Fluid Dynamics for Astrophysics
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Lecture - 56
Magnetohydrodynamics [MHD]: Waves in MHD – Alfven waves and magnetosonic waves

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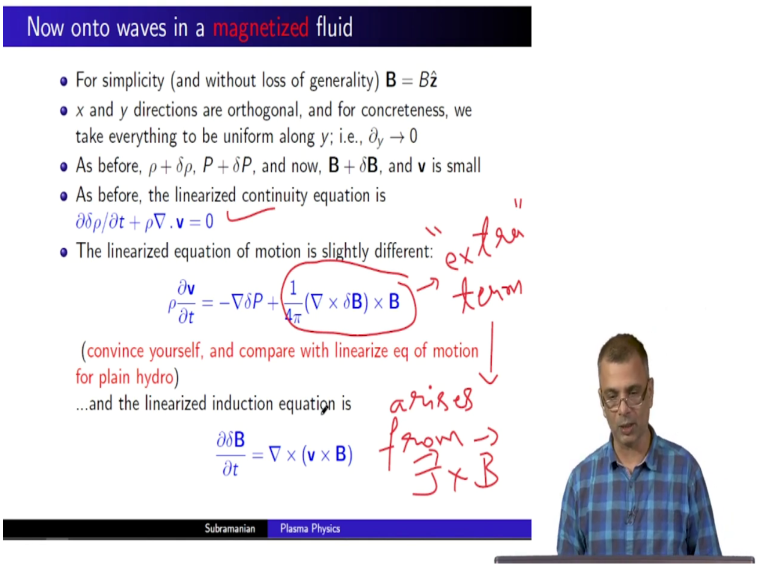

So, we are back to talking about waves in a magnetized fluid. We have already looked at sound waves which are waves in an unmagnified fluid and now we looked at the slide yesterday last time we met. And as we remarked the continuity equation is exactly the same, no difference at all. This is the continuity equation and this in particular is a linearized continuity equation, it is no different from how it was earlier.

The linearized equation of motion which is the momentum equation that contains this extra part. This is the extra term extra with respect to an unmagnetized situation that is all, right and it arises from the Lorentz force term which is J cross B right, but this is not J cross B itself.

This is something that needs to be kept in mind. This is a linearized version of J cross B and J is simply curl of B right. So, so that is the extra term and so, the linearized equation of motion, neglecting viscosity of course, is that and the linearized induction equation is this ok.

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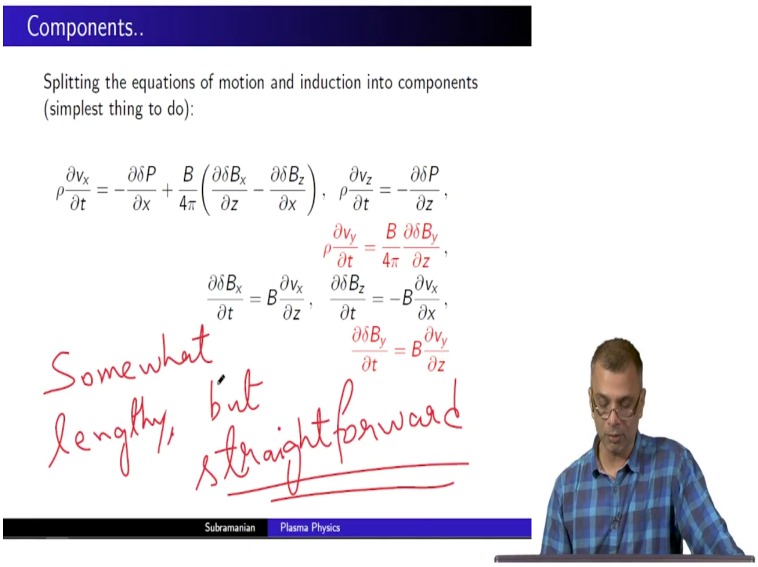

So, now before going on what we did was we said it is simple to split up split up these vector equations in components x, y and z right. Recognizing that the magnetic field is only in the z direction ok. So, the magnetic field is just in the z direction and all we are writing now all we are doing now is writing down the equations in blue this, this and this in components that is all.

So, it is somewhat lengthy, but straightforward really and the first thing to realize now is that you know these two equations in red right. So, that was the comment that I made last time. It is somewhat lengthy, but straight forward. It is really no big issue to write down these three equations in component form. The remarkable fact right now is we will consider magneto sonic waves in due course.

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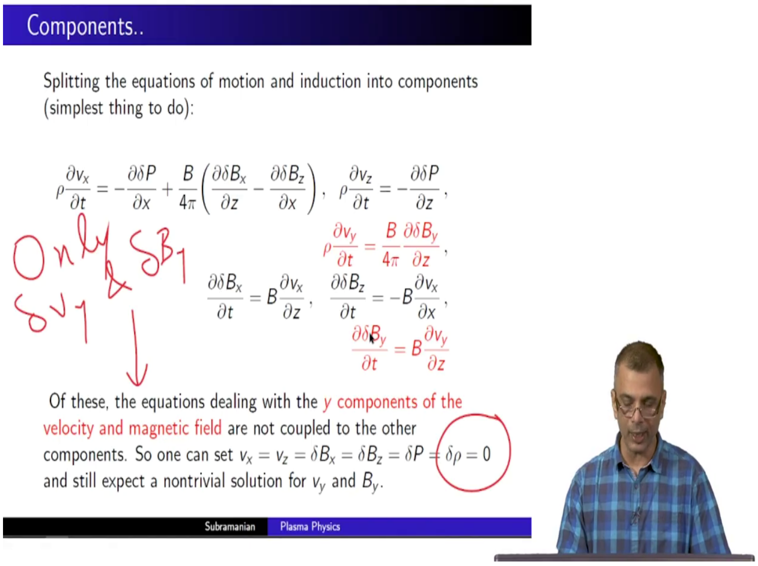

But at the moment the remarkable thing to recognize is that the y components you know of these two equations this is a this arises from the induction equation and that arises from the continuity equation that y components of the velocity and magnetic field are not coupled to any of the other components.

