Fluid Dynamics for Astrophysics
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Lecture - 54
Magnetohydrodynamics [MHD]: Dynamo theory

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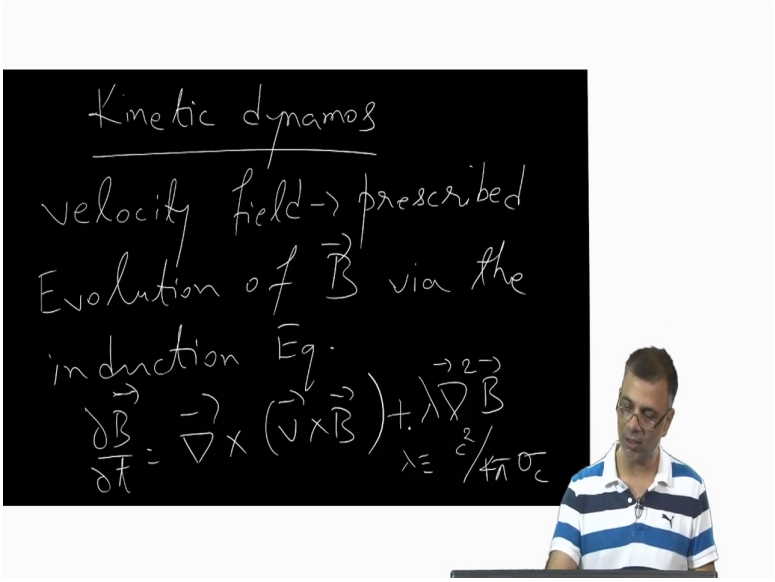

So, let us resume a discussion of kinetic dynamos. Kinetic dynamos as we said earlier or once, where the velocity field is prescribed is prescribed is given to us and through some other means ok and we look at the evolution of the magnetic field, evolution of B via the induction equation ok.

Induction equation which we have seen many-many times dB dt is equal to the curl of V cross B right plus possibly a resistive term and we will talk about that in a minute right. So, now

the two things about any dynamo you should show that the presence of a small seed magnetic field renders the system of equation, equations unstable so that the seed fields grow.

Because we are interested in knowing the origin of large scale relatively stable magnetic fields. That is really the problem of a dynamo ok. So, the two things in a dynamo number 1 is to show that the presence of a small seeds seed magnetic field renders you know the system of equations unstable making the seed field grow into a large scale magnetic field that is point number 1.

Point number 2 is to show that the fields having grown to appropriate values can sustain against that decay, because you know the it is not as if there is always resistive term here. There is no matter what you do? There is always you know a resistive term, this kind of a resistive term with where the lambda is related to the conductivity all this is in CGS mind you. So, see ok where this is the conduct is the conductivity.

So, this resistive term might well be small, but it is there ok. And remember so, in the face of resistive decay there dynamo has to somehow keep regenerating the fields so that the fields do not just die away ok and it turns out that this is a nontrivial problem ok. What we the second term this term, this term, it contributes to the decay of magnetic flux. And so, it turns out that the decay problem is not such a simple problem, it is not such a trivial problem.

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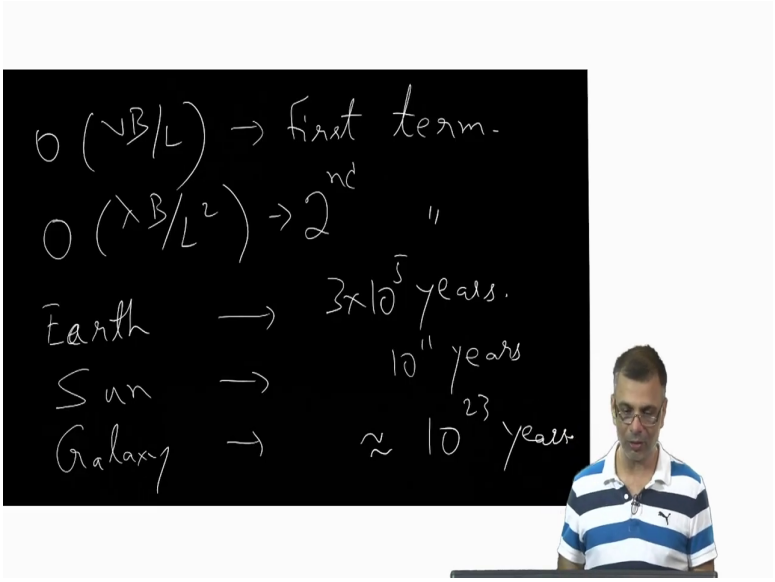

