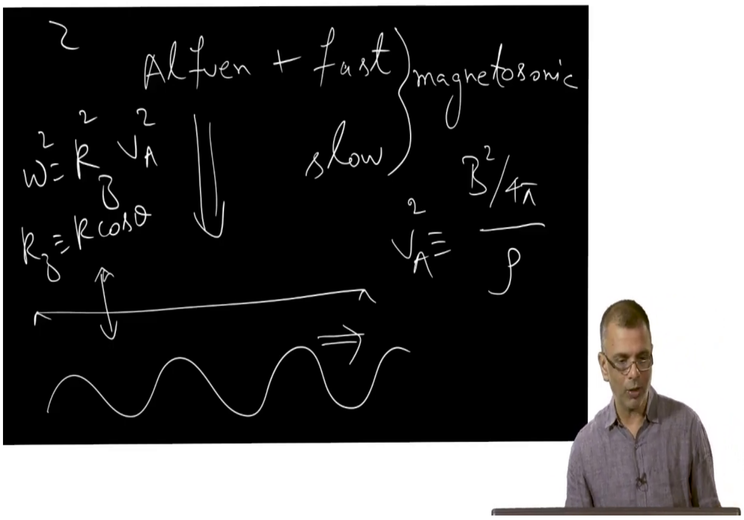
Fluid Dynamics for Astrophysics
Prof. Prasad Subramanian
Department of Physics
Indian Institute of Science Education and Research, Pune

Lecture - 58
Magnetohydrodynamics [MHD]: Shocks in MHD

(Refer Slide Time: 00:16)



So, we are here now to sort of wrap up our discussion of Alfven modes. Well, to be precise modes in a magnetized plasma which include Alfven modes as well as the Alfven modes plus fast and slow magnetosonic modes ok.

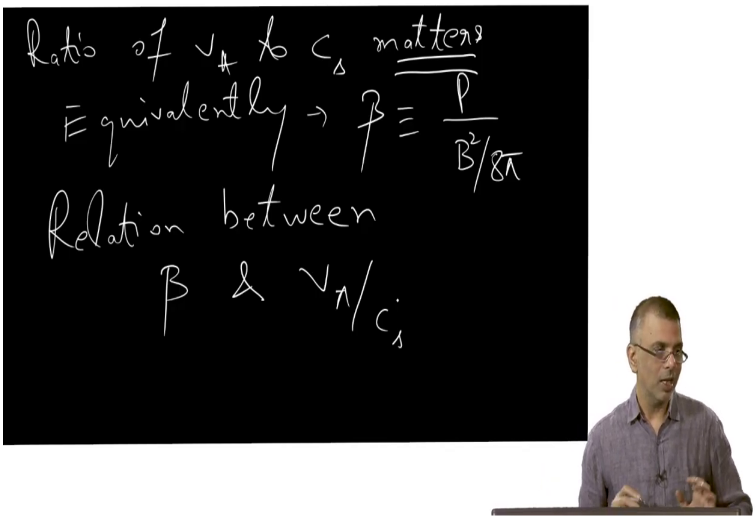
And so just to reiterate, the Alfven mode is very much like a wave on a string, so that would be a stretch string which is essentially the magnetic field line here. And you twang the string like this, you know you have transverse displacements transverse to the direction of the string and also to the direction of propagation.

And the string starts you know exhibiting undulations like this. And the wave travels in this direction with a velocity V A which is B square over a 4 pi over rho this might be 4 or 8. I, am not entirely sure, please check. And so these would be the Alfven waves, they are transverse waves with no density perturbations.

However, these two modes the fast and the slow magnetosonic modes which we have discussed at length they are you know they are sound like waves ok, and they are a little complex. It all depends upon their precise characteristics; depend upon the direction of propagation with respect to the background magnetic field.

And that is true for the Alfven mode also you know dispersion relation is omega square equals K z squared K z squared V A square ok. Where the K z is essentially K cosine theta where the theta is direction between the k vector and the direction of the magnetic field. So, that is true for the Alfven wave also the fact that the propagation is (Refer Time: 02:32) well, you know that the theta matters.

