Fluid Dynamics for Astrophysics
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Lecture - 59
Magnetohydrodynamics [MHD]: Shocks in MHD - Shock jump conditions

So, having conceptually enumerated the way in which the conservation relations for magneto hydrodynamics from which we will derive the jump conditions of course, differ from the hydrodynamic case let us now proceed to write them down right.

So, but before that I think it is useful to sort of very briefly review how we did things in hydrodynamics ok. The very same thing just so, we can move rapidly when we come to higher magnetohydrodynamics ok.

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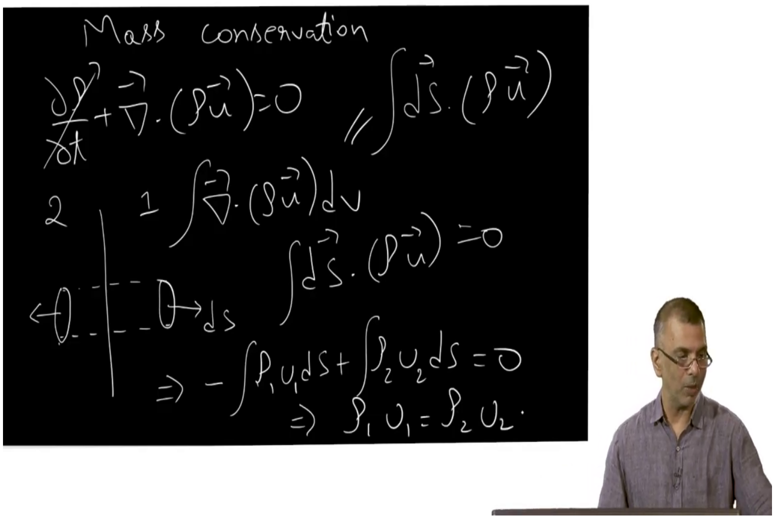

So, consider the mass conservation right, mass conservation once again mass conservation ok. What we really are saying is that the rho u is constant right, but what do you really mean I mean how did we this actually come about right?

So, because the mass conservation equation is essentially d rho dt plus divergence of rho u is equal to 0 and if we consider a steady state situation where this goes away and so, we are only left with the divergence of rho u equals 0 ok.

And applying the divergence theorem the volume integral of this can be the volume integral of the divergence of rho u becomes a surface integral of you know this becomes a surface integral of right. So, the volume integral of this becomes equal to the surface integral right. Now, the question is how do you choose the surface cleverly.

The thing is here is here is the shock or some kind of discontinuity and here is a region 1 and here is a region 2 and let us consider a little cylinder like this. So, which is the dS you are talking about? This would be the dS ok. So, therefore, this thing this dS dot rho u this one being equal to 0.

So, you see the volume integral of the divergence of rho u equals 0, means that by the divergence theorem the surface integral of this is equal to 0 implies if you consider just this little thing with the length of the cylinder being shrunk so much that you know this phase of the cylinder and this phase is of the cylinder are arbitrarily close to the shock front ok.

So, this essentially becomes minus rho 1 U 1 dS plus rho 2 U 2 dS. Why the minus here? Because the dS here is facing that way and the dS here is facing the other way, right. This is equal to 0 ok, which gives you rho 1 U 1 equal’s rho 2 U 2. This is how the mass conservation came about if you remember.

So, the key is this diagram. This little diagram where you have you know a little cylinder with the dS you know like this and the length of the cylinder being a shrunk to an arbitrarily small thickness.