The first term in this equation is order is of the order of you know VB over L, this is the first term. And the second term is of order lambda B over L square, 2nd term and as we know the ratio of these two terms is the Reynolds number right. Now, turns out that if you calculate the time scale of decay ok, the time scale of decay by considering only this term and this term ok.

With this term not being there, this gives you the time scale of decay. Just look at this and this ok, you can figure that out only from the length scale and the conductivity. You can figure that out just from this ok. So, let us look at the time scale of decay for certain well known objects for the earth it turns out that the decay time scale is something like 3 times 10 raised to 5 years, the resistive decay time scale given the values of the conductivity and the length scales and everything.

The time scale for resistive decay for the earth is something like 3 times 10 raised to 5 years, for the sun it is something like 10 raised to 11 years if we put in the appropriate numbers ok and for the galaxy, for our milky way galaxy it is a something like a 10 raised to 23 years, really, really huge.

Now, this is really large so as such there really should not be a problem, but it turns out at least for the sun ok. Sun has been around for you know considerably more than this and so, you do need a dynamo. So, as such the field would have decayed away ok before this.

And so, but you do need a dynamo to regenerate the field and at least for the sun it becomes a bit of an issue, because you know this we have seen that it there is observation of evidence that the field is not only regenerated, but it also flips every 11 years ok.

So, these are challenges a kinetic dynamo theory should address rather the version of the kinetic dynamo that we will be adopting, addresses mostly the second question which is how does the field you know retained its identity in the face of resistive decay? ok.

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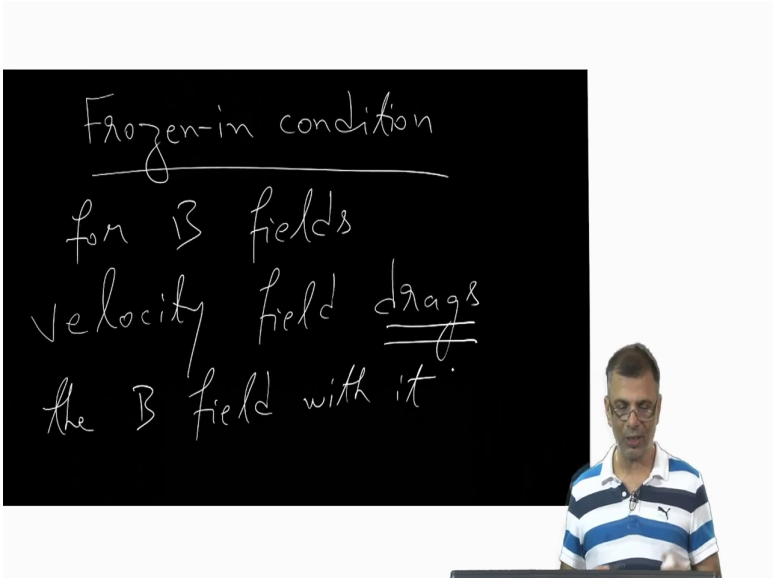

Now, the main thing to remember is the frozen in condition, which we discussed at length a little while ago, the frozen in condition for B fields. In other words, the fact that whatever the velocity field does the B field obeys ok so, this is the one of the main things to remember. In other words, the velocity field drags the B field with it, this is the sense of the frozen in condition ok.