You see you have v y here and you have delta B y here; you have you have delta B y here and v y here. They do not talk to there is no appearance of delta B z or anything or for that matter v z in these two equations. So, that is what a one means by saying that the y components of the velocity and magnetic fields are not coupled to the other components.

Another way of thinking about this is to say that you can simply set the x and z components of the velocity as well as the magnetic field you know perturbations and for that matter even the pressure perturbation and the density perturbation equal to 0. And still you can expect a nontrivial solution for v y and B y ok.

So, this is also pointing to the fact that here we are concerned only with only delta v y and delta B y that is it.

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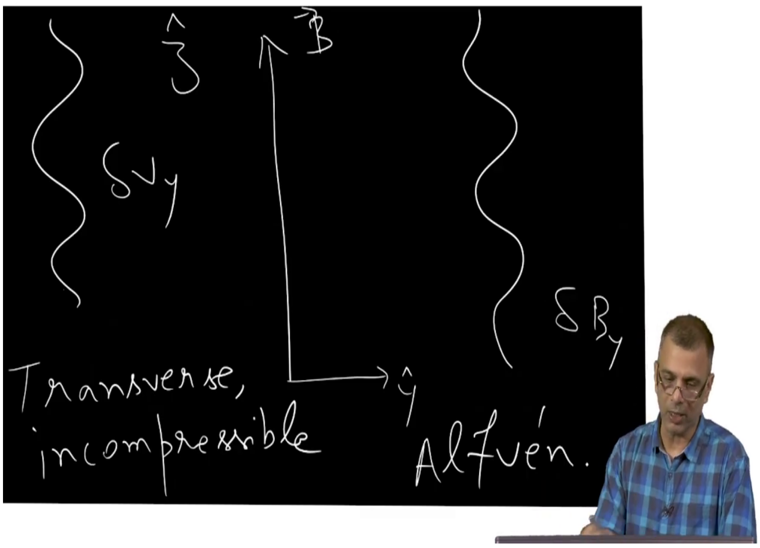

In other words you have you have a magnetic field pointing in the z direction right z hat and so, so this would be there right and suppose this was the this was the y direction ok. So, we are considering a perturbation of the magnetic field that looks like this right.

So, in other words you only have, so, this is a y direction right. So, so the magnetic field is perturbed in the y direction you see and the velocity also the velocity is also perturbed in the y direction. So, this is the situation that we are considering ok.

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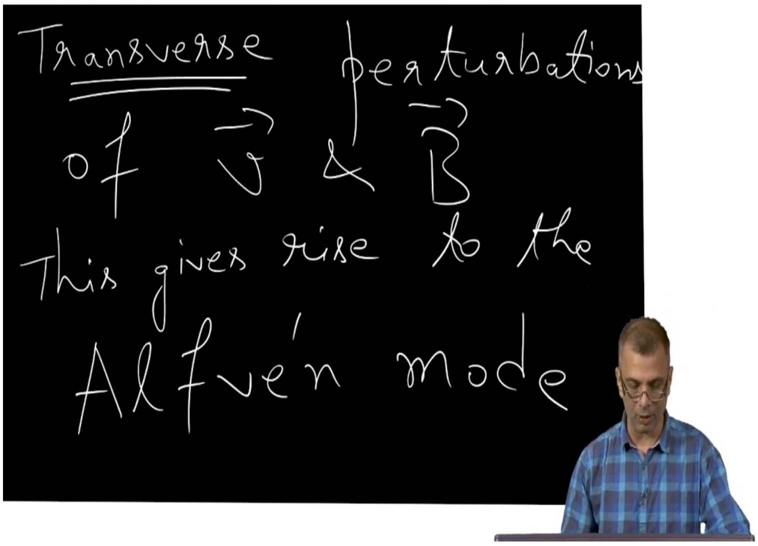

In other words transverse perturbations; we are talking about perturbations of v and B that is what one means by this right. Transverse to what? Well, transverse to the to the uniform magnetic field direction ok. So, that is the main thing I want to I want to emphasise here right and so, we are considering a mode that arises only from transverse perturbations of v and B.

This gives rise to the important to the Alfven mode. This is a very important mode of wave propagation and let us now go ahead and see what the properties of this of Alfven mode are right.

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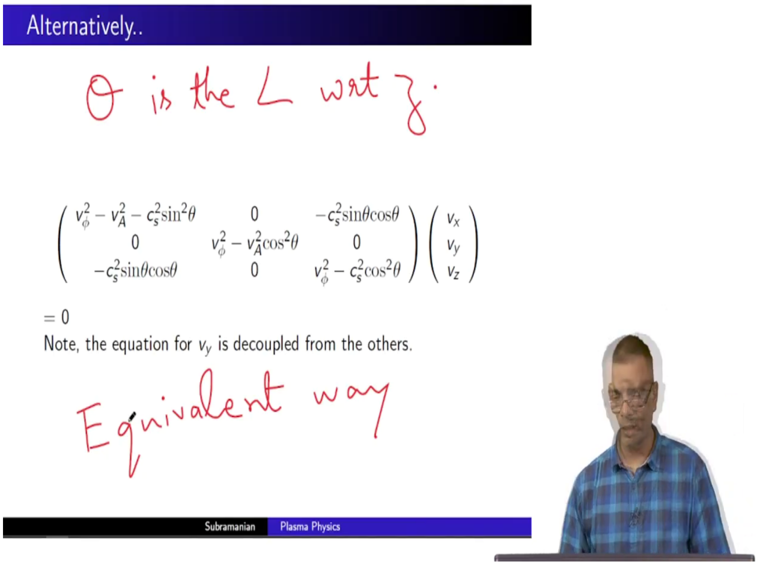

