 0 temperature. So, I can equally write down twice m over h bar square half and mu to the power half. So, this expression, I can also write down as g beta over twice pi square h square over 15 m times twice m over h bar square raised to the power 5 by 2 and mu to the power 5 by 2. So, you see the beta P has this expression.

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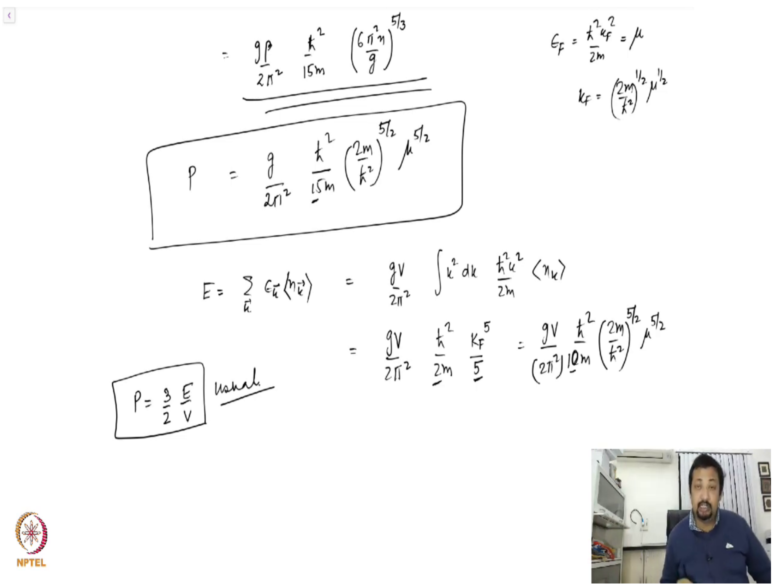

Now, beta beta is going to cancel out from both side right. What about the energy? Energy was sum over k epsa k n k which I know as g V over 2 pi square k square d k h square k square over twice m. So, sorry there is an average occupation over here and n k.

So, this is going to be g V over 2 pi square h square over twice m and I am going to have k F to the power 5 over 5 right and once again, this is going to be just g V in terms of the chemical potential, the energy is going to be twice pi square h square over twice m.

And I am going to have twice n by h bar square raised to the power 5 by 2 nu to the power 5 by 2 and there is going to be a this factor is going to be 10, 5 multiplied by 2; this factor is 15 and this factor is 10 and you immediately realize that pressure is 2 by sorry pressure is 3 by 2 times E over V which is the usual case that for a non relativistic gas and which we derived very generally in our earlier classes.