Fluid Dynamics for Astrophysics
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Lecture - 61
Non-ideal MHD: Magnetic reconnection - The Sweet - Parker Model

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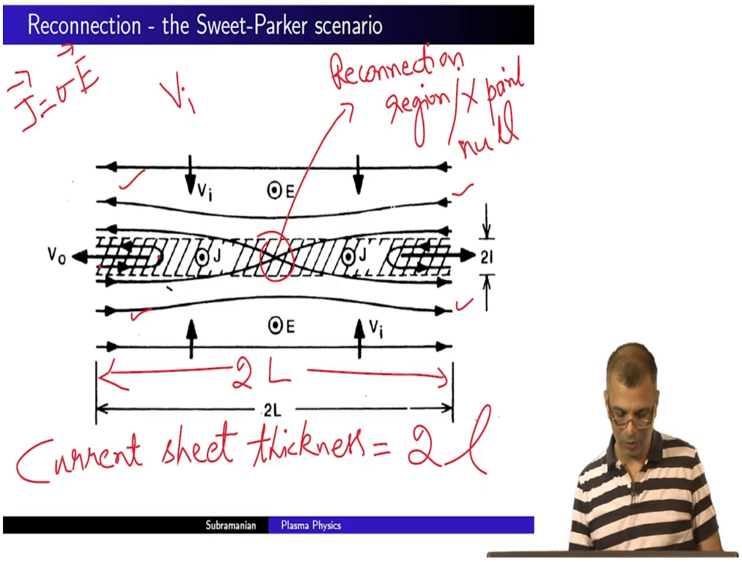

Now, we are back and let us delve into some more details of reconnection. Specifically, start thinking about something called the reconnection rate ok. In other words, the rate at which magnetic field magnetic flux in particular is annihilated, because as we said you know as we pointed out the whole point of reconnection conceptually.

Yes, it is cutting and pasting field lines which is not you know permitted in MHD and so, you have to; you have to invoke fancy non ideal effects such as finite resistivity and everything. Where the finite resistivity comes from that is another matter, but we will see, but the bottom line is the reason reconnection is so interesting to astrophysicists is that it is a potential way of converting magnetic energy into in into heat.

Also, you know bulk kinetic energy of the flow and so, essentially the amount of magnetic energy that is converted is essentially, it depends upon the amount of magnetic flux that is annihilated that is eaten up right. and so, the rate at which this magnetic flux is annihilated is very important, this is called the reconnection rate or it is related to what is called the reconnection rate and we let us embark on a little bit of discussion of this topic right.

So, this is what is called the Sweet Parker scenario after the two scientists Eugene Parker and Sweet who almost independently came up with this particular kind of calculation and it is all explained in this figure here. The basic details are the same as what we have seen earlier.

So, you have got two anti parallel field lines. You see one going this way and the other going this way right, two antiparallel field lines which are being brought close to each other at a velocity V i ok, at a speed V sub i right. As they come close to each other the straight field lines get a little curve like this right.

And finally, what happens is this field line reconnects with this field line to give a thing like this and this one reconnects with this one to get give a configuration that looks like this and right here this is the reconnection region or whatever the reconnection variously called the reconnection region are often called the X point for obvious reasons you see, the magnetic field configuration looks like an X or null point so on, so forth.

It is in this small region that all the action happens ok. Now, from Amperes law we know simply, because you have got; you have got oppositely directed fields you know he field pointing this way and a field pointing this way, right here in between you need to have a current.

In this case the current that is coming out of the plane of the screen. So, this dot would be the head of the arrow ok and you can either think in terms of current density or you can think in terms of electric field ok, because you know from Ohms law you just have J is equal to sigma E.

Now, you have a finite sigma not an infinite sigma right. This is the conductivity and so, the J and the E go hand in hand they are the same essentially, except for a factor of sigma and yeah. So, you remember I mean you know the whole point of you know of ideal MHD was that the sigma was infinite.

Therefore, there was no way you could have any electric field in the bulk fluid. In ideal MHD there is no scope for an electric field whereas, here you see at least in this very local region you have you know the appearance of an electric field. So, this is yet another way you can understand this off invoke statement of stated thing that ideal MHD is violated in reconnection.