Alternatively, I would say never mind this slide. TThis is an exact thing as this ok written in a slightly different form. I have written in terms of theta which is the angle with respect to the z axis and we have already introduce the Alfven velocity and the sound speed is exactly the same ok. So, you have this, this matrix multiplying the velocity vector and that is equal to 0.

And so, in order for a nontrivial solution to exist the determinant of this matrix needs to be 0. So, it is an equivalent wave of representing it. We will not I just wanted to show it you, but we will not concern ourselves with this wave of representing it anymore.

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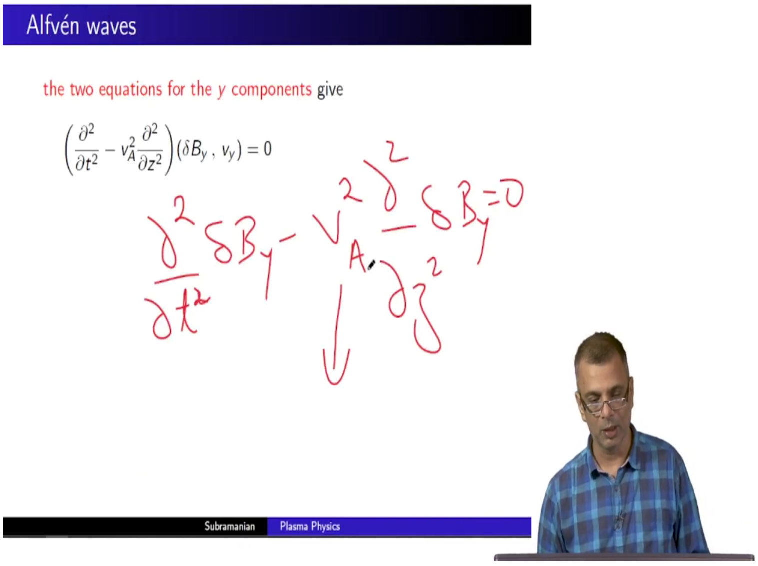

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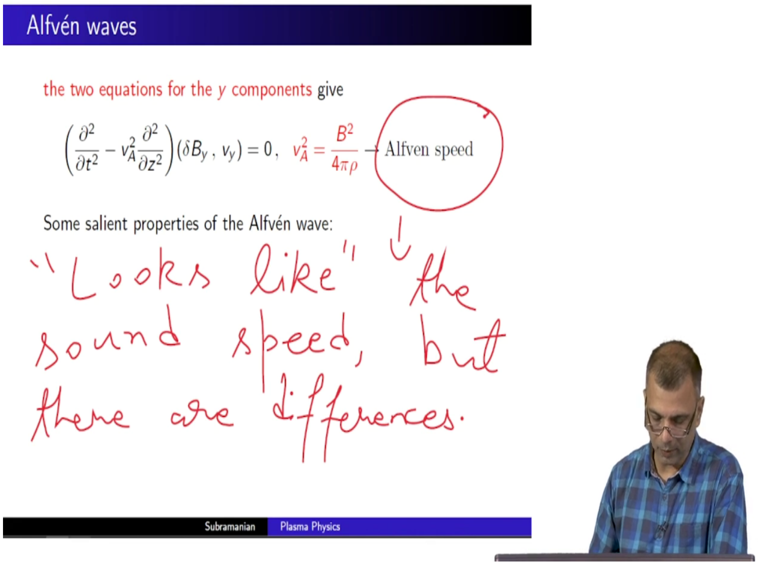

So, the thing is when you ah look at these two equations right these are two coupled first order equations and you can you know you can parlay them into one coupled second order equation exclusively in delta v y or exclusively in delta P y. And that is what this is.

So, you have an equation which looks like d square dt square of delta B y minus this quantity v A squared times d squared d z square of delta B y equals 0 or the very same equation with delta B y replace by v y that is it ok, where this all important ah thing v A squared this is the Alfven speed ok. It looks like the sound speed, but there are important differences. And let us see what those differences are right.

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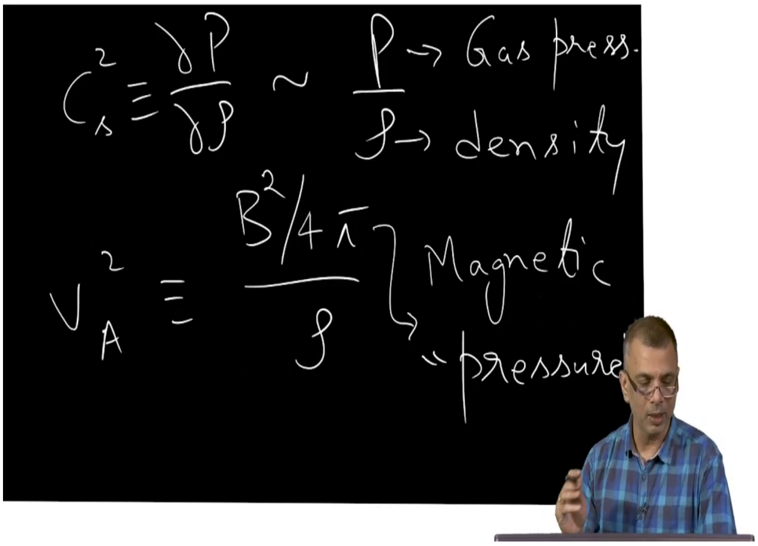

So, you see the sound speed if you remember was written as you know C s squared was dP d rho which we sort of carelessly can write it as P over rho ok, where this was a gas pressure and this is the density gas density. Whereas the Alfven speed which we just define is given by B squared over 8 pi.