(Refer Slide Time: 02:52)

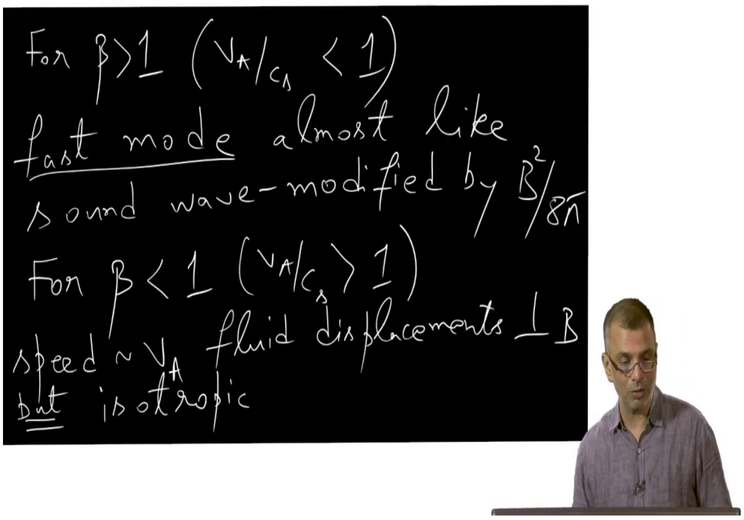


But let us now since the magnetosonic waves the fast and the slow magnetosonic waves were slightly different let us summarize having discussed them so far, let us summarize their properties a little bit. First of all the main thing is the ratio of the Alfven velocity to the sound speed this matters greatly; or if you will you can equivalently the ratio of V A to C s, and you can also you know write it in terms of the plasma beta ok.

I will leave you to figure out how. If you remember the plasma beta is essentially the ratio of the magnetic pressure to the sorry the particle pressure gas pressure to the magnetic pressure ok. So, when the beta is very low, that means, the magnetic pressure dominates; and when the beta is high, the gas pressure dominates.

And I leave you to figure out the relation between beta and V A over C s ok. It is quite easy. You can just go from here and figure this out.

(Refer Slide Time: 04:10)



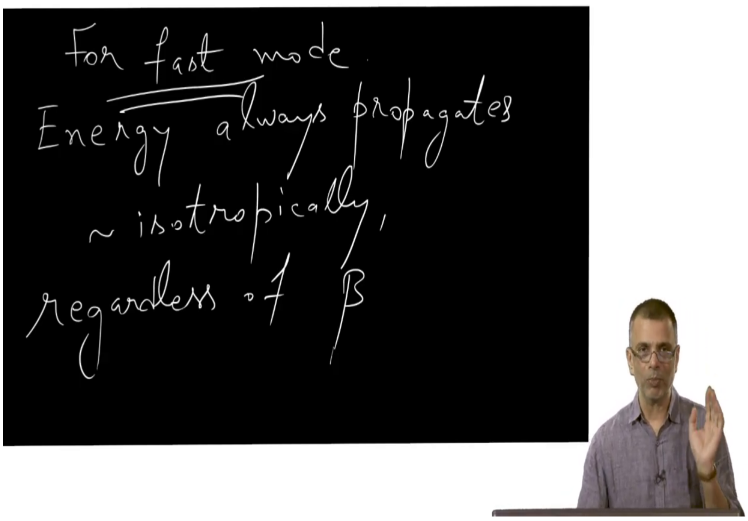
Anyhow, so the deal is for low beta for beta greater than 1 ok. In other words, V A over C s less than 1 ok, the fast mode fast mode the fast when I say fast mode I mean fast magnetosonic mode ok, it is almost like a sound wave almost, like a sound wave ok, modified a bit by magnetic pressure, almost like a sound wave except is modified by B squared over 8 pi, modified by the magnetic pressure ok.

For beta less than 1, in other words V A over C s greater than 1, yeah, it propagates the speed the speed of the fast mode is so right now we are discussing only the fast mode ok. We will, after this we will discuss a slow mode ok. The speed is almost the Alfven velocity ok. And the fluid displacement, the fluid displacements are perpendicular to the magnetic field very much like the Alfven speed ok.

But you know it is almost isotropic. In other words, dispersion relation the dependence of this of omega and k does not contain a theta as in the dependence on theta is very weak ok. So, in these two as in the beta less than 1 limit, it is like the Alfven waves in these two respects in that it propagates at the speed at the Alfven speed almost, and the fluid displacements are also perpendicular to B.

