Fluid Dynamics for Astrophysics

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Lecture - 53
Magnetohydrodynamics (MHD): Magnetic flux-freezing (contd.), magnetic dynamos

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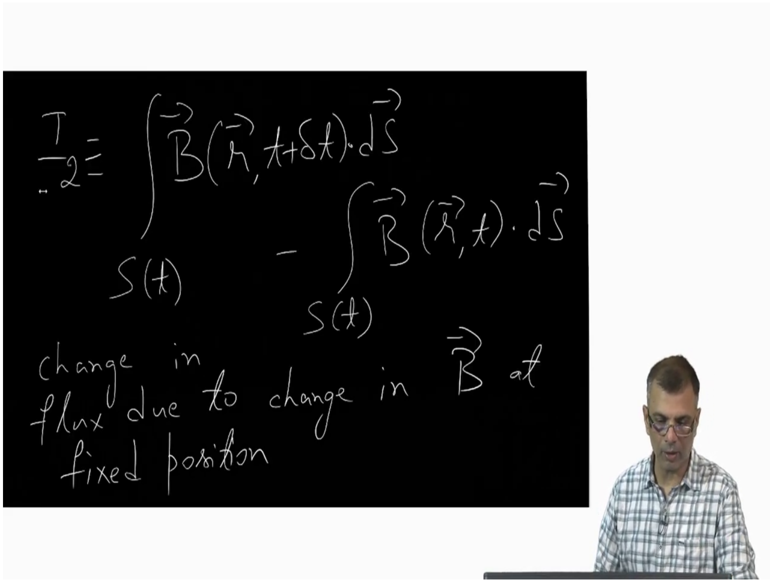

So, you see we talked about the I 1 and the I is sorry, not I this both are I 2.

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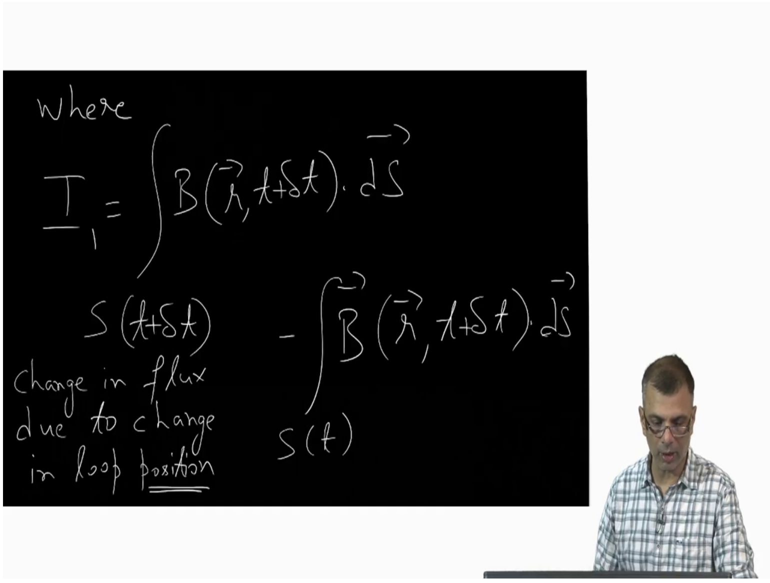

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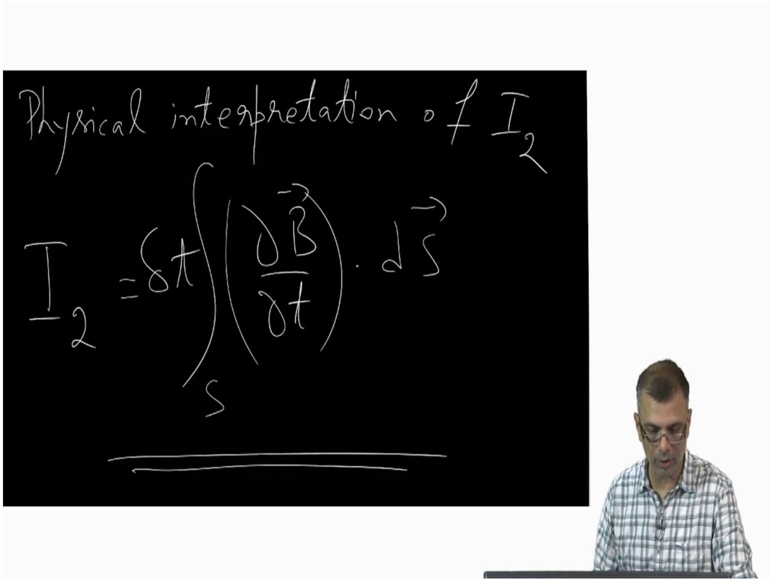

We talked about this I 1 and I 2 and we assigned a physical meaning to I 2 right, but the thing is we are really talking about the change in flux through the curved cylinder not only through these cross sectional surfaces you know, but through the entire curved cylinder ok.