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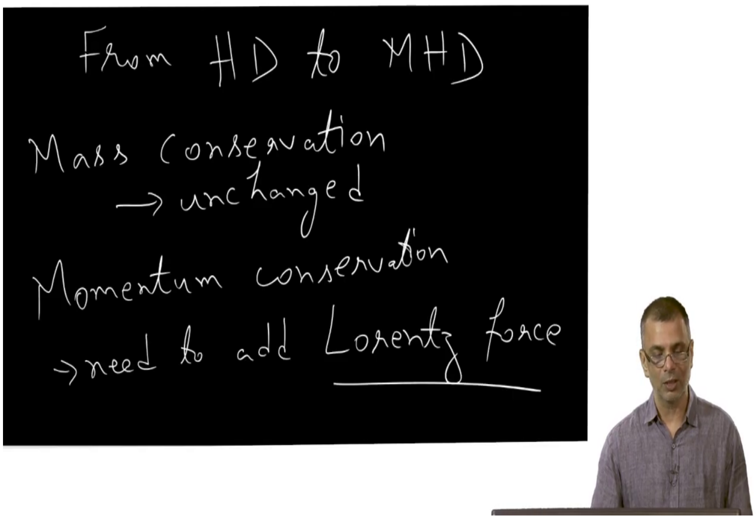

Similarly, momentum conservation which is very important because you know that is where magnetohydro dynamics starts becoming significantly different from hydrodynamics.

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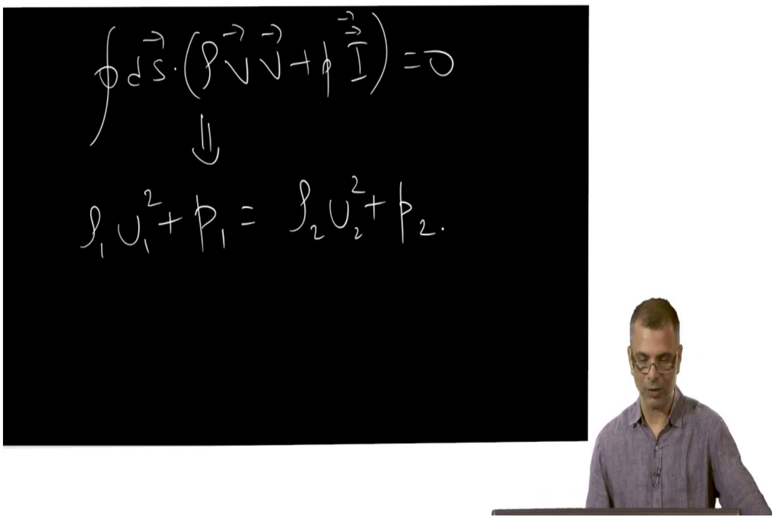

And the momentum conservation equation becomes now, we are still doing hydrodynamics is essentially dot rho, where this is the outer product plus p I equals 0. This is just the dyadic tensor with a simply a tensor with the diagonal elements being 1 and the rest of the elements being 0.

And this is the outer product, the outer product of these two velocities ok, not the inner product the outer product ok. So, now, you know you evaluate this over the same cylinder over this over this same cylinder and you carry out this integral.

And this gives you the momentum conservation equation that we are we were familiar with. 1 U 1 squared plus p 1 is equal to rho 2 U 2 squared plus p 2 right. So, this is how we got the you know the momentum and flux conservation equations. Now in MHD though, now we are talking in MHD right.

So, in MHD the as we said the mass conservation equation is the same no need to repeat it now.

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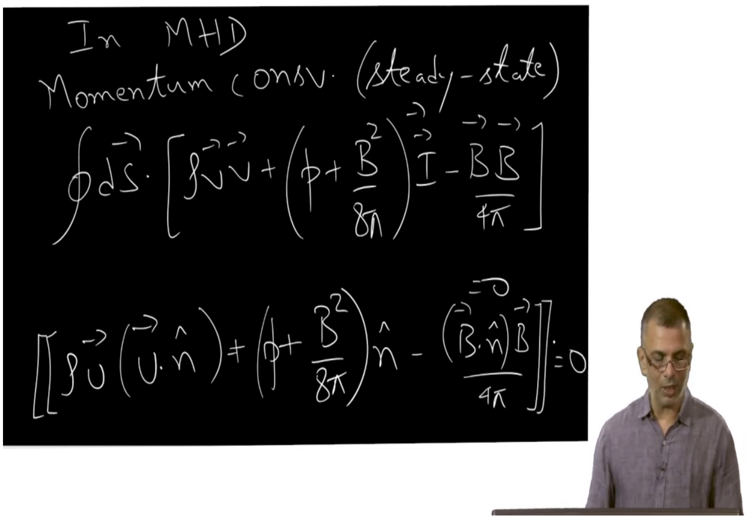