What one really should say is ideal MHD is violated in the reconnection region right here ok, via why? Well, via finite resistivity effects or via the appearance of an electric field like that in this case the electric field the is pointing out of the plane of the screen and so this would be the head of the arrow ok right.

So, let us now by way of dimensions let us think of the thickness of this current sheet as 2 times small l ok. So, the current sheet thickness is 2 times small l whereas, the macroscopic dimensions are 2 times large L. Why the 2? Well, you cut it in between and so this would be large L this would be large L and even here you cut it like this and so on one side you will have small l, other side you have you would have small l.

The other thing this tells you is that this figure attempts to show to you is that you have these field lines approaching very-very close together, they come very close together and then finally, post reconnection what happens is they snap like this. This is the po this is the after kind of field line like this, this one and this one ok.

The field lines snap apart and they move out at velocity V naught. So, initially this would be the velocity at which the field lines are being brought together and this V naught would be the velocity at which the field lines are snapping apart ok. So, having done this.

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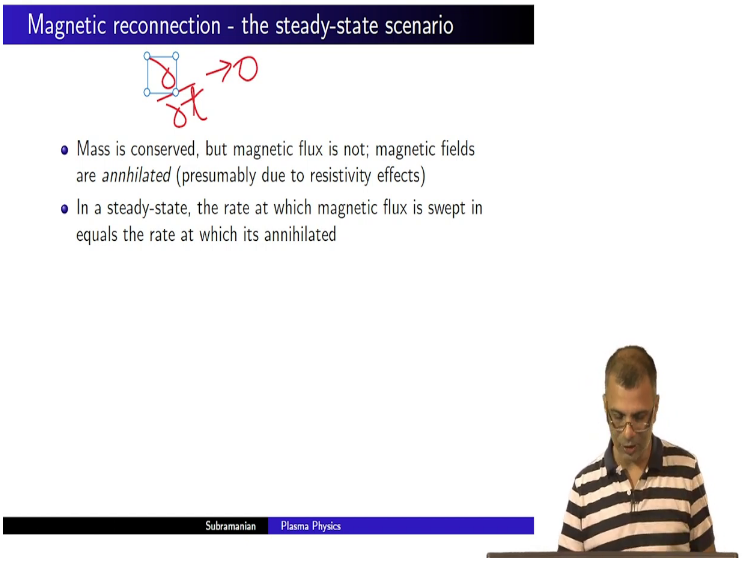

Now, let us see like we said mass is conserved, but magnetic flux is not. That is the whole point magnetic flux being not being conserved is yet another way you can understand the violation of ideal MHD. In ideal MHD magnetic flux always conserved right. So, in this case, in at least in the reconnection region magnetic field is not conserved magnetic fields are annihilated ok.

So, let us presumably due to resistive effect ok. So, right and by the way we are talking about a steady state scenario in which there is there is no in other words, you know everything appears steady. In other words the entire, the rate at which mass is swept in is equal to the rate at which mass is leaving.

So, in steady state the rate at which magnetic flux is swept in is equal to the rate at which its annulated. There is a whole definition of steady state ok. There is no difference in these rates otherwise, this would be violated ok. So, this is another way you define the steady state.

You are sweeping in magnetic flux at a certain rate into the reconnection region ok and it is being eaten up ok, the magnetic flux is being destroyed or annihilated at a certain rate and these two rates are equal ok, that is another definition of steady state ok.