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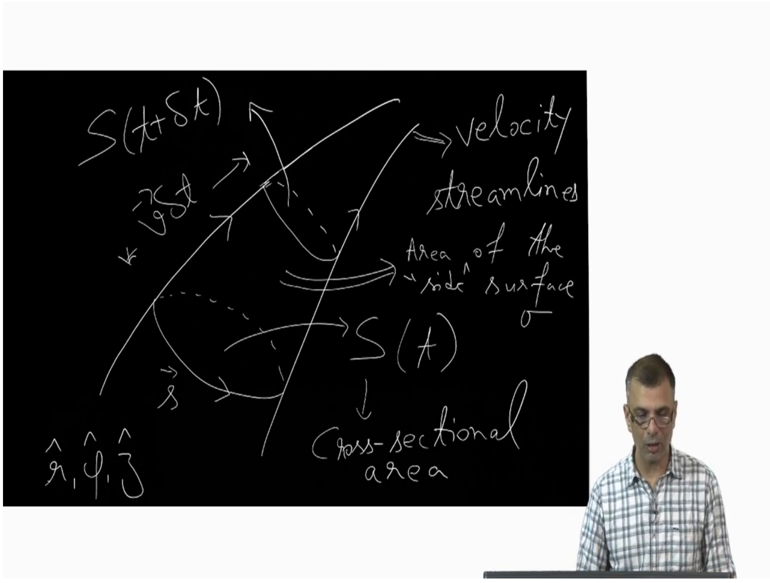

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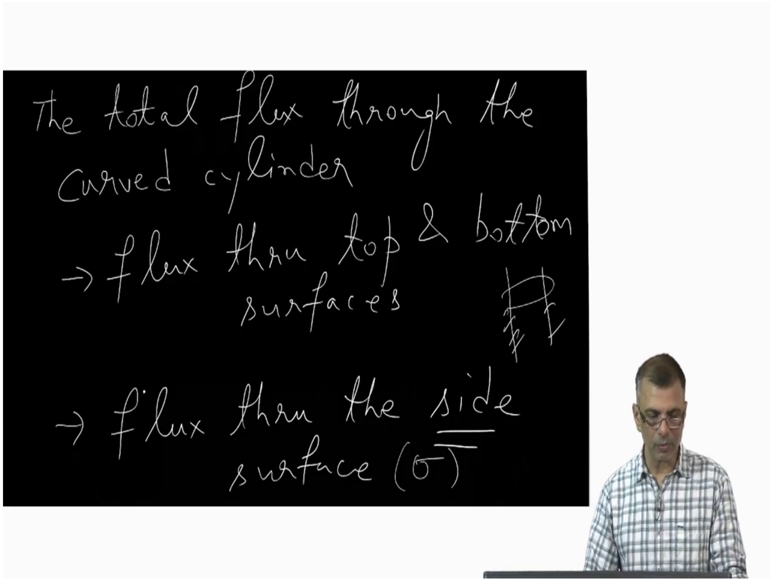

So, in other words the total flux, the total flux through the curved cylinder, which curved cylinder? Well, this one right. I mean the this, this, this curved cylinder. It includes, it has two pieces.

So, to speak flux through top and bottom surfaces which is what we have been talking about so far and also flux through the side surface. In other words the side surface this thing, this stuff. Not this not the you know cross sectional. Let the side surface be called sigma ok.