So, in magneto hydrodynamics momentum conservation as we said momentum conservation is pretty much the same except for the addition of the Lorentz force which now manifested. So, the J cross B manifests itself in the form of magnetic tension and pressure ok; magnetic tension along the magnetic field and magnetic pressure perpendicular to the magnetic field.

So, the momentum conservation equation steady; state steady state as always. In other words that d over d t partial d partial t goes to 0 and it becomes dS dot this chunk is the same as before and this chunk is also the same as before except now you add magnetic pressure right and this is the dyadic.

So, this is the same now, you added magnetic pressure, but there is one more thing ok. This is a magnetic pressure in addition to that you have the magnetic tension with a negative sign here where this is an outer product ok. This is equal to 0 ok. And just like you got this from this the jump condition now becomes from this the jump condition now becomes rho U times U.

I am writing this down without too much derivation , but I i i am hoping that you know the details you will work out the algebra it is not that hard ok. Um The algebra is not that hard and then you will be able to work it out yourself. It follows directly from here.

B squared over 8 pi in the n hat direction right minus B dot n hat, where n hat is the wave propagation direction of course, or the shock propagation direction I must say. So, as you can see it is a little more complicated, it is a little more. So, but you can see the same things.

You can see the rho U squared, you can see the p except now you have got a B squared over 8 pi and you have got this magnetic tension. These are the two pieces that are new in the momentum conservation for MHD ok.

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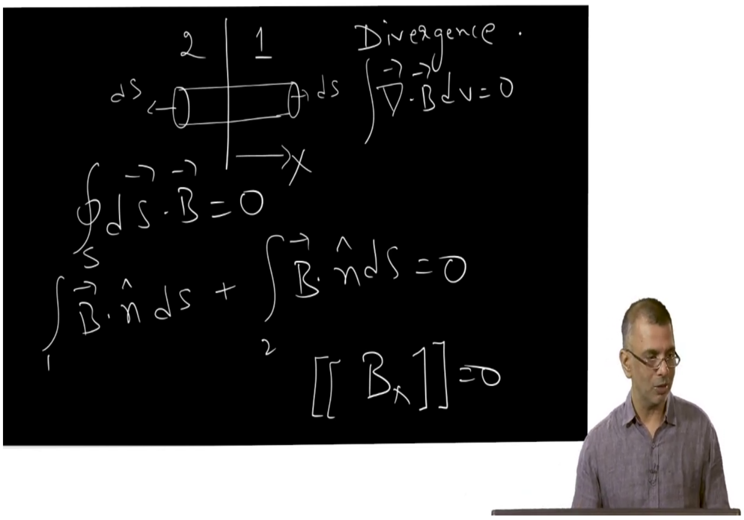

The divergence constraint again we will draw this thing once again ok. So, the divergence constraint in so, and we will draw this little cylinder again. This guy the dS is this way and the dS here is this way and the length of the cylinder can be you know arbitrarily shortened right. So, the divergence constraint in delta B equals 0 can be rewritten using the divergence theorem as dS dot B equals 0. You agree? This is the same as this right.

Now, we perform this integral over the cylinder right. So, essentially what happens is you have from 1 and 2 so, you have over one B dot n right dS where n is the outward normal right plus B dot n dS. So, this is over you know and this is over the phase in medium 1 or on side 1 and this is over the phase on side 2 .