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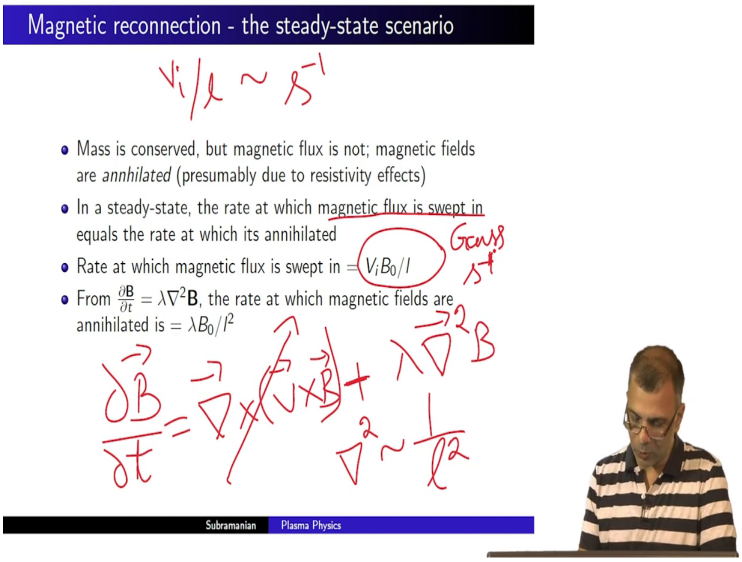

So, what is the rate at which magnetic flux is swept in? You see, V i is the is a speed at which you know the magnetic flux is brought in ok and this whole annulation is happening over the thickness of the current sheet you agree 2 L and therefore, the rate would be V i over l like that. In other words, the dimensions of V i over l would be per second, you agree with that. So, this gives you Gauss per second ok. Now so, this is the rate at which magnetic flux is swept in, this thing.

Now, what is the rate at which is annihilated? Now, annihilation follows from finite resistivity effects. So, now, what happens is you have the induction equation which now looks like this d B d t equals lambda where lambda is in includes all the resistive terms del square B ok.

The generally you would have seen the induction equation is this, curl of right and that is all, because the other the other resistive term was generally not important, this term was generally not important. In this case the opposite is true, this is not important ok and so it is only this term which is important.

So, you have d B d t equals lambda you know del square B and from you know order magnitude analysis you see the d B d t is like a Gauss per second and the Gauss per second is what? So, you have you know a lambda let us keep lambda as it is ok and this del square B is something like right. 1 over l square, because it is like a d over d x kind of thing d over d x d over d y.

And where is you know why are we writing this as one over small l squared, because the whole point is that d over d x tells you the rate of change of magnetic field in space. Where is the magnetic field changing? The magnetic field is changing only in this region is not it? It is really or rather is changing over this length scale.

So, it is correct to replace the d over dx by something like 1 over l and therefore, d square d square over d x square is something like 1 over l square, where l is a small l the dimensions of the reconnection region because that is where the d over d x is appreciable, outside of that there is really no change d over d x is as good as 0; that is why we write the magnetic fields are annulated as lambda we keep it as it is times B naught over l squared the l squared comes from the del square B.

And if you look at the dimensions of lambda if you look at the dimensions of l it turns this works out to be B naught B per second and it has to the left hand side is B per second. So, it works out. Now, what you do? The whole point of the steady state assumption is that you equate the rate at which magnetic flux is swept in to the rate at which magnetic flux is annihilated. So, these two quantities need to be equated.

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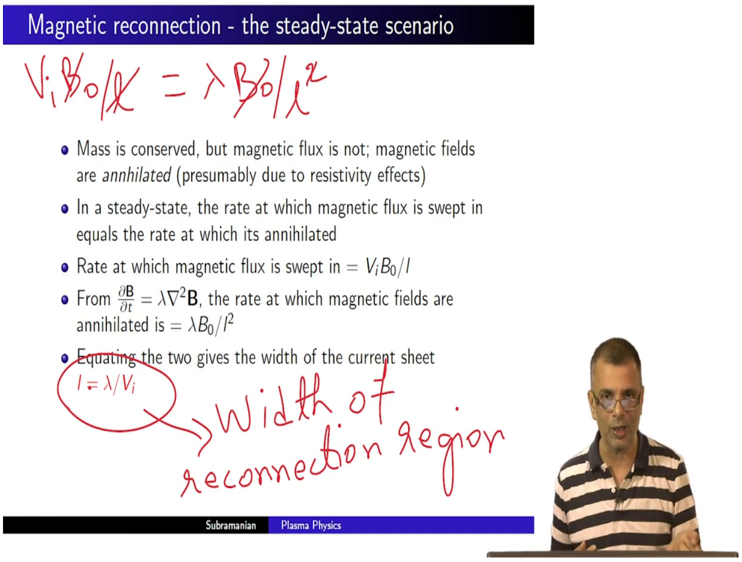

So, you equate this to this ok you write V i B naught over l equals lambda B naught over l squared ok. This gives us the all important width of the reconnection region, very-very important. How thick is the reconnection region? From this I strike off B naught and I strike of one factor of l right, I strike off this and I get l equals lambda over V i.