I beg a pardon I wonder if its B squared over 8 pi B yeah 4 pi sorry. So, its B squared over 4 pi divided by rho. So, I say this and that look very similar. So, the rho appearing in the denominator is the same and this is the magnetic pressure. In what sense is the magnetic pressure?

If you remember we wrote down the magnetic stress tensor right and this would be the diagonal term in the magnetic stress sensor ok. And you know that in any kind of pressure the diagonal term is interpreted as the scalar pressure ok. So, in that sense this can be interpreted as a magnetic pressure. So, this and that the expressions for the Alfven speed and the sound speed look quite identical.

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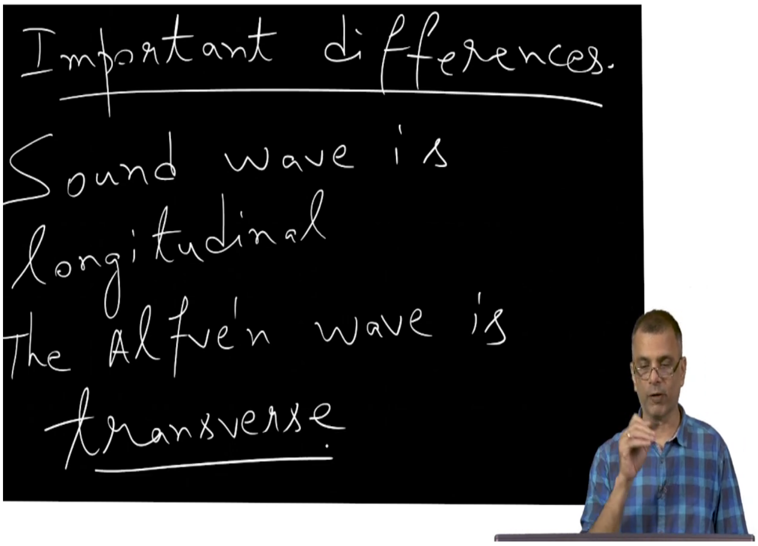

Except there are differences there are important differences important differences. The sound wave is longitudinal, right. In other words the k that the propagation vector of the wave is a long pressure and the sound wave essentially let me finish this.

In other words the density and pressure perturbations are along the direction of propagation ok, whereas, here you see and Alfven wave by definition that is the direction of propagation along. So, this wave you know it travels along the z direction along the direction of the a unperturbed magnetic field, but the perturbations are transverse. You see the perturbations are in the y direction.

So, the Alfven wave, the Alfven wave is transverse. So, this is one very important difference between the sound wave and Alfven wave. Although the expressions for the sound speed and the Alfven speed look quite similar with simply with the magnetic pressure replacing the gas pressure ok, but the nature of the waves are quite different. So, this is something that I wanted to sort for emphasise and is vary the Alfven wave.

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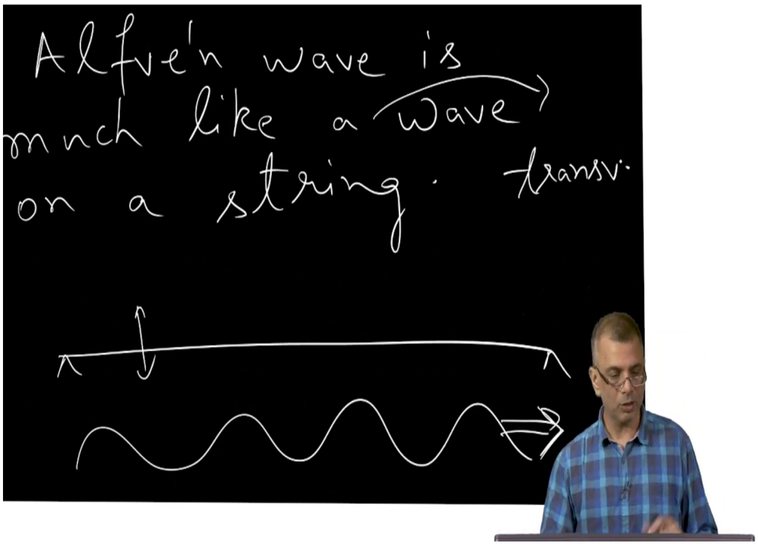

ah Alfven wave is much like wave on a string right that is exactly what you know. Say you have a string like this and you know I fasten at both ends and so, it has a certain tension a on it and you are toying it, right. You give you know that kind of a perturbation to it and you have you know as a result you have waves propagating like this.

Those would be waves and a string and that looks exactly like the Alfven wave. You see the magnetic field is like a string in the z direction and is as if the magnetic field which is an elastic kind of a thing, the magnetic field line which is like a rubber band its being twanged and transverse perturbations on this rubber band or propagating along the magnetic field direction right.

So, in that sense the Alfven wave is much like a transverse wave I should say transverse wave on a string ok. So, this is something important to keep in mind as we go ahead.