This is equal to 0 which gives you which is which essentially tells you that this is the jump can or rather. This and X is of course, the outward the direction of propagation of the shock like that. So, the difference in B X on the two sides of the shocks has to be equal to 0 ok.

So, this is what comes from the divergence constraint this is one; so, normal component of B remains constant across a shock . This is the you know one of the main things right.

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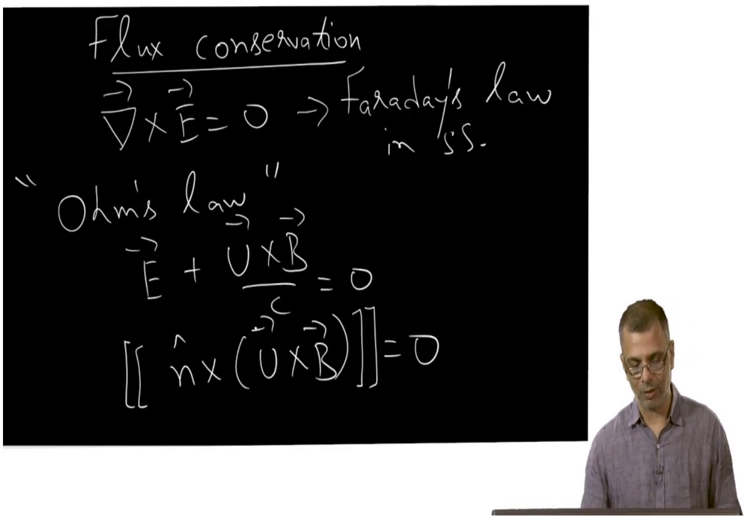

So, next one is conservation of flux conservation magnetic flux conservation. How does that work out? Well, in steady state Faradays law simply tells you and we will soon write E in terms of B and you know that . So, this is Faradays law in steady state right.

And Ohms law you know gives you I would put this in quotes. We have seen this earlier how to relate the electric field to the magnetic field, we have seen this earlier in terms of. So, E plus we have seen this earlier right. So, now, what we do is we integrate this over a thin strip including the shock just like this just like this ok.

So, what we are saying is that is a component of E tangential to the shock this guy must be continuous that is what it boils down to. And so, integrating after integrating over this kind of a thin cylinder you will get cross. This is the jump condition that arises from flux conservation ok.

In other words the this quantity n hat cross U cross B which is you know this the difference on the two sides of the shock has to be equal to 0. So, this is the new constraint that arises from flux conservation.

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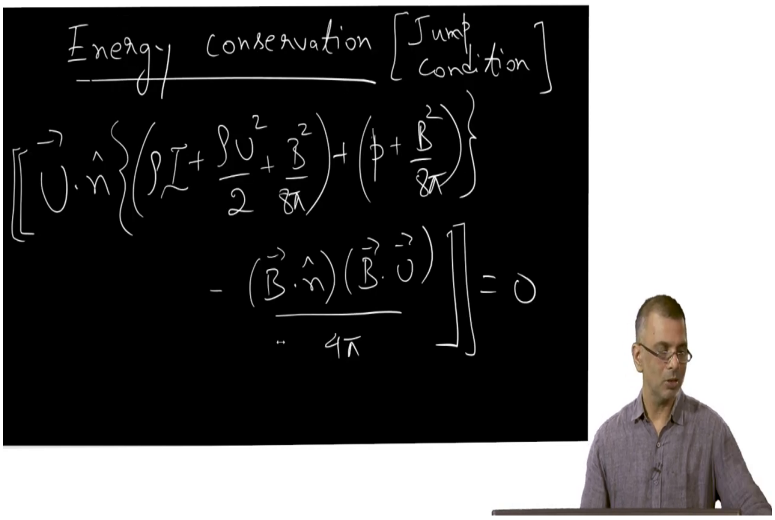

Mass conservation is the same and conservation of energy. We will simply write this down. We will not bother about energy conservation. The last one gives you it is a little long, but it is worth writing it down and so. As such it is no its not nothing terribly different its yeah. So, rho times this is the same internal energy that we have written down earlier.