So, one tries to see what follows we will discuss the magnetic field of the sun a turns out we that the we have a little more of a basis for understanding the basis, how the magnetic field on the sun is generated then for the earth itself simply, because we have more regular observations.

So, it is a bit ironic, but it is true. Not to imply that you know everything is known about how the solar dynamo works. No, no not all there are big gaps in our understanding, but one thing

I would like to make clear. There is no see, the point is the solar plasma through plenty of observations, you know things like the temperature, the conductivity and the time scales over which things vary in the solar plasma.

You know these things very-very well ok and especially, with regard to the timescales in other words, we are mostly interested in low frequency phenomenon ok and so, there is very solid ground to believe that really whatever, whatever phenomenon controls a solar magnetic field is governed by the magneto hydrodynamic equations ok.

In other words, the basic assumptions that go into magneto hydrodynamics are well satisfied for anything any mechanism that is supposed to be generating the you know magnetic field on the sun and if that is so, we know that one of the central corner stone's magneto hydrodynamics is that the only current there is, is a displacement current ok in other words B, in other words j the current density is simply the curl of B ok.

There is no other physical current being driven by batteries or anything, because you know at these scales the plasma is essentially neutral ok. There is no reason to believe that this neutrality is destroyed ok and even in the solar wind, even out in the solar wind outside of the solar atmosphere where near the earth we have detailed observations of the solar wind plasma. And there is really no reason to believe there is that the large can neutrality is in any way destroyed.

So, there is no net current anywhere. And so, it is if one sort of thinks that you know the there is some kind of a net current that gives rise to magnetic field in from Ampere's law that is not true for simply, because there is no scope for a net current ok. So, one has to look for alternative ways of how to generate the magnetic field.

It simply, certainly not via you know any kind of a net current that is simply not present in MHD and the conditions appropriate for MHD are amply satisfied both in the solar interior and the atmosphere and further out in the solar wind ok. So, which is why we have to resort to slightly more complicated the pictures like the kinetic dynamo which is successful and

explaining I would say 60-70 percent of them of the solar magnetic field puzzle, but not all ok.

So, let us see how much we understand ok. So, the frozen in condition essentially says that the velocity field drags the B field with it and so let us try to see given this what components of the magnetic field can be generated and how the overall magnetic field is sustained ok. So, right now as it happens there is something called you know an anti-dynamo theorem and I am not going to you know elaborate and what this is you can Google it.

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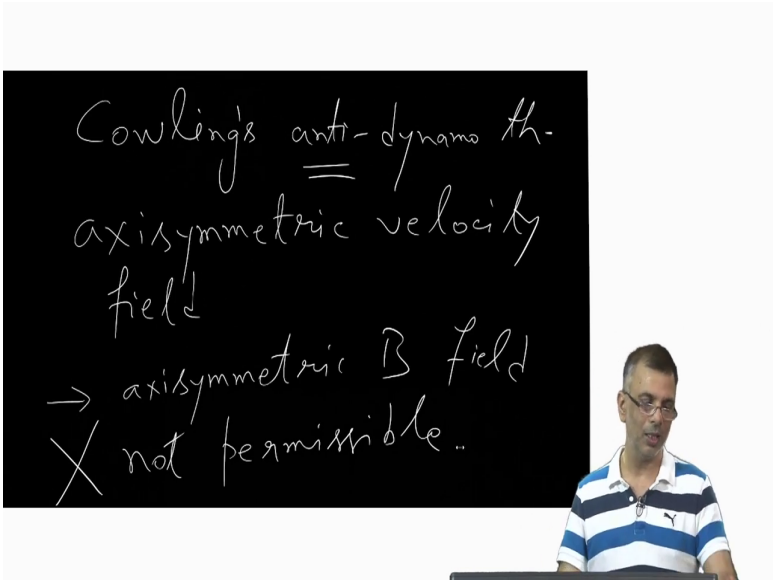

What it says it is called the Cowling's anti-dynamo theorem which says that an axisymmetric field, an axisymmetric velocity field generating and an axisymmetric magnetic field is not permissible. Hence, the name anti-dynamo ok and I am not going to go into exactly what the Cowlings anti-dynamo theorem is. So, so, this would be a very simple situation where you

come up with, this would be the simplest of explanations unfortunately, the simplest of explanations is not permissible, what can one do ok.