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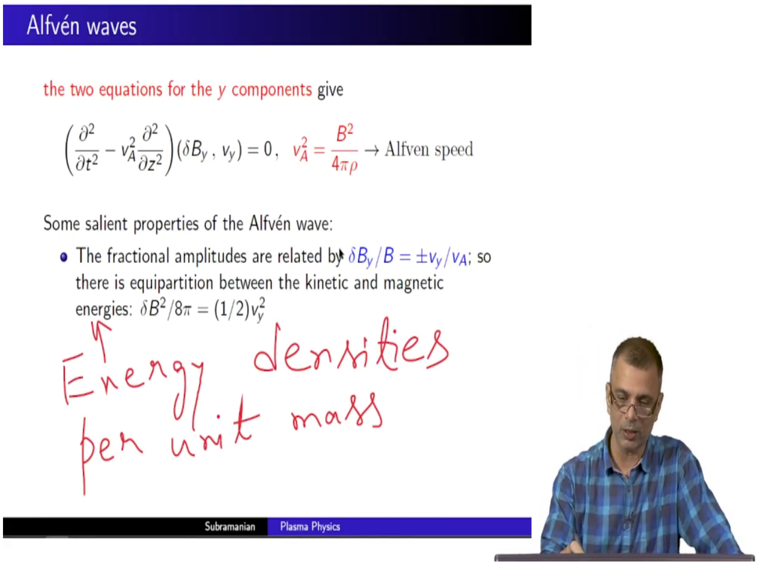

The other things some other salient properties so, the Alfven waves that the fractional amplitudes are similar in otherwise delta B delta B y over B is the same as v y. I simply write v y because this really no need to write that v y there was no average v to start with.

So, the small v y is indeed a delta quantity ok and an immediate consequence of this is that there is equipartition between the kinetic and magnetic energies. In other words the energy in the magnetic field perturbations is delta B squared over 8 pi right and energy density. Actually I should say energy density energy densities to be precise. I should not I mean I have been a little it is not so much energies, but it really should be energy densities.

So, the this will bit of a lose way of saying it. So, the delta B squared over 8 pi is the energy density in a magnetic field perturbations and half v squared energy densities again per unit gram per unit mass. So, this as you can recognize is the kinetic energy kinetic energy you know per unit mass right.

So, so this would be energy density specific energy densities and there is equipartition ok. The magnetic energy density and the kinetic energy density are the same and this is a simple consequence of this fact ok.

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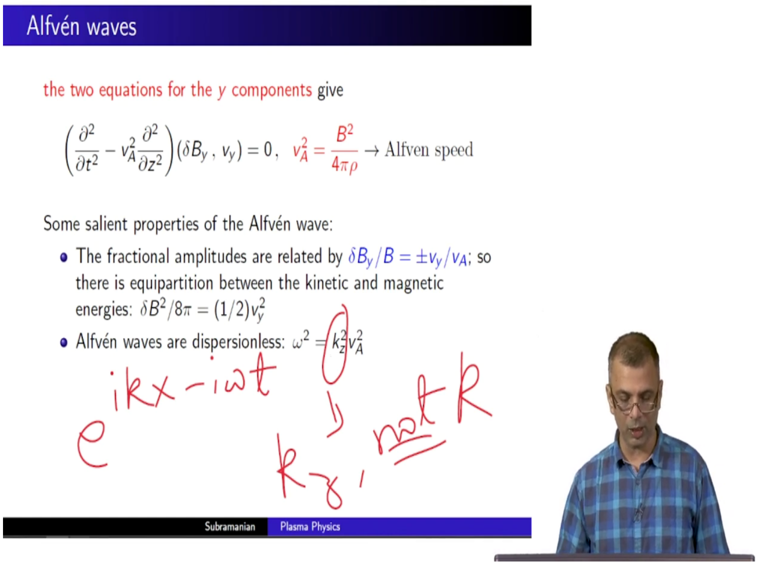

Alfven waves are also dispersion less. If you assume e raise to i k x minus i omega t this kind of a dependence a for the delta B y as well as v y. What happens is this d square over dt square becomes minus omega square right and sorry omega squared just becomes omega squared and the d square of d z square becomes you know minus k square.

And so, just from this you can you can figure this out ok. This is the dispersion relation and as with again this is similar to the sound mode. As with the sound wave rather the sound waves are also dispersion less and so, are the Alfven waves. The omega over k z or the d omega over d k z are both not functions of omega itself. They simply involve the fame velocity.

However, please remember this is k z not k, not k itself. This is k only along the z direction only along the magnetic field direction. So, please this is one important difference between sound wave and the Alfven wave ok.

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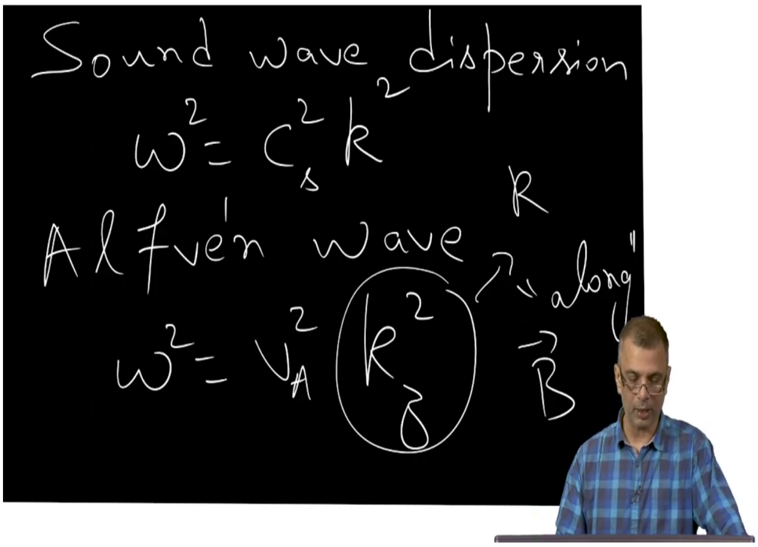

So, so the dispersion relation for a sound wave is omega square equals C sub s square k square like that. Alfven wave is omega square equal v a A k z square, very important ok. First of all the whole point is that the Alfven wave is like a transverse wave on a string and the string is the magnetic field.