This is the same kinetic energy density that we had also written down earlier the additional chunk is the magnetic energy density right here. That is the additional chunk plus again you had this earlier and this is the additional piece minus because of the magnetic tension you have one more one more slightly complicated looking thing and this way its like that over 4 pi.

And you the difference in this on the two sides of the shock has to be equal to 0 that is the energy conservation energy jump conditions really. So, this completes it really. So, you had the mass conservation, you had the divergence constraint, you had the momentum conservation which was this and you had the divergence constraint like we have said before. You had flux conservation and you have energy conservation.

So, this is pretty much it. Now, what about this right? How are we going to this? This all looks a little forbidding and how does one proceed right. The way one proceeds is by doing the following.

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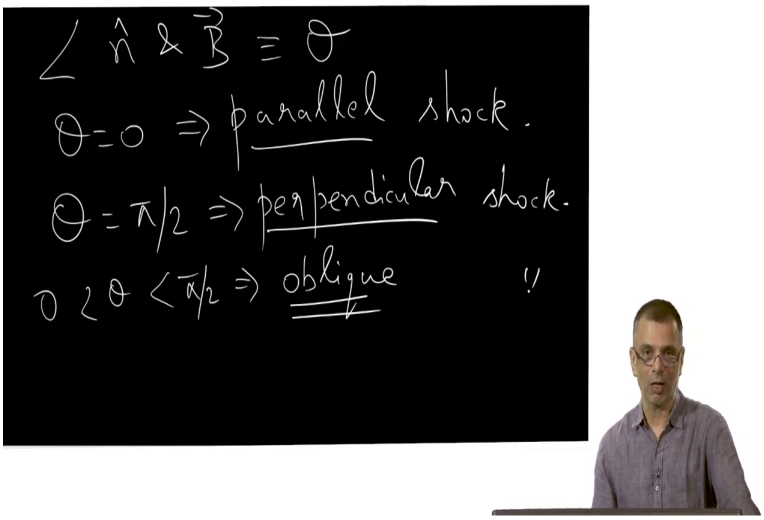

You first of all what you do is you n is the shock propagation direction of course and the angle between n and the magnetic field is we define as theta ok, right. So, theta equals 0 is what we call a parallel shock ok.

Theta equals pi over 2 is what we normally would call a perpendicular shock ok. And of course, in between it is a theta less than pi over 2 to greater than 0 is the more complicated case of a oblique shock. We will not bother too much about this oblique shock ok.

In addition to shocks we know that there are other kinds of discontinuities. We had we had you know encountered those in hydrodynamics as well. We had something called a contact discontinuity which involved a jump only in density not in a velocity or a pressure ok. So, that was a contact discontinuity and that appears in magnetohydrodynamics as well.

And then you have two other kinds of discontinuities in magneto hydrodynamics which are called the tangential discontinuity and a rotational discontinuity. We will sort of talk about this, but if time permits, but let us first enumerate the difference between a parallel shock and a perpendicular shock and essentially what these are really ok.

So, let us talk about parallel shocks first. So, this is how one. So, essentially this U dot and these guys this is how you know things simplify ok. These are the two extremes and the angle between n and beta is taken to be theta and if that is equal to 0, it is called a parallel shock if that is equal to pi over 2 it is called a perpendicular shock.