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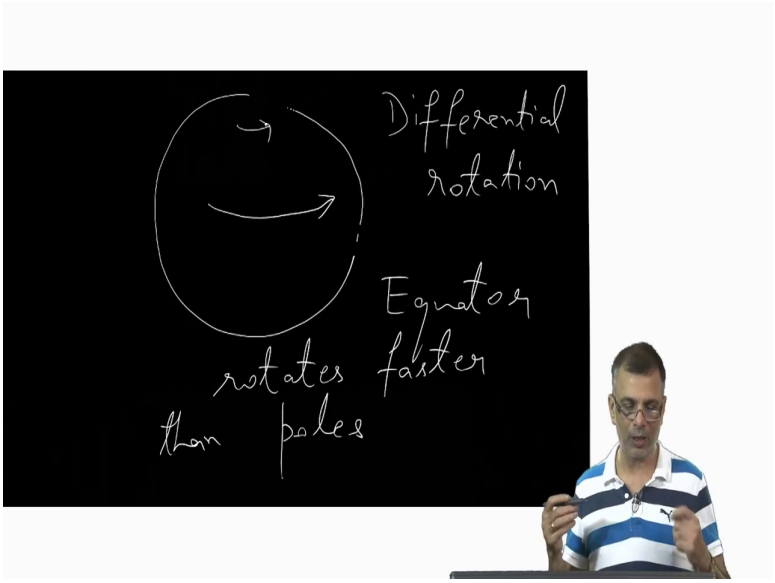

So, let us look at kind of a pictorial idea of how solar dynamo might possibly work. So, let us draw a cartoon of the sun like so sorry, this is terrible yeah, anyhow I mean so something like this ok and let me erase this, I do not know why this is not working right ok.

Now, so, that would be the surface of the sun one of the main things about solar observations that its mean it that it is been observed from surface observations now what we are talking about is magnetic field generation in the interior of the sun not on the surface. Let us keep that firmly in mind, but none the less we have to make do with only surface observations, but from surface observations it is now, it is spanning on nearly 100 years. It is evident that the equator rotates faster than the poles ok.

So, this is called differential rotation ok. The equator and you might wonder rotates faster than the poles like so. So, this would be you know a sketch of the velocity of the five directed velocity vector at the equator and that would be a sketch of the five directed velocity field near the pole. So, this is longer than that ok. So, this is one of the main observational pieces. Now, what does this do for us? ok. What this does for us is the following.

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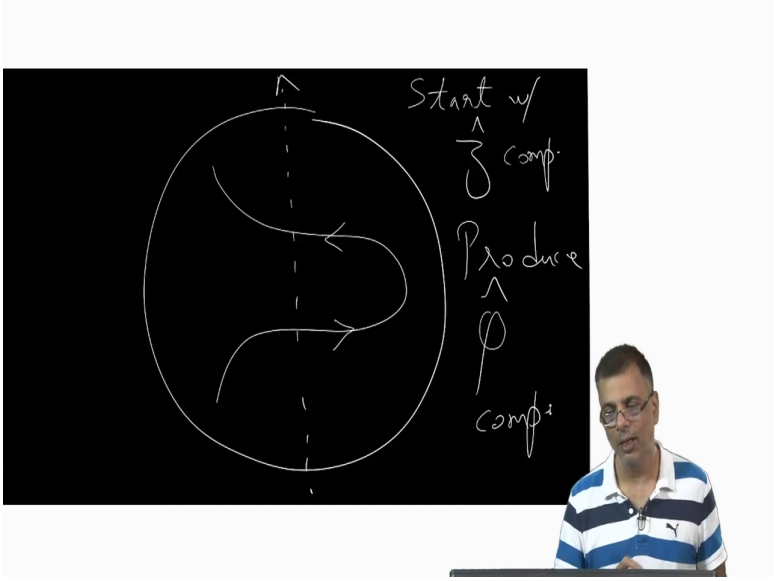