In other words, lambda is directly proportional to the resistivity ok. So, the more resistive ok, the larger the value of resistivity in this region, the thicker the dimensions of the region where all this reconnection action is happening ok. The more the resistivity, the thicker this region, the lesser resistivity, the thinner this region ok.

Maybe, I should use the word anomalous resistivity, because in the bulk of the flow the resistivity is technically 0 ok. It is only in here that some magic resistivity some due to some non ideal effects resistivity is starting to appear. It is kind of an anomalous effect. So, it is really an anomalous resistivity we are talking about here ok.

So, the larger that anomalous resistivity the thicker the dimensions of the you know of the reconnection region also, a smaller the inflow velocity right, the thicker the dimensions of the of the reconnection region that is what this formula is telling us ok.

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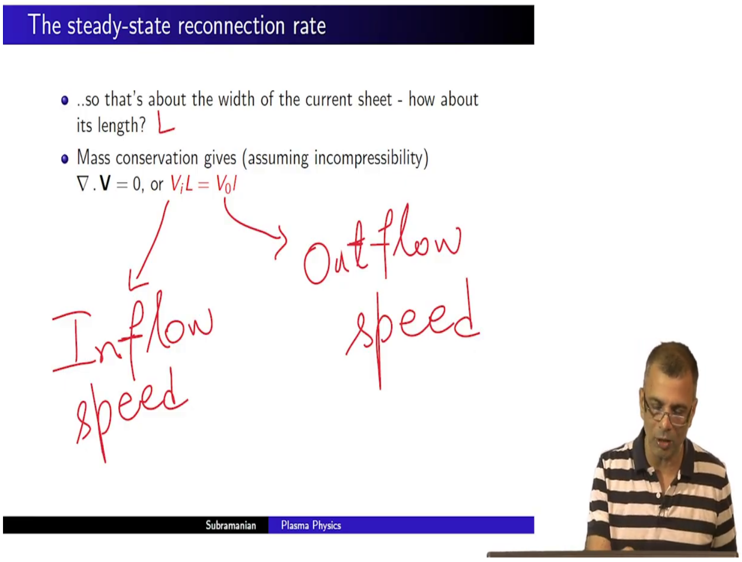

Yeah, so that is about so that is about the you know the width of the reconnection region this quantity very important ok, but we also need to consider the macroscopic length, the larger dimension. What about this capital L? As you guessed we derived this small l from a conservation or non conservation of magnetic flux ok.

So, the capital L will be derived from conservation of mass ok. Assuming so, how about its length? In other words, how about its length capital L? This is what we are talking about, you know. What is capital L? So, in order to do that we use mass conservation and assuming incompressibility we let us not, let us neglect compressibility effects for a minute just to make life simple.

You know otherwise that introduces yet another that throws a spanner in the works as it were and it introduces yet another complication. So, let us not, let us not bother ourselves with that complication. So, let us say and we know in the incompressible condition is given by this divergence of V equals 0.

And so, if divergence of V equals 0 is equivalently expressed as V i times large L is equal to V naught times small l, where V i is inflow velocity or as shall we say to keep it simple inflow speed and this is called this is the outflow speed and what outflow are we talking about?

Well, this outflow, this outflow right here, this is what we are talking about and this would be the inflow ok right. So, V i times the this is essentially the you know the one dimensional equivalent of divergence of V equals 0 ok. So, now, so, the ratio of the large L to the small l is given by the ratio of V naught to V i ok that is what this is the telling us.