So, the appearance of a magnetic field breaks a symmetry. Here that the k is the same in all directions you see. There is nothing to distinguish one direction from the other. Here the presence of a magnetic field immediately sets a preferred direction ok. So, it breaks symmetry and so, the k in one direction is not the same as a k in other directions ok.

Specifically, the k along the magnetic field is different from the k perpendicular to the magnetic field and here the we are referring to I know k z, in other words the k along the magnetic field ok. So, this is very important to keep in mind. So, k z, not k right.

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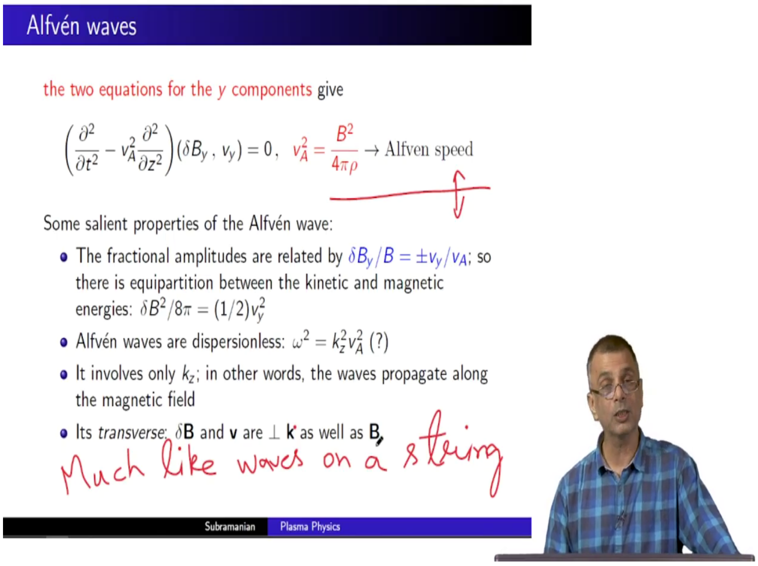

It involves only k sub z yeah. So, so that is what that is clearly the case. In other words the waves propagate along the magnet field like we have said and its transverse the delta B and v are perpendicular to k as well as to B as we have already remarked they are much like waves on a string ok.

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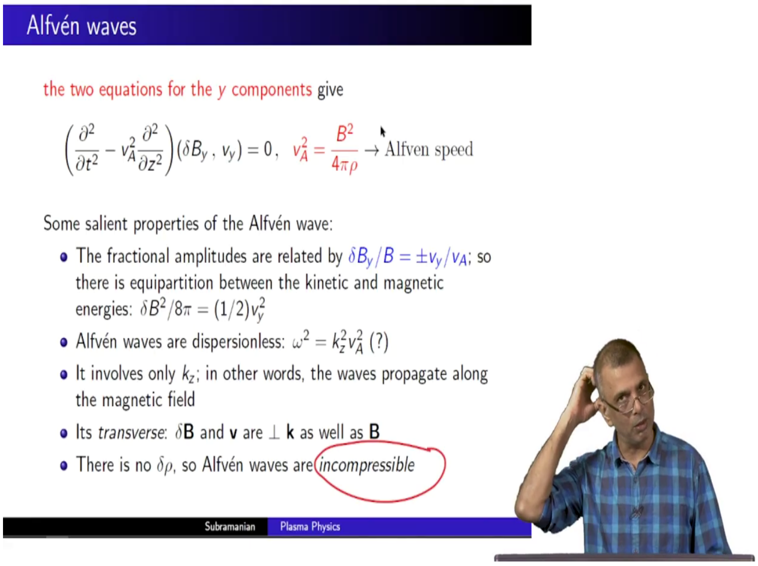

The other thing is that the other important differences that there is no appearance of delta rho. There are if you remember we could hear you see we had said we could get nontrivial solutions to these two equations with delta rho actually being equal to 0. No density perturbations that is one way of looking at it the other way of looking at it is that nowhere in this equation thus delta rho make an appearance right.

So, therefore, Alfven waves do not involve density perturbation and all. therefore, Alfven waves are incompressible; very very important point and this is a important point of distinction with the sound waves. Sound waves are compressible that is a whole point of sound waves ok. Sound waves involve compressions in density as well as delta rho and delta P.

Here neither delta rho nor delta P makes an appearance. So, Alfven waves are incompressible ok. So, although the expressions for the Alfven speed and the sound speed looks similar there are several points of difference between the sound speed and the Alfven speed and these are important to note right.