So, again let me try to draw circle like that ok. Imagine and initially now, you always have to start with a seed field. Imagine an initially z directed magnetic field yeah. Now, the equator this part is rotating faster than this part. So, what will happen is, this field well kind of get distorted into something like this.

So, this is what it used to be ok. It used to be from bottom to top, but because the central part is moving faster than the top part. What is happening is that the central part is getting dragged

out more as compared to the top part ok and this is why is it getting dragged out, because the magnetic field is frozen into the flow ok and now, the this entire thing remember this is not happening at the surface. It is happening well beneath the surface ok.

So, you started out with a z directed field, start with z and you produce from dragging of the field due to the fact that the field is frozen into the velocity flow, which is which exhibits this differential rotation. It enables us to produce a azimuthal component. You can see that an azimuthal component is present in its, this is no longer a purely z directed field.

It is a z directed field plus a phi directed field produce phi hat, a phi hat component start only with a z hat component and you produce a phi hat component you see. So, this is one thing. Now, but this is not all, right. The point of the whole dynamo, if the dynamo should work then we should find some way of reinforcing this z hat component.

You started out with a seed field and somehow the dynamo needs to go back and strengthen the z field so that it can remain strong in the face of resistive decay. So, this is our main thing. Now, turns out that there is one more issue here, one more important component and that is a little shall we say that that stretching things a bit. If this was not there the whole thing would not work and that is called the issue of a helical turbulence.

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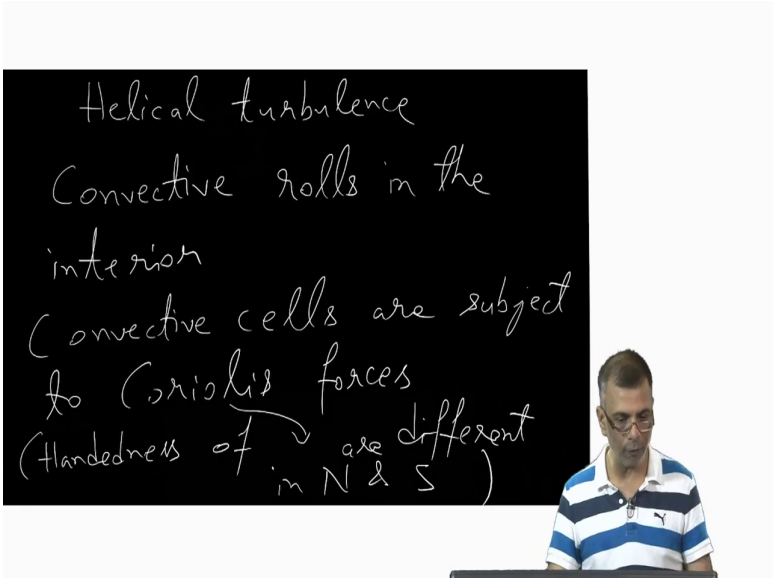

And let me try to explain what this is in a minute, turns out the interior of the sun, if I remember correctly the last 30 percent of the suns depth ok, it is convectively unstable ok. What this means is that the there are always convective bubbles in the plasma which are flowing ok.

So, these convective bubbles you see are so, you have to; you have to accept the fact that there are convective motions, a convective rolls ok, convective rolling motion in the interior. This is one important part ok. And these convective motions are subject to Coriolis forces.