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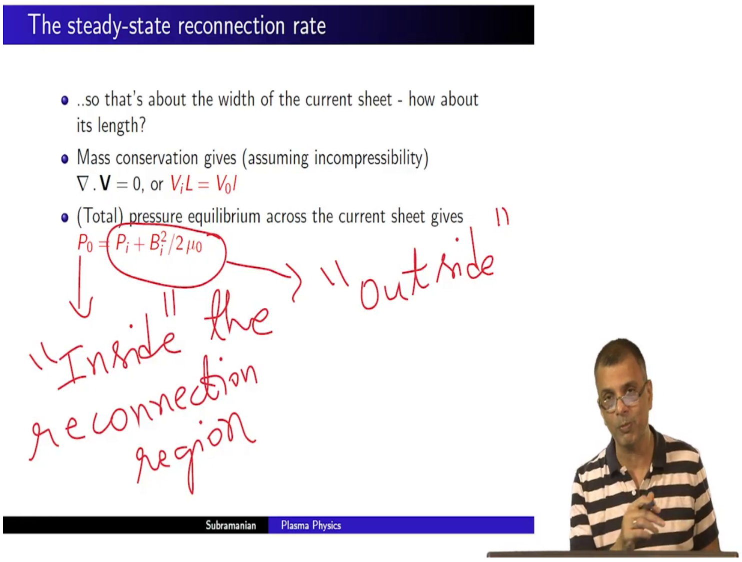

Now, total pressure, now there has to be pressure equilibrium across the current sheet ok otherwise, the current sheet will either keep expanding or it will keep collapsing if there is no pressure equilibrium. It is only, if there is pressure equilibrium that you will have a steady state scenario ok. So, across the current sheet so you have P naught which is kind of inside the current sheet, are inside the reconnection region right and this whole thing is outside.

So, the pressure inside the reconnection region and the pressure outside are balanced, of course, there are different contributions to the pressure. Outside the reconnection region you have the gas pressure which is P i and the magnetic field pressure which is in this case we are writing it in S i units. So, instead of B squared over 8 pi you have B squared over 2 mu naught ok. It is just a change of units as such physically nothing different is happening, it is just the way we are writing it.

Anyhow this gas pressure and magnetic field pressure whereas, inside the reconnection region you notice, there is no magnetic field pressure, there is only gas pressure very curious is not it? Why is that?

Well, remember we were saying that inside the reconnection region the magnetic field is annihilated, the magnetic field is essentially driven down to 0. It is eaten up, magnetic flux is eaten up there is no magnetic field right there in that very small reconnection region that is why you do not have any appearance of a B naught squared ok.

There is no B naught ok, it is a null point it is an X null as a null of magnetic field. So, there is no magnetic pressure, there is only particle pressure. So, inside the reconnection region there is only particle or gas pressure and outside you have gas pressure plus magnetic field pressure. So, the these are these have to be in equilibrium.

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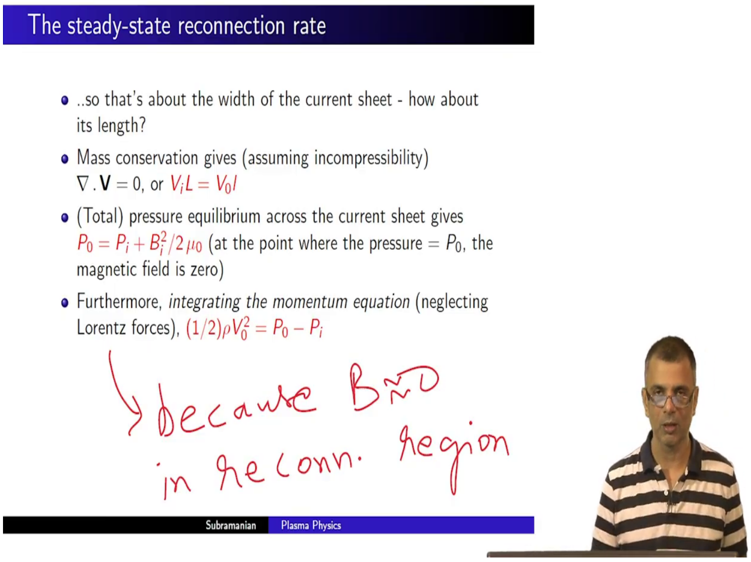

Yeah, at the point where the pressure equals P naught the magnetic field is 0, this what we just said. Now, integrating the momentum equation you get the difference in pressures is simply one-half rho V naught squared this is we neglect the Lorentz forces why? Because B is nearly 0 in the reconnection region.