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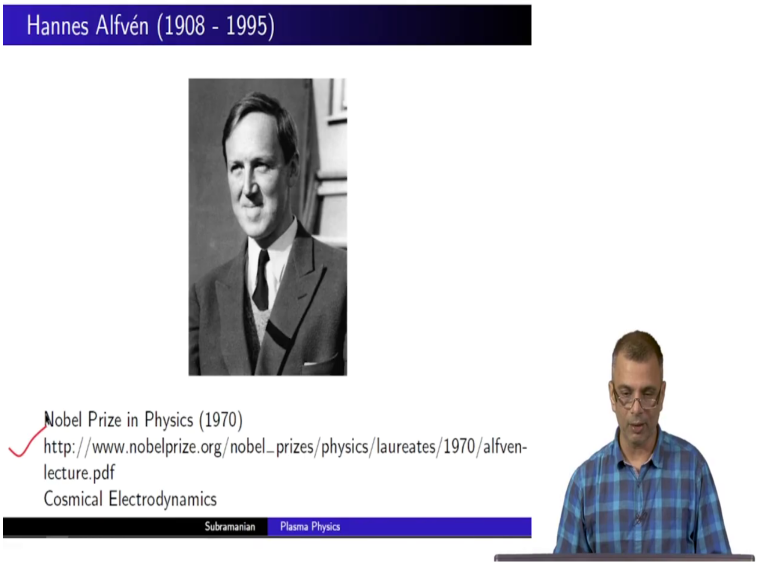

So, let us go ahead. This is a picture of Hannes Alfven. He won the Nobel Prize in Physics in 1970 for many things, but you know and I am urge you to you know look up this website very interesting website which contains his Nobel Prize lectures, where the lecture he delivered while accepting the Nobel Prize and in its called Cosmical Electrodynamic. It is its very instructive. I would strongly urge you to go through it at least glance through it once right ok.

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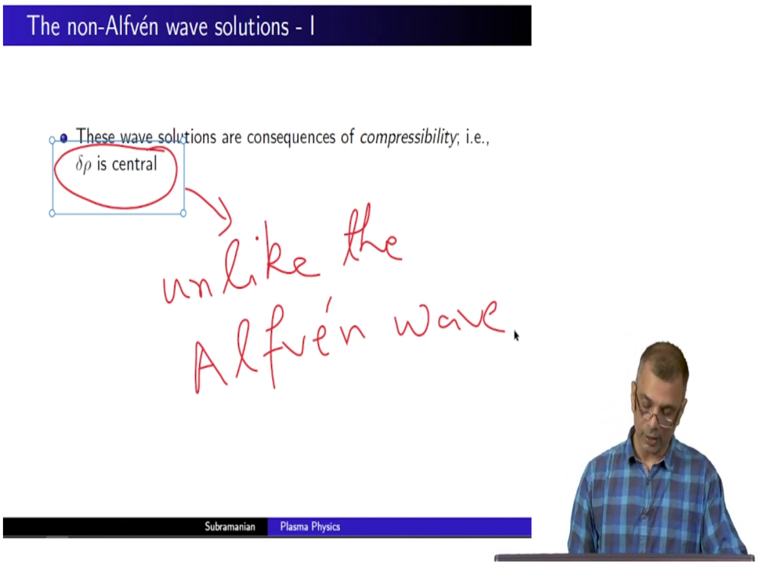

So, now let us now move to the non Alfven wave solutions the remember we split up the three equations in components and we marked only the ones which involve only the y components we mark them in red. Now, how about the other solutions right? So, we marked the once which involve only the y components we mark them in red and we found that they represent Alfven waves.

They represent a particular kind of wave which is transverse which in other words the perturbations or transverse of velocity and the magnetic field perturbations or transverse like this right and involved involves the they are transverse and involve no delta P and delta rho. So, they are incompressible right and so, these are the this is just to you know a these are Alfven Alfevnic perturbations ok.

Now, turns other the other solutions are consequences of compressibility. In other words unlike the Alfven solution the appearance of delta P is very important unlike the Alfven wave. The Alfven wave is incompressible there is no need to invoke a delta rho and a delta P. Here the delta rho is central as we for these particular kinds of waves.

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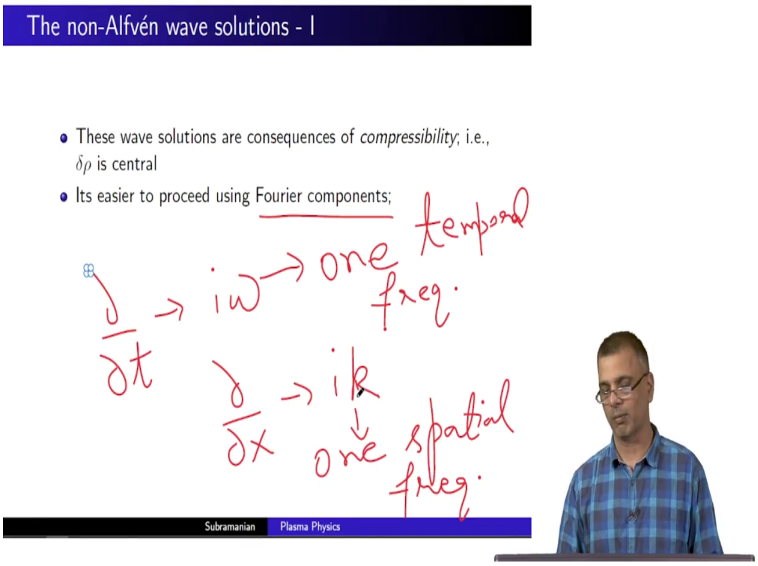

It is easier to proceed using Fourier components. In other words wherever you see d over dt you write it as i omega, wherever you see d over dx you write it as a i i k ok. And there can be a minus i omega or there can be a minus i k depending upon exactly how you whether you are writing e raise to i omega t minus k x or e raise to i omega plus k x. So, it does not matter.

So, this is essentially what I mean by this ok. So, instead of the differential equations you will get algebraic equations and the only the only slight disadvantage there is that you will be dealing only with one temporal frequency and one spatial frequency or spatial wave number ok.