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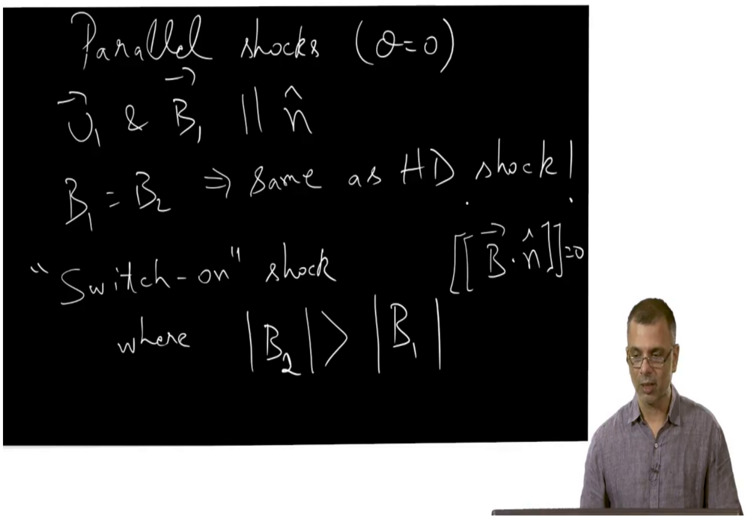

So, let us now discuss parallel shocks separately ok. In this case U 1 and B one both are parallel to n hat right and so, in the simplest case where B 1 is equal to B 2 on the two sides the magnetic fields are the same ok. If the magnetic field is parallel to the shock velocity and is constant in front of the shock as well as behind the shock then this happens and then this becomes same as a hydrodynamic shock. So, this is comforting ok.

The everything that we have done for hydrodynamic shocks remains valid here too ok. The other possibility other possibility is a switch on shock in which we will not bother too much about this. A switch on shock where B 1 can be at least the magnitude of B 1 B 2 sorry B 2 can be greater than the magnitude of B 1 ok.

And in this case some of the flow energy is indeed converted into magnetic energy ok. The only thing is the normal component of B is conserved. In other words this B dot n this is conserved, whereas, the tangential component is not. In this case the tangential components are the same and. So, in one particular case the parallel shock is the same as a hydrodynamic shock ok.

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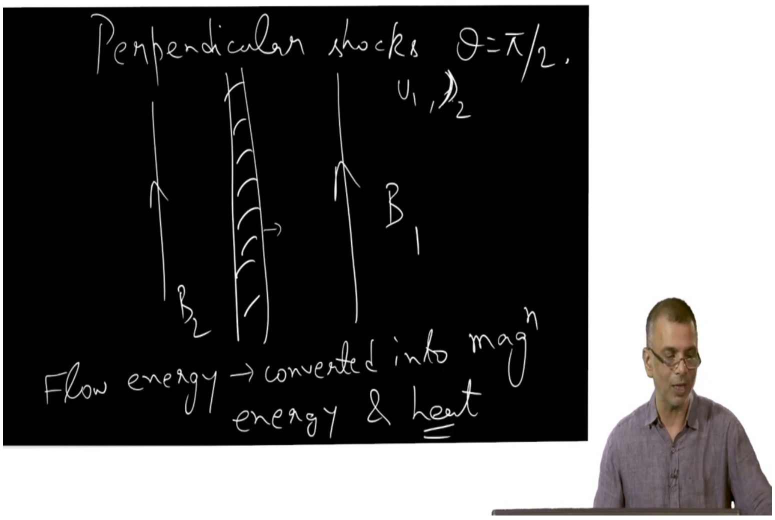

So, now let us now discuss perpendicular shocks, where what this means is theta equals pi over 2 ok. And so, essentially what this would mean is that if this is a shock if you know this is the shock and so, on one side you would have U 1 rho 1 and everything and what the perpendicular essentially means is that the B 1 is like so, and the B 2 is like. So, that is what it means.

So, the magnetic field is perpendicular to the shock normal ok. The shock normal is like this right. So, it is perpendicular to the shock normal, this is what it means. So, in this case the flow energy the main thing about perpendicular shocks is that the flow energy which is the which is essentially the kinetic energy associated the rho U squared so to speak.

The flow energy is converted into magnetic energy and heat while heat if there is a means of viscous dissipation of course, that needs to be kept in mind ok. But, the main thing is there is a there is a chance for flow energy to be converted into magnetic energy.