These convective cells are subject to Coriolis forces, why? Because they are in a rotating frame of reference ok and the Coriolis forces are of opposite directions in the two hemispheres; in the southern and the northern hemisphere. You see if you are standing on the

north pole you have your head pointing this way whereas, if you are standing on the south pole you have your head pointing this way.

Whereas the rotation of the sun is in the same direction it might be differential, but it is in the same direction. So, the handedness is different the handedness of the rotation for the person whose head is pointing this way is different from the person whose head is pointing this way.

So, the handedness of the Coriolis forces are different in the northern hemisphere and the southern hemisphere. So, the handedness of the Coriolis forces are different in N and S hemispheres ok. So, the so, and therefore, what is going to happen is this guy is going to get distorted by the fact that there is little inside the plain of the paper there are dis-convective rolling motions and I will try to you know sketch it.

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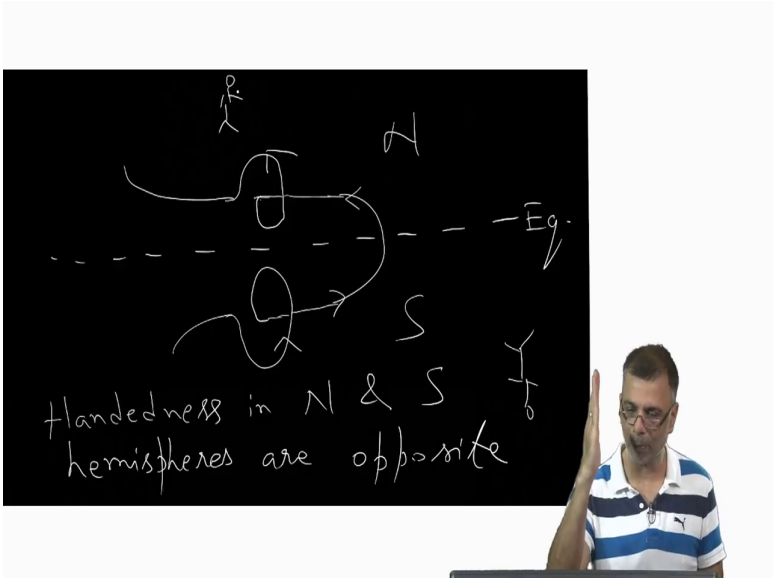

So, let me first sketch these guys and let me sketch these guys. If you remember this is pointing this way and this is pointing this way. This is exactly the same as this exactly except what is happening in between, I have not yet done and I will do it in a minute right. So, what is going to happen here is that this guy is going to get modified like this get connected here and this guy is going to get modified like this and get connected here. It looks like this and this also looks like this ok.

So, this thing is simply connected like this and this thing is simply connected like this ok. Remember, these rolls are in a plane perpendicular to the plane of the screen in a meridian of point ok. This is very important to realize ok. And the handedness, you see this guy is pointing this way and the rolls are this way and this guy on the other hand is pointing so this would be the north hemisphere and this would be the south hemisphere and this is the equator, is the equator ok.

So, the handedness in the northern hemisphere and the southern hemisphere are opposite, handedness north and south hemispheres are opposite. You point your thumb this way and do a right hand rule and here you point your thumb along the direction of the field and you do a right hand roll, you find that the sense of rotation is opposite in the northern and the southern hemisphere that arises from the fact that the Coriolis forces are in opposite directions in this northern and the southern hemisphere ok.

Because the direction of rotation is the same ok the five direction is the same, but in the northern hemisphere a person standing on the north pole for instance has their head pointing this way and on the south pole on the other hand, they have their head pointing in the opposite direction like that ok.

So, that that is that persons head or there ok. So, the handedness are different now what happens? So, these rolls are in the plane in the meridian of plane that is perpendicular to the plane of the screen.