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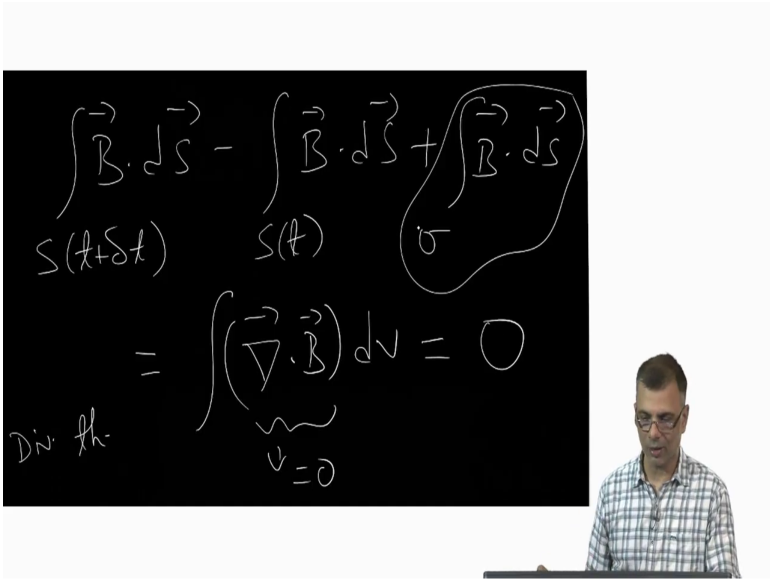

So, therefore, I can symbolically write it as the, I can symbolically write it as B dot dS S t plus delta t right minus well, it is it is really I am not writing explicitly r and t and everything here. It would be here would be r t plus delta t here, it would be simply r and t, but so, therefore, I am I am keeping this a little less just to avoid too much complication.

So, the difference in flux through the through you know the top and bottom surfaces plus whatever flux is passing through the side surface ok, through this side surface, through this surface here, the side surface of the cylinder right and now these are all a B dot dS kind of thing is not it. Now, this is all B dot dS and by the divergence theorem we get the integral of B dot dS is simply ah, we know that this is equal to the integral of divergence of B right dv.

And we know that this is equal to 0, always equal to 0 divergence of B is always equal to 0, no matter what right. So, this has to be equal to 0. So, this connection between the surface integral and the volume integral like this is through the divergence theorem ok. So, there is that. So, this is one important thing.

Now, let us talk a little bit about this guy. We have mostly been concentrating our attention on this fellow and that fellow. Now, let us concentrate our attention on the flux through the side surface. So, what is it? Right. So, first of all what is this sigma? The sigma is the side surface of the cylinder is not it.

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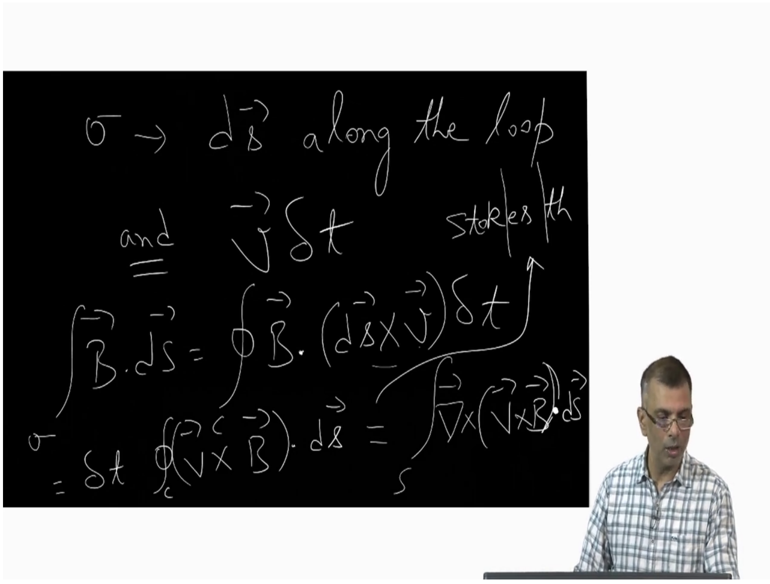

So, sigma comprises length element along the loop I will go back to the figure in a minute and show you what I mean. It comprises, it comprises this right and of course, this yeah.

So, it comprises the length element along the loop like I have written and which is the V dt is essentially. So, it would essentially be sort of a you know integral this times V dt which is this ok. So, so, that is what I am writing here ok. So, over the side surface this B dot dS is equal to the contour integral ok of B dot. What is this side surface? The side surface is the d small s which is the length element cross V delta t.

Now, it is important to figure out why I am writing this cross right. So, essentially what I am saying is it is a dS, it is a dS cross V. Well, the reason I am I am writing this cross is because you see the V is along this direction and the ds is along this direction. So, what would be the cross product of azimuthal times z directed field in cylindrical coordinates, it would be a radial right.