So, Lorentz forces are j cross B. So, it is not so important. So, therefore, we so, this gives you something about that. This is the outflow velocity, this tells you how fast the outflow velocity will be. It depends upon the difference in gas pressures in the reconnection region and outside. Of course, on the mass density, no doubt ok.

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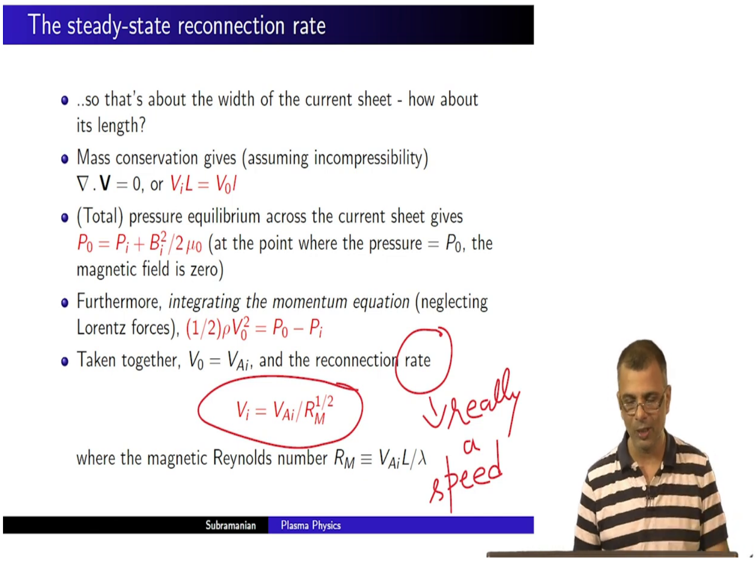

So, now taken together what will happen is you will find out if you solve these, you will find out that the outflow velocity is essentially the Alfven velocity. The Alfven velocity far away from the reconnection region right. In the reconnection region the elephant velocity falls to 0, because the magnetic flux is 0 right.

Taken together this is what we come up with and this is all important reconnection rate. It is really a speed ok. So, what this is? This is what is called the reconnection rate ok. It is a self limiting process really. It is not as if you see this, it is not as if I can V i is this velocity right.

It is not as if I can drive these oppositely magnet magnetic oppositely directed magnetic fields together at any arbitrary speed. No, I cannot. I can only drive it with the speed that is commensurate with the fact that whatever magnetic flux I am bringing in here needs to be eaten away immediately ok.

The rates have to balance the rate of you know bringing together of magnetic flux has to balance the rate at which they are eaten away number 1, number 2 the mass fluxes has to have to be equal, number 3 there has to be pressure equilibrium otherwise the current sheet will grow thinner and thinner.

If you want of a current sheet thickness that stays constant with time there has to be pressure equilibrium, if I have to satisfy all of these three conditions ok I find that there is a very specific number for the speed at which I can bring these oppositely directed magnetic fields together and that is equal to the Alfven speed outside in very far from the reconnection region divided by the magnetic Reynolds number, a square root of the magnetic Reynolds number which is defined by this ok.

The magnetic Reynolds number is defined by the Alfven speed times the macroscopic length divided by lambda ok. Now, if the lambda is technically almost close to 0 the magnetic field Reynolds number goes to infinity and vice versa.

If the lambda is large the magnetic Reynolds number is small ok. So, this is the; all important reconnection rate and this is just something, this is just a quirk of the terminology. It is what is normally called the reconnection rate, is really a reconnection speed ok.