Whereas, that possibility did not exist with parallel shocks because especially if B 1 is equal to B 2 there its it reduces to the to being the same as a hydrodynamic shock ok. So, that is that is the one thing.

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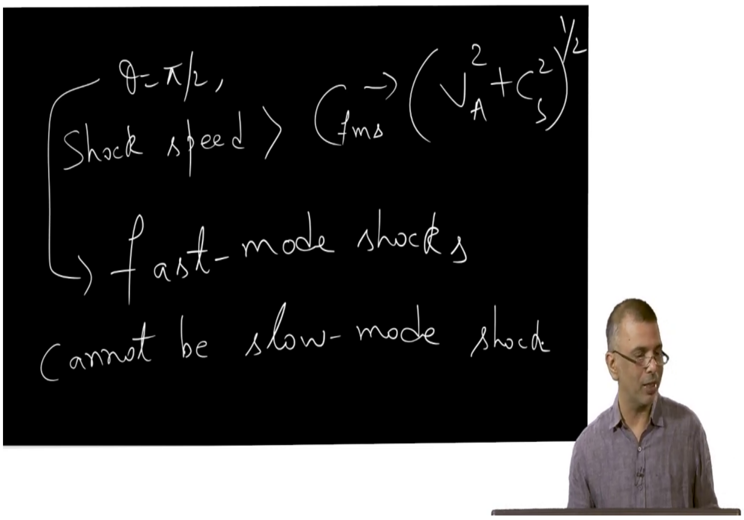

The other thing of course, is that the shock speed for theta equals pi over 2, the shock speed is shock speed exceeds should be greater than the fast magnetosonic speed ok, which is in particular the well the fast magnetosonic speed also has a theta in it. So, what I really mean by that is V A squared plus C S squared one half that is what I really mean by this ok.

Now, the so, that is the one thing. The other thing is that the in fast shocks the magnetic field is constant in direction and increases in magnitude by the same ratio as a density. So, you see here the magnetic field does not change direction. You know it is constant in direction except you know the density ratio increases across the shock, the magnetic field also increases in the same ratio ok.

And perpendicular shocks are essentially these kinds of shocks are essentially fast mode shocks, slow mode shocks and that is obvious from this statement. The shock speed has to exceed the fast mode speed and so, these are perpendicular shocks essentially ok. No cannot be it cannot be slow mode shocks.

In other words if the shocks propagation speed is greater than the slow mode speed and you insist that I want a perpendicular shock; in other words I want a shock that looks like this that is not possible ok. So, this is another small nugget to be kept in mind and oblique shocks are of course, you know a little more complicated and we will not bother with it too much ok.

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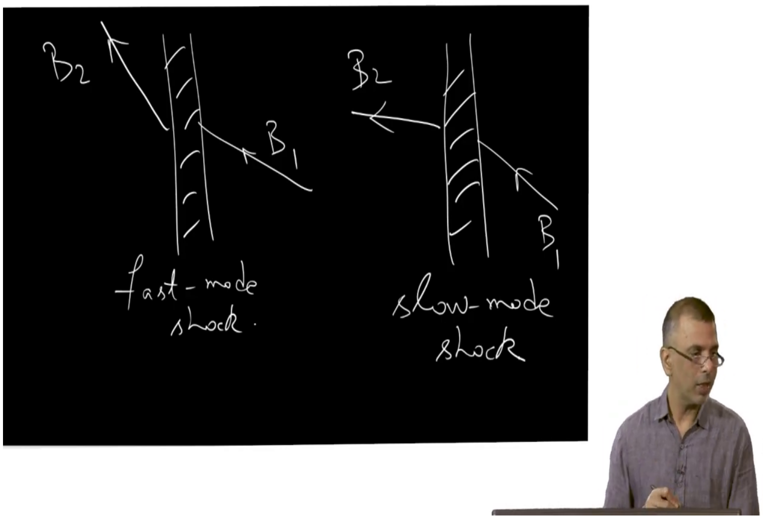

Now, let us very briefly look at the slight difference between fast mode and slow mode shocks. So, suppose this is a fast mode shock. The main thing about a fast mode shock is that if this was B 1 on one side, the B 2 looks like this; it is refracted away from the shock normal.