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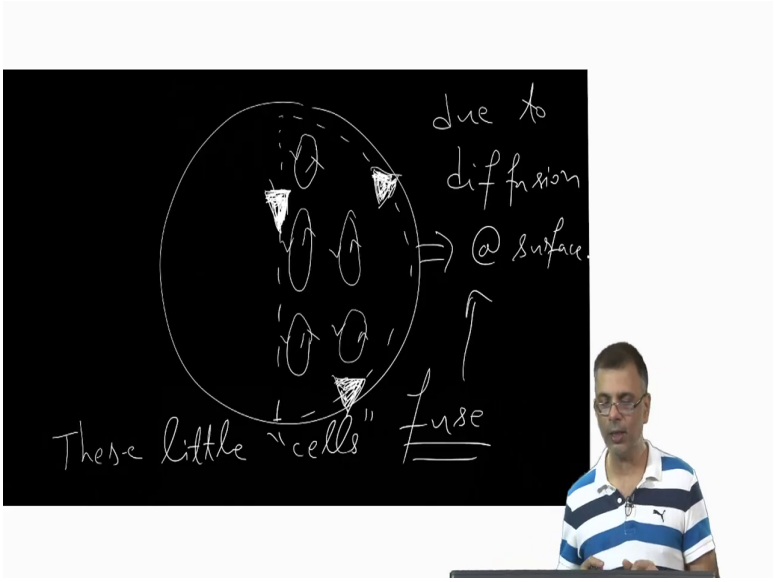

So, let us now try to look at the meridianal plane and draw the thing again like and in the meridianal plane like this what I have is several little rolls like this ok and the sense of the roll is like this yeah, like this yeah and all the rolls are like that. These are little convective cells, deep inside the solar atmosphere and what happens is these convective cells, these little cells which are each of these guys, fuse due to diffusion.

In other words what will happen is the boundary between this cell and that cell kind of merges, they fuse ok. Now, let us look at what is the general direction in all along this boundary. The general direction of the field is like this ok. So, this is a arrow pointing, this way and what is the general direction of the field here? It is like this, then arrow pointing this way yeah, and the general direction of the field is like this here, then arrow pointing this way ok. So, what is the magic that is happened now?

Two things are very-very crucial here one is that. So, this arrow was pointing this way, this arrow is pointing downwards and so on so forth ok. And so, this would be at the very surface ok. This is well in the interior and this is well at the surface the this is a cross section, this is a cross section that is there is perpendicular to the plane.

So, it is that cross section that we are showing here ok. So, this would be at the surface so, to speak at the surface so, what is going on here? What is going on is that so, there are two things just to remember the two things that are crucial one is that the handedness should be correct.

In other words, helical turbulence should have the proper handedness and that is reasonable to expect, but is still somewhat contrived, I will admit to it and these little cells will have to field due to diffusion so that the overall when you add up these little cells on the periphery the field looks like this.

And on the periphery what is happening the large scale field is pointing from the bottom to the top and that is exactly what we started out with. You see, the large scale field was pointing from that bottom to the top ok.

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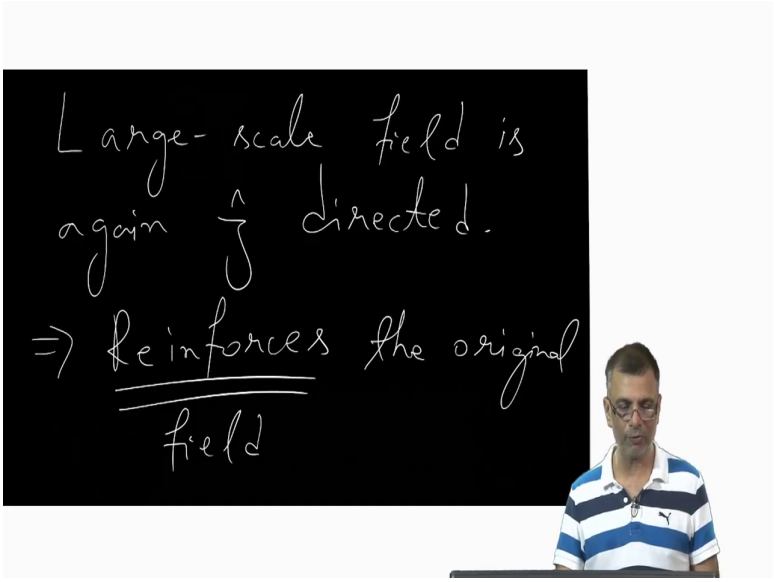