So, it starts looking like a transverse Alfven wave. However, the dispersion relation is such that the theta dependence is very weak unlike in the case for an Alfven wave which you know the dispersion relation does have a very strong theta dependence ok.

(Refer Slide Time: 06:44)

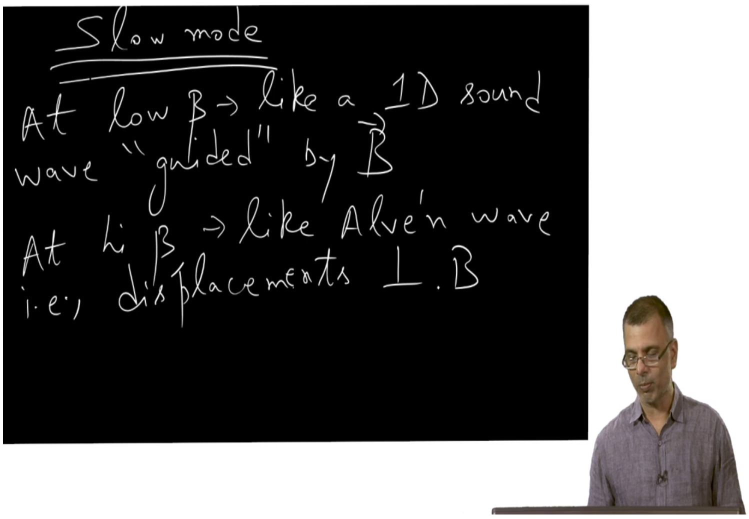


And for the fast mode, for fast mode, the group velocity which is the velocity at which the energy propagates, the energy always propagates isotropically not along the field or perpendicular to the field propagates almost isotropically regardless of beta regardless of beta or regardless of the ratio of V A over C s ok. So, that completes our little wrap up of the fast mode.

Now, by the way all of these relations, all of these deductions simply follow from the dispersion relation that we have seen earlier ok. What I am saying here is nothing it is you stare enough at the dispersion relation you will be able to figure all these things out ok.

These are just sort of all these things that I am telling you right now are just convenient you know markers to maybe for to remember, but other than that everything is contained in that dispersion relation that comes out of the quadratic equation that we have seen earlier ok.

(Refer Slide Time: 08:15)

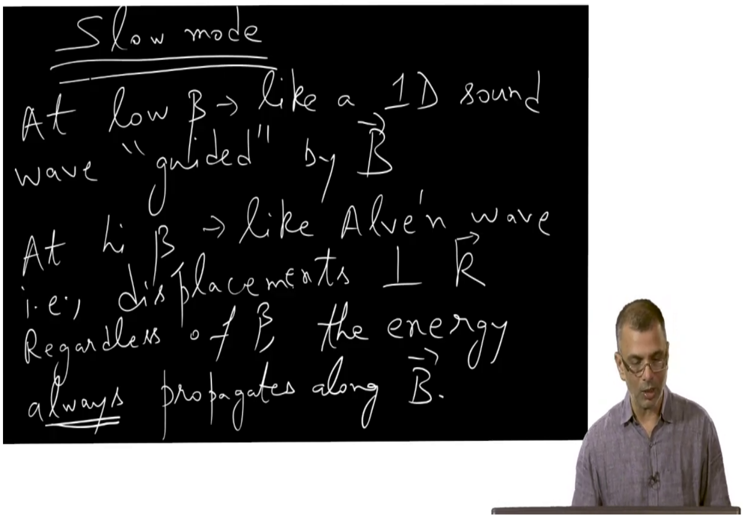


Now, as regards to slow mode, so from now on everything that we will be saying has to do with the slow mode ok. As regards with the slow mode at low beta, it behaves like a sound wave 1D, it is like a 1 D sound wave except it is not isotropic sound wave, so isotropic guided by the field ok.

So, it is very much like a sound wave in many respects in that you know this propagation speed is almost the sound speed and so on so forth, except it is not isotropic its guided by the field ok.