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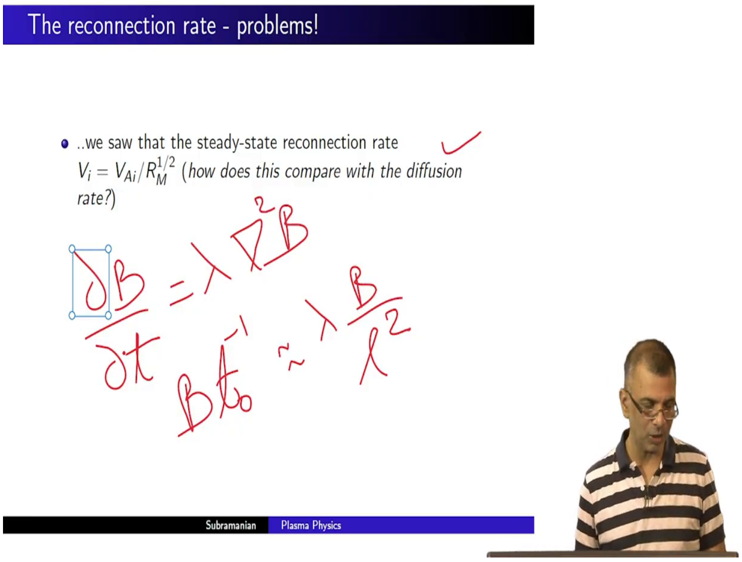

Now, why are we so concerned about reconnection rate? You see, this is the steady state reconnection, this is the reconnection rate that we derived in the previous slide ok, but now this is another thing to ponder. How does this compare with the diffusion rate? In other words, if I write down the ok and if I dimensionalize these things B times some t naught inverse B over some l square, if I plug this in this will give me the diffusion time scale.

This t naught inverse is this, this t naught is a diffusion time scale. How does this compare with the diffusion rate? It is useful, I urge you to work this out. How does this compare with this t naught? Well, you have to you have this is a this is actually a rate, in other words this is a time scale. So, if you want to convert it into in into a speed you have to invoke a length scale maybe, you should invoke the macroscopic length scale ok.

So, I urge you to compare it with the diffusion rate turns out that the diffusion rate is always very-very small ok. The reconnection rate is substantially larger ok, but the thing is you know in astrophysical and we hinted at this a little earlier.

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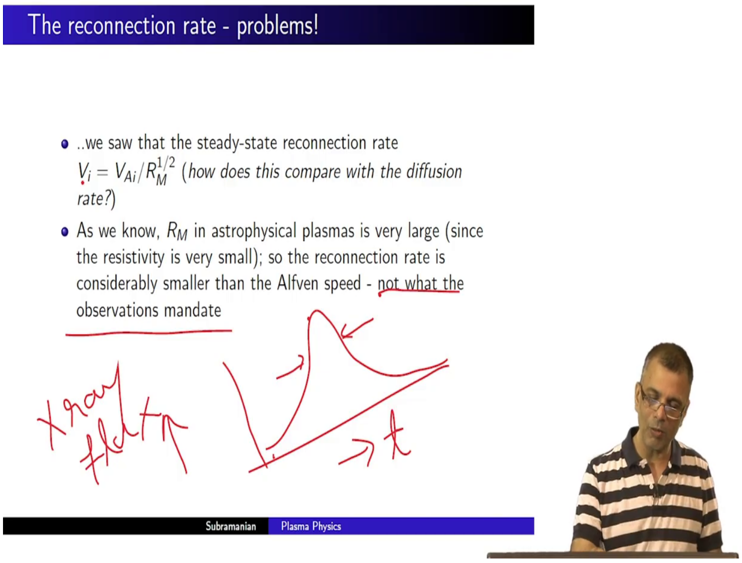

Problem here is that the magnetic Reynolds number in astrophysical plasmas is very large since the resistivity you see, this the this is the definition of the Reynolds number if that if the resistivity is very small the Reynolds number is very large and unfortunately, the Reynolds number appears in the denominator here, the dependence is a little weak, because there is a square root ok.

So, the dependence is a little weak, but still the Reynolds number appears in the denominator. So, if the Reynolds number is large, the reconnection rate is small, this is the thing ok.