In cylindrical coordinates you have the radial, the azimuthal, and the z directed. So, phi cross z gives you r and so, in some sense this ds cross this thing that we wrote down ds cross V this would be in the phi phi hat direction and this would be in the in the z hat direction and the crossing these two gives you r hat and that is exactly that is say.

So, when we talk about the side surface of a cylinder the outwardly directed area element is indeed along the radial direction that is the reason we write this cross product and this cross product is very-very important ok. This one I can also write as, I take the delta t outside ok and I can write this as a contour integral of V cross B dot ds and this comes from vector algebra identity a dot B cross c ok and this is c cross a dot B ok and this delta t is simply comes outside.

You should try to figure out why this is true ok and this is a contour integral right of V cross B and using the stokes theorem. The contour integral can be turned into a surface integral and so, I would have a curl here curl of V cross B dot ds ok, where there is a dot ok, here and there should be a dot here.

The dot is outside of the bracket ok. So, this comes from stokes theorem right and so, this equality comes from stokes theorem ok and you already start recognizing where this, this is an important part of the of the induction equation right. Curl of V cross B you recall how it appears in the induction equation.

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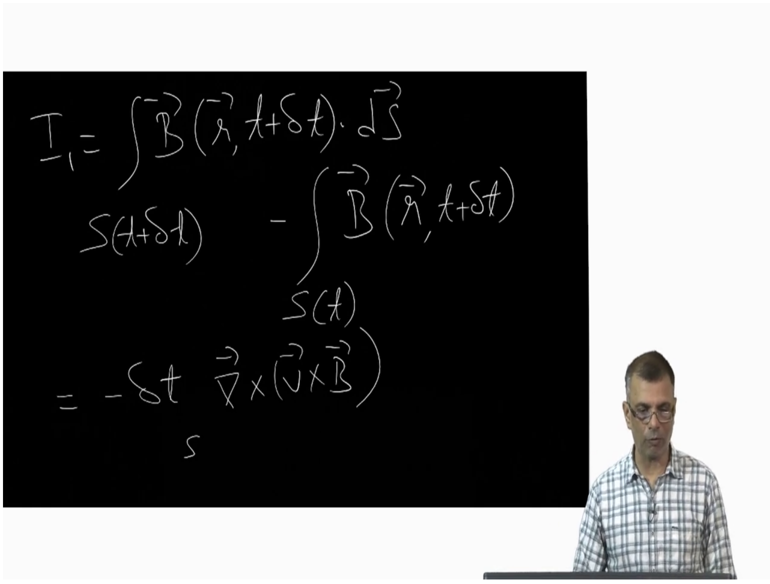

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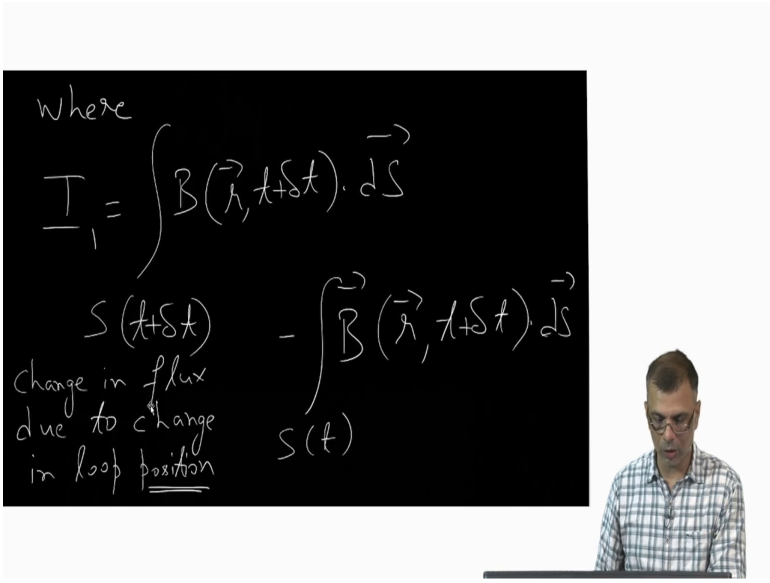

So, therefore, we have so, we have the third part now. So, the I 1 which we defined here, which is the change in flux due to the change in loop position which we wrote down as this yeah.