At high beta, in the other limit, at high beta it starts looking like a Alfven wave in that displacements i.e., what I mean by that is that displacements the fluid displacements are perpendicular to the field perpendicular to B ok, perpendicular to the wave vector actually, perpendicular to strictly speaking I really should say a wave vector, perpendicular to K to the wave vector ok.

(Refer Slide Time: 09:53)



And regardless of the beta ok, the energy the group velocity d omega d K, the energy always propagates along the magnetic field propagates along B ok, this is how. So, you notice the big difference between the slow mode and the fast mode with regard to energy propagation.

For fast modes, the energy always propagates isotropically regardless of beta ok; in other words, it has nothing to do with the magnetic field. Whereas, for the slow mode regardless of beta the energy always propagates along the magnetic field ok. So, this completes our short survey of you know the different modes of propagation in a magnetized plasma ok.

So, now you might ask all of this is fine you know we talked about the sound wave, we talked about the Alfven wave, we talked about two kinds of magnetosonic waves. And what about it I mean in practical terms why is it important in astrophysical studies? Well, really one of the reasons this is I can give you a couple of answers to that.

One is many times you know these waves are important it is important to identify the waves ok, because any arbitrary disturbance is really a superposition of waves ok. It is a super position of waves, it is just not just one kind of wave of the other many times there can be a mix of wave. So, it is important to identify the dominant kind of wave that is one thing.

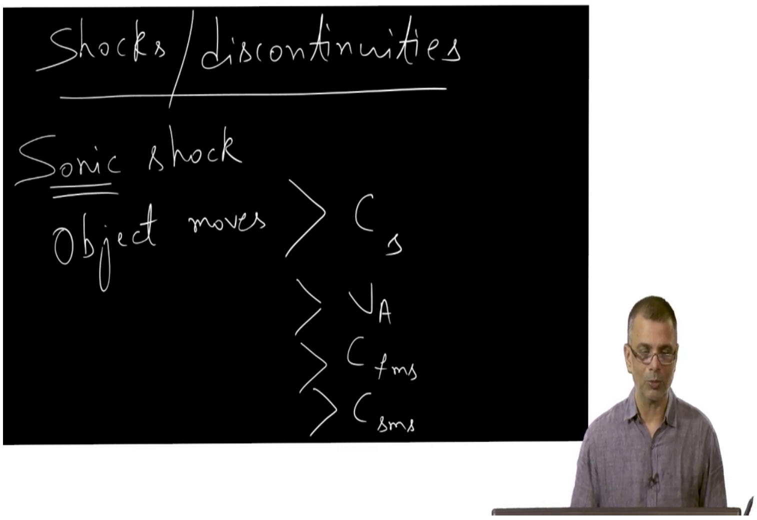
The second thing is waves dissipate although in this case where we were talking about I am ideal MHD, so there is no scope for dissipation, but nonetheless you know in non ideal cases where finite resistivity is present, our finite viscosity is present waves to dissipate ok.

And the dissipation leads to heating of plasma. And this is often a very important question in astrophysics, for instance, the heating of the solar corona of the outer atmosphere of the sun this is afterward like shall we say two decades of research this is still unsolved problem.

And so it is important to know many times you know people resort to the existence of different kinds of waves in order to understand how the solar corona is heated. Well, dissipation apart dissipation mechanisms apart, in the first place you need to identify which kinds of waves are you talking about.

And so which kinds of waves are you holding responsible for the dissipation ok. So, this is one reason we need to understand waves ok. The other reason of course, are shocks as we have said you know shocks are very very important in astrophysics.

(Refer Slide Time: 13:14)



And so shocks slash discontinuities right. So, the question is, what kind of shock are we talking about? Are we talking about for instance a sonic shock? A sonic shock meaning a sonic shock arises from a situation where a piston or a driver or some object exceeds the speed of sound, and that can possibly lead to steepening non-linear steepening and the formation of a shock.

So, what was the characteristic speed the object? The this can happen when an object moves at a speed greater than the sound speed. Similarly, you might you might now that we have identified two or three other kinds of speeds, three other kinds of characteristic speeds.

You might say that in this case r n in you know the C f m s or C s m s. So, all of these are now characteristic speeds this. This refers to the fast magnetosonic speed, and this refers to the slow magnetosonic speed. So, now if an instead of just one kind of shock you have the possibility of all kinds different kinds of shocks ok.