Only with one value of omega and one value of k at a time when you are doing Fourier components, but that is ok the equation is linear. So, the behavior for other frequencies; spatial and temporal can be then superposed linearly superpose. So, you have the solution for one, the solution for the others look exactly the same, the functional dependencies are exactly the same; that is the whole advantage of doing Fourier language right.

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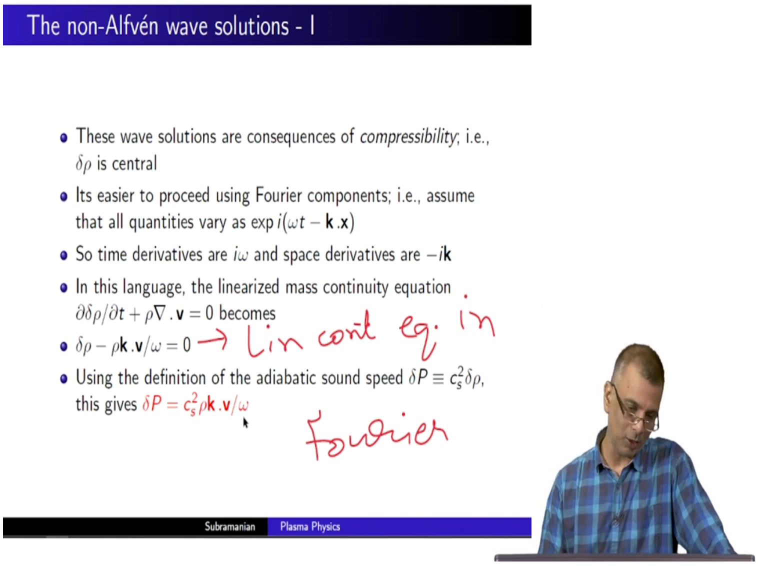

So, I i assume that it is an exponential i omega t minus k dot x right. So, therefore, all time derivatives of i omega and all space derivatives are minus i k right. So, in this language the linearized mass continuity equation which is this becomes that right. So, it is quite simple.

The d over dt became an i omega and the divergence of v became i k times v that is its. So, that is how you get this the this is the same thing as the mass continuity equation. So, this is really the mass continuity linearized continuity equation in Fourier language that is what this is.

Instead of rho we can we can you know rather instead of delta rho if we prefer to write things in terms of delta P then you have to invoke the sound speed. So, so, this is the same as this except you have we have now invoke the sound speed to eliminate the delta rho in favour of delta P. So, this again is the same equation linearized continuity equation in Fourier language ok.

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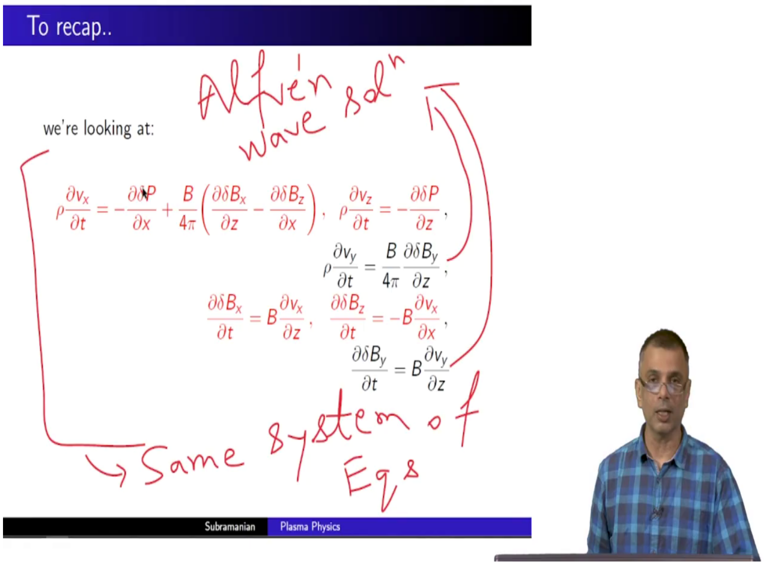

So, just to recap we are looking at the same system of equations, except we had originally looked at this and this. These gave the Alfven wave solution right. This equation this equation they together they gave the Alfven wave solution. Now, what we are going to do is we are going to look at the remaining equations which is this, this, this and this ok.

And what is more? We are not going to bother about; just for convenience rather, we will not look at the differential equations themselves. We will look at the Fourier analyzed versions of the differential equations where the d over dt becomes an i omega t and the d over dx becomes an i k ok.

So, and other important thing to remember is that you have the appearance of delta Ps here. You see delta Ps or delta rhos for that manner you have a delta P here you have delta P here. So, therefore, clearly these modes do involve pressure and density perturbations and these are central to the understanding of these modes and therefore, these kinds of modes are necessarily compressible ok.

And these modes are called magneto sonic mode. Sonic because they are like sound waves in that they involve compressibility and magneto sonic in that there sound waves modified by the presence of a magnetic field. And so, these magneto sonic waves are split into two varieties. One is a fast magneto sonic wave and one is a slow magneto sonic wave and we will discover soon enough that they have very interesting properties in some particular situations depending upon the angle of propagation of the wave with respect to the large scale magnetic field.

The magnetic field again is always along the z direction ok that can always be assumed without loss of generality depending upon the direction of propagation of these magneto sonic waves with respect to the magnetic field of course. Sometimes they start looking like the sound waves sometimes they do not. Sometimes they look very much like the Alfven wave. So, we will analyze these solutions when we meet next. So, for the time being there is it.

Thank you.