Whereas if it is a slow mode shock, it is the opposite. If it is a slow mode shock if this was the B 1, it is like that this would be a slow mode shock ok.

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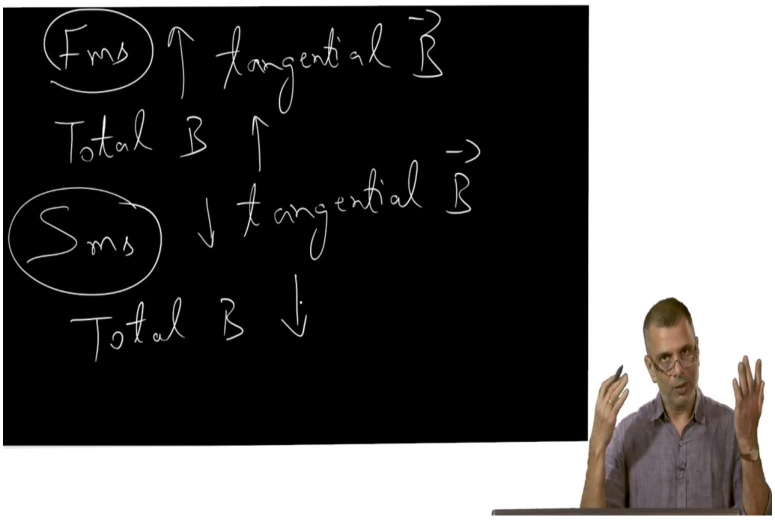

So, fast mode shocks F Fms, fast mode shocks they increase the tangential component of B and the magnetic field is refracted away from the shock normal as shown here and the total magnetic field also increases. Total B increases for fast mode shocks, ok.

For slow mode shocks on the other hand for slow mode shocks ok, they decrease the tangential component of B and the magnetic field is refracted towards the shock normal as so, the shock normal will be either this way or that way. At any rate that would be the shock normal. So, the magnetic field is refracted towards the shock normal as you can see here as I sketched here and the total B the total magnetic field decreases.

So, this way the fast mode and slow mode shocks are kind of opposites of each other. And of course, you can have a fast mode shock when the speed of the shock exceeds the fast mode speed the C S squared plus V A V A squared and you can have a slow mode shock when the shock speed exceeds the slow mode speed; obviously, right.

So, we will not bother too much because this is to expand these things because it is not our intention to go into the gory details of magnetohydro dynamic shocks right. I just wanted to give you a brief sketch of how the fact that you have three extra characteristic speeds, how it you know it results in different kinds of shocks.

And the fact that you are now dealing with magnetohydro dynamics instead of instead of hydrodynamics how it affects the jump conditions. The jump conditions are clearly different slightly different, but nonetheless in different in very important ways from the hydrodynamic jump conditions right.

So, as you can see the introduction of a magnetic field essentially disturbs the symmetry ok, disturbs the isotropicity I would say. It introduces a preferred direction and so, whether or not the shock is propagating perpendicular or parallel to the magnetic field it makes a big difference and. So, it results in two different kinds of shocks. So, or two different well the characteristics are quite different.

And so, I hope I have conveyed you know some flavor of the of the richness of the physics that arises because of the presence of the magnetic field in the fluid. And for the rest of the lectures what I will do is I will now sort of change gears and move away from ideal magnetohydrodynamics to start talking about something called reconnection ok, which deals with cutting and pasting of field lines which is prohibited in ideal MHD.

The reason we are we will talk about this is because reconnection is held responsible for a whole lot of energy conversion processes which are important in astrophysics. So, for the time being we will stop here.

Thank you.