So, again why is this important? Well, it is important because you know if you are talking about most of the time as we have seen as we have discussed in astrophysics, you hold shocks responsible for particle acceleration right. If you are going to be holding it responsible for particle acceleration.

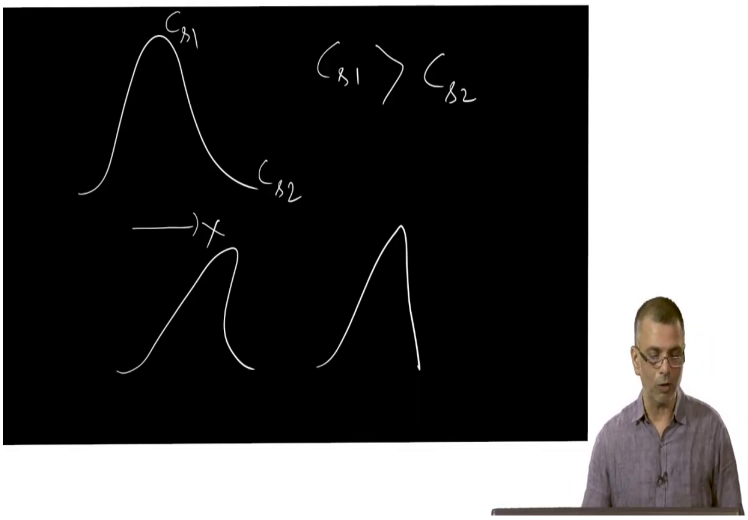
Yyou need to know what kind of shock you are talking about, only then you will know what kind of jump conditions you to expect, and then you can go on from there ok. So, because things depend upon the jump conditions, apart from the intrinsic you know interest in knowing what kind of shock it is right. And now having identified three mode characteristic speeds.

We know that there can be there can be many different kinds of shocks, there can be a sonic shock, there can be an Alfvenic shock, there can be a fast magnetosonic shock, there can be a slow magnetosonic shock. Sometimes, an object can exceed more than one of these speeds, and then the question is what kind of shock will form, or what kind of shock is most likely to form ok.

So, now these are fairly you know involved questions. And we cannot possibly I just wanted to you know motivate it, and we cannot possibly go into the details of all these, but I will try to sketch some of these details some of these issues before we you know go on.

Or at least give you a at least some idea of what to expect from the fact that we now have several different kinds of characteristic speeds ok. So, before that just by way of very quick you know review you know how does the shock form.

(Refer Slide Time: 16:35)



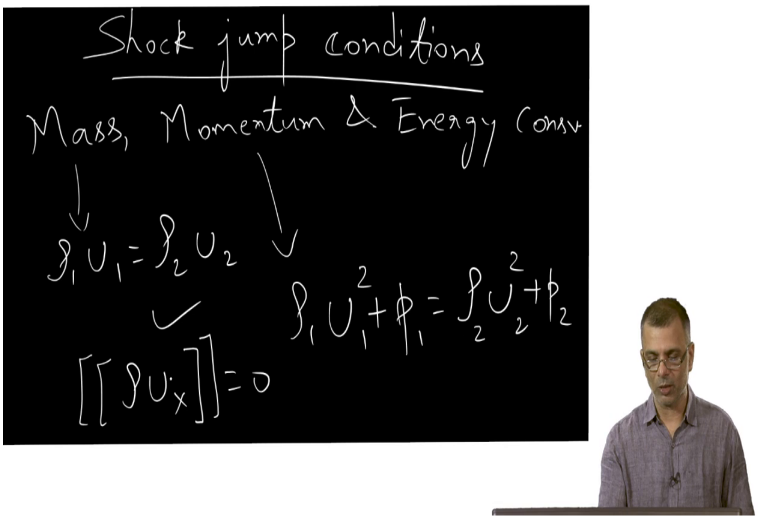
The shock form suppose you have a pressure pulse right. And we are simply talking about hydrodynamics, now we are simply talking about how hydrodynamic shock forms. So, this would be a pressure pulse in X. So, here what happens is simply because the pressure here is larger than the pressure here.