So, the reconnection rate is small, small in comparison to what? Small in comparison to the Alfven speed ok. So, and I want to; I want to you know talk a little bit about this sentence in a minute. So, this is not what the observations mandate. What the observations, for instance what do you really mean by this sentence?

Well, let us back up a little bit let us consider a specific phenomenon like a flare. A flare on the sun in the solar corona which is a flare is simply you know an increase in if you if you were plotting the say the X-ray flux as a function of time ok and this can be X-ray flux, this can be ultraviolet flux, this n B flux at any wavelength ok as a function of time. It a flare is just this it increases and then it decreases.

It need not be symmetric like this, many times the increase is very abrupt and then and the dying off is very gradual either way. This is a flare, you know you striker matchstick flame flares up that is a flare ok. It is just an increase in the number of photons as a function of time X-ray photons are plotted on the y axis and the number of photons increases, this is what a flare is.

The question is what is this time scale? Say you know one half of maximum time scale, what is this time scale? And so, this is what the observations mandate, but mind you this has got nothing to do with I mean as such it is a very indirect thing to relate this time scale to anything like the reconnection time scale, but the thing is its as follows, the line of reasoning is as follows, well you know who was responsible for heating the plasma, we are holding the reconnection process to be responsible for hitting the plasma, is not it.

So, if the reconnection if the magnetic flux is annulated at a certain time scale the heating also happens at approximately that time scale, because it is annihilated magnetic flux that leads to heating. If the magnetic flux is annihilated over a certain time scale the heating also happens at that time scale and therefore, once the plasma is heated over a certain time scale. It will also radiate, hot bodies radiate, it radiates and this is observed radiation plotted on the y axis.

So, the since the heating occurs at a certain time scale and then after the heating is turned off, in other words after the reconnection stops well, the you know the flux has to fall, the plasma has to cool of course, there are conductivity effects ok to be worried about. It is not as if you know that the region which is you know radiating is completely thermally isolated from its surroundings.

You have to worry about conductivity, but roughly speaking you know once the heating is turned off the plasma has to cool and so, the observed you know radiation also falls down. So, it is in that sense that the time scale the observational time scale is roughly this and you compare this with the Alfven speed that you expect in the medium and turns out that the time scale over which this happens is roughly similar to the Alfven time scale or something like you know yeah.

So, whereas, if you plug in typically assume numbers for the Reynolds number, the magnetic Reynolds number in astrophysical plasmas turns out that the V i the reconnection rate is actually very-very small. It is nothing close to the Alfven time scale whereas, the observations kind of mandate what is called an Alfven time scale.

So, there is a problem with this nice scenario and how to solve this problem? Petsheck came up with a very innovative solution to solve this problem and we will talk about this and when we meet next so for the time being.

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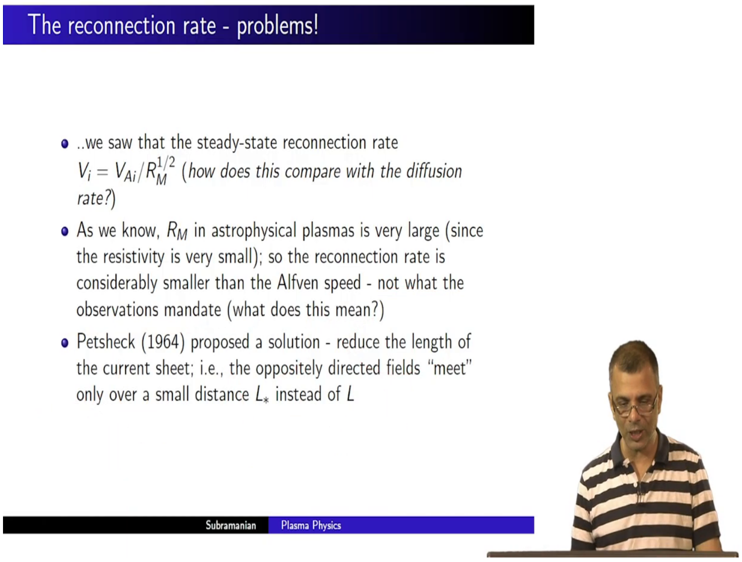

Thank you.