So, the large scale field is again z directed. So, it reinforces the original field, it reinforces the field that we started with. We started out with a large scale z directed field, we generated a phi component and from the phi component we twisted the phi component appropriately in a certain manner and we let these little convective cells fuse and infusing they produce a large scale field with which adds to the original large scale field, it reinforces the original large scale field and so our cycle is kind of complete now, alright.

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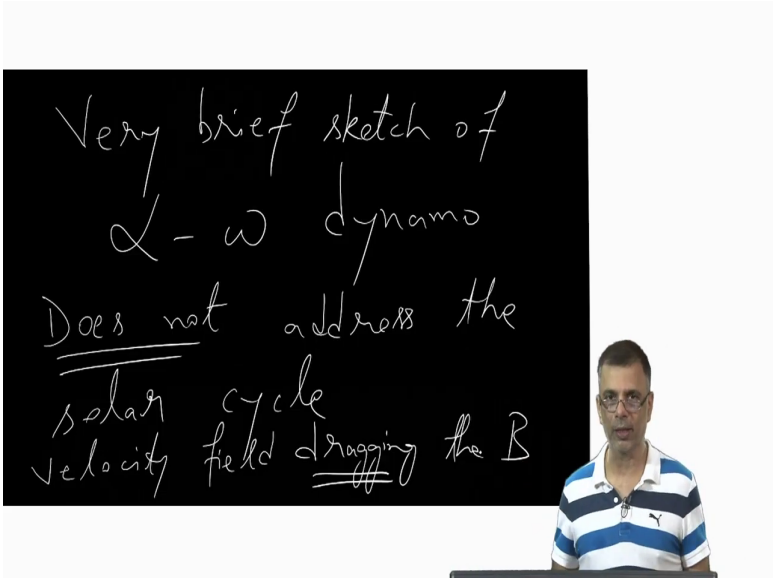

So, this is a very brief sketch of how the solar dynamo of how the this is a very not so much so solar dynamo, it is a very brief sketch of what is called the alpha omega dynamo. The omega refers to the differential rotation, because omega is generally you know and alpha refers to this kind of this helical turbulence source big. And there are several things this is a very brief sketch of the alpha omega dynamo.

It does not address, at least the way we have sketched it. It does not address the solar cycle 11 year change in the magnetic field, this is just a very-very basic that is all we know. The only reason I said this I brought this up in the context of this course is that it realize crucially this whole thing you see, the this whole sketch, this guy the fact that the fact that the z directed, the initially z directed field is dragged and to produce a phi component and the fact that these convective rolls ok.

What are the convective rolls? Convective rolls are essentially velocity field that is what they are and the velocity fields also drag the magnetic fields to form little loops in very specific directions ok and then the little loops you know fused together and so on and so forth, but all of this, the entire phenomena is very crucially dependent on the velocity field, dragging the magnetic field.

It is very-very crucially dependent on that ok and what is this a consequence of? This is a consequence of the induction equation and the induction equation is something that we have spent a fair amount of time on.

So, I just wanted to you know end the section by illustrating how the centrality of the induction equation and how one of the key predictions of the induction equation results in explaining at least in giving at least a very skeletal over view kind of sketch of the alpha omega dynamo right ok. So, that is all we have time for right now and we will get into a completely different topic the next time.

Thank you.