The sound speed here C s 1 is greater than C s 2 right. So, the crust yeah, so the crust starts overtaking the leading edge, so it starts doing this ok. And so eventually there is steepening and like that like that. So, eventually what will happen is when the crust starts over to simply because the sound speed is here is larger than the sound speed here.

Why is that? Because the pressure here is significantly larger than the pressure here, these are not small amplitude waves, these are there is not there is not a small amplitude disturbance rather there is a large amplitude disturbance ok. So, the wave front steepens this, this steepens and a shock may form.

So, this would be the steepening of the wave front and the possible formation of a shock.

(Refer Slide Time: 18:04)



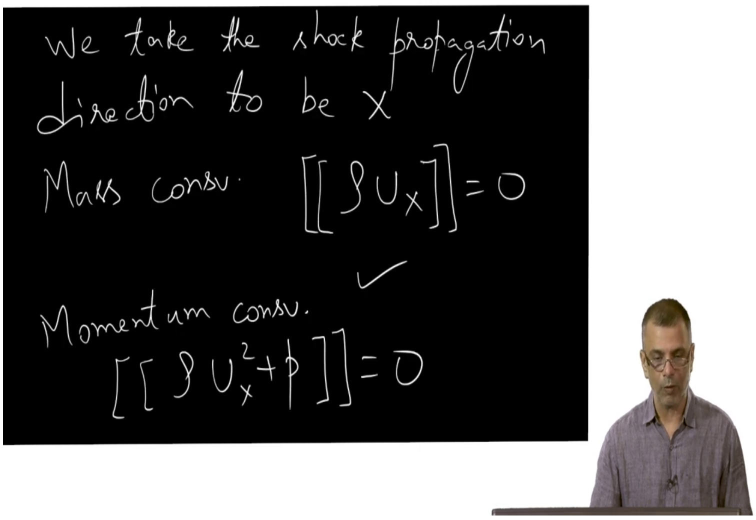
And so this was like a one slide review of when a shock might form. And then what we did for sonic shocks was we evaluate we wrote down the shock jump conditions; in other words, what to expect on either side of the shock right. We used the mass, momentum and energy conservation equations to figure this out right.

To figure out the jump conditions in other words what how much of a jump to expect in various quantities like density, pressure, velocity, and so and so forth on either side of the shock. Now, we have done this earlier, but I thought I would introduce this to you in slightly different language.

For instance, when we employed mass conservation we said rho 1 U 1 is equal to rho 2 U 2. Where you know the 1's represents the upstream condition; 2 represents the downstream conditions so and so forth.

We did this and we you know so this is essentially what we did. And when we conserve momentum, we said rho 1 U 1 square plus P I urge you to go back and look at this is rho 2 U 2 square plus P 2. This is how we did it right.

(Refer Slide Time: 19:52)



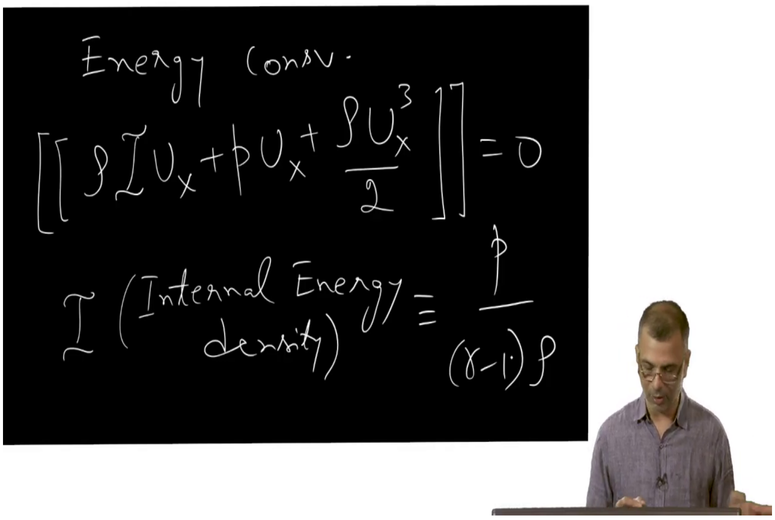
And when it came to energy, so now, another way of writing this another way of writing this very same thing is to say that now first of all we take we take the shock propagation direction to be X, just for simplicity this shock is propagating in the X-direction.

Now, mass conservation can be written as these two this is the just notation ok, this is same as this ok. So, this is what I saw instead of writing this I might simply write which is to say this is simply to say that rho 1 U x 1 minus rho 2 U x 2 is equal to 0 that is what this notation is telling you.

The reason I am saying this is I just wanted to introduce this notation here that is all ok, and nothing different. So, for sonic shocks mass conservation can be written as that. And momentum conservation momentum conservation can be written as rho U x squared plus P equals 0.

Again this is the same as this ok. So, far we are not saying anything new, we are simply reviewing what we have already done ok. We are just you know yeah. So, so there is nothing terribly new in what we are saying here.

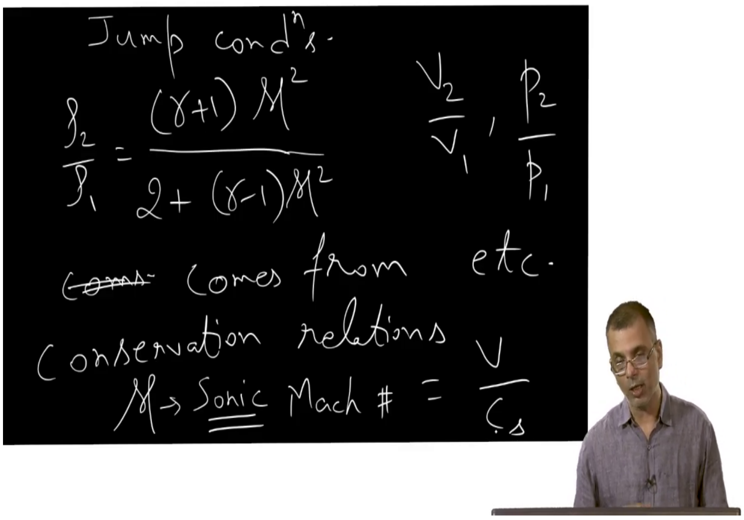
(Refer Slide Time: 21:52)



And for completeness the energy conservation is written as this can be written as rho. And now where we are introducing a new quantity, and this is again this is nothing different from what we have seen earlier, I will explain this in a minute plus it just looks different it is really the same thing.

Where I is what is called the internal energy density, and is equal to P over gamma minus 1 rho that is the definition ok. So, now, we have what we have done so far is simply write down you know the mass, momentum and energy conservation for hydrodynamic shock.

(Refer Slide Time: 23:16)



And from these, from the mass, momentum, energy conservation equation, we can figure out the jump conditions. The jump conditions rho 2 over rho 2 over rho 1 was equal to I will simply write this once, and write one of these expressions and then leave it there. This is simply review I do not want to you know repeat everything now this is just to just to make a familiar with the previous result.

In the same spirit as what we did, we first reviewed you know sound waves before going on to Alfven waves, and fast and slow magnetosonic waves right. So, it is in the same spirit, we are now reviewing you know sonic shocks which we have already done before, before going on to other kinds of shocks ok.

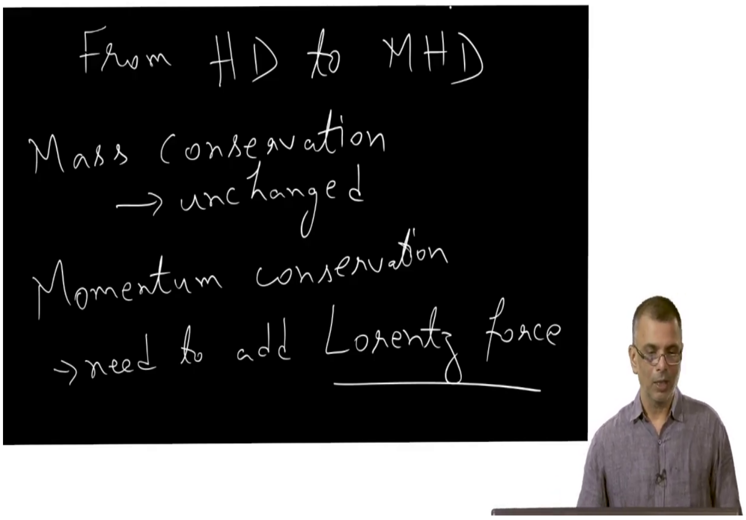
So, now, so these jump conditions this for instance the ratio of the densities is given by this. This comes from this comes from the conservation relations, conservation relations, and which are those the conservation relations are just these are this, and this.

So, just like the density jump conditions, you also have the velocity jump conditions V 2 over V 1 for instance, P the pressure jump conditions and so on so forth ok. So, this was just to sort of give you an idea, and so and M of course, is the sonic Mach number equal to velocity over just a speed of sound ok.

So, now as you can suspect with the other kinds of characteristics speeds which is the Alfven speed, the slow and fast magnetosonic speeds, you will start having different kinds of Mach numbers also not just a sonic Mach number, you will have an Alfven Mach number, you will have a slow magnetosonic Mach number.

You will have a fast magnetosonic Mach number right. So, things start getting a little more complicated, but that is I mean it is it is nothing to you know so the it is the philosophy is still the same ok. So, it is on that kind of philosophy that we focus right here.

(Refer Slide Time: 26:17)



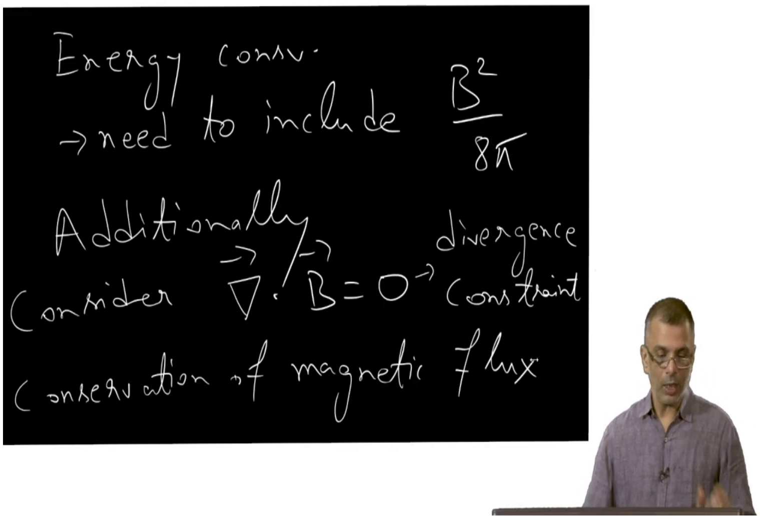
Now, from hydrodynamics from while going from hydrodynamics to magneto hydrodynamics ok, what happens, what changes? Well, as we have said as we have remarked a few times earlier now there are three more characteristic speeds that happen, but as far as conservation relation goes ok.

These conservation relations that we just wrote down what changes? Well, mass conservation is unchanged, thankfully unchanged. It is the same as what it was you know for hydrodynamics even when you do magneto hydrodynamics right. Momentum conservation.

Well, what was new in the momentum conservation equation when we did magneto hydrodynamics? Momentum conservation you remember that in while talking about momentum conservation the one thing that we that was new was you need to add the Lorentz force the J cross B force right.

So, this is the new thing that is the new wrinkle or the new complication if you will when you start doing magneto hydrodynamics right.

(Refer Slide Time: 27:51)



And also so energy conservation well energy conservation, you need to include magnetic energy magnetic energy density ok, so that is one thing. Also additionally, additionally since we are doing since we are dealing with electric and magnetic fields, you need to consider the divergence of B equals 0, you need to consider right.

And consider conservation, so you this is the divergence constraint, and conservation of magnetic flux which comes from the in the induction equation of course, conservation of magnetic flux. So, in addition to mass conservation which is unchanged momentum conservation which is technically the same equation except we need to add Lorentz forces ok.

Again also with energy equation which is pretty much the same except we need to include magnetic energy density. We also have two new constraints when we are dealing with magneto hydrodynamics. One is the divergence constraint which is you know divergence constraint on B.

And we also need to take into account conservation of magnetic flux. So, you have to take these two new things plus the extra bits in the in the old conservation equations into consideration when trying to derive jump conditions in magneto hydrodynamics ok.

So, that is it for the time being. And when we resume, we will start a writing down the conservation conditions for magneto hydrodynamics.

Thank you.