So, that is equal to change in loop position right. So, initially the loop was at St plus delta t right and the flux is right dot d rho. You remember we are now fussing about I 1, because you know I 2 we have already gotten a nice physical interpretation of I 2. So, so we do not worry too much about that ah. So, so let us now get back to I 1 and. So, I 1 is equal to this minus S of t B r plus delta t.

You see I am keeping the magnetic field the same, I am only changing the loop with the cross sectional area which is changing, because of the change in loop position ok and this is equal to minus delta t curl of V cross B right. This is this comes from here of course, this comes from here ok. It actually comes from here and here.

So, what we did and was we identified that this is equal to this quantity this B dot dS is equal to so that is what we did right and I 1 we wrote it down like this and so this is where we have this important thing yeah and we know that I 2 is equal to we already written this down this is equal to delta t S dB dt dot dS.

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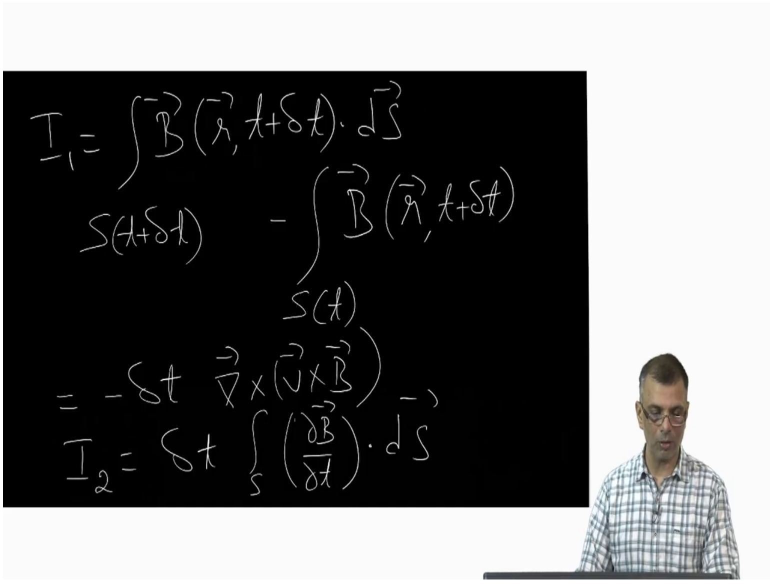

We would we are already you know identified the I 2 to be this. So, essentially this becomes I 1 and this becomes I 2 and I 1 plus I 2 is what we are concerned with of course, right.

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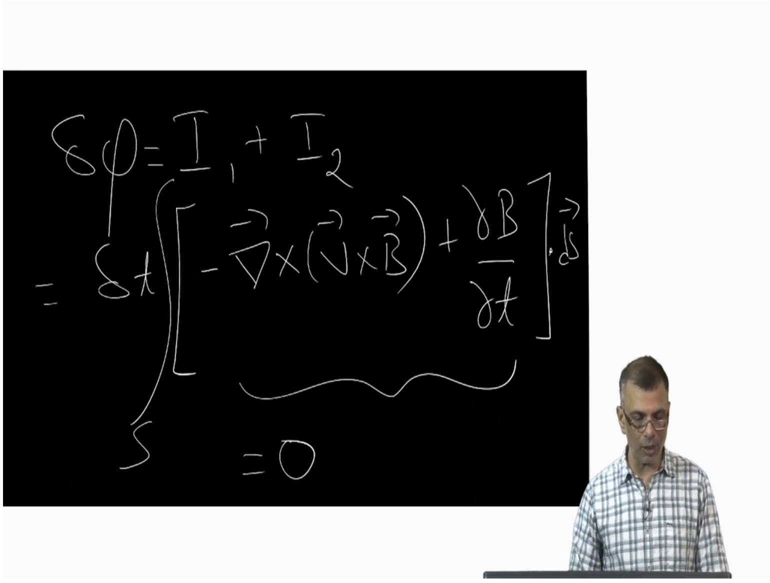

So, therefore, delta phi which is what you know, at the end of the day we are concerned with delta phi and we wrote that wrote down delta phi as I 1 plus I 2 right is equal to delta t times, because we identified I 1 as curl of V cross B times delta t we write down this as delta t times minus this is I 1 curl of V cross B, this is done yeah.

And I 2 we wrote down as the same delta t dB dt right ok and also plus like this and of course, I already always have a surface integral. So, I should have a surface integral here too and what is this? This is what we started with for an ideal you know, if you if you recall this is what we started with dB dt is equal to curl of V cross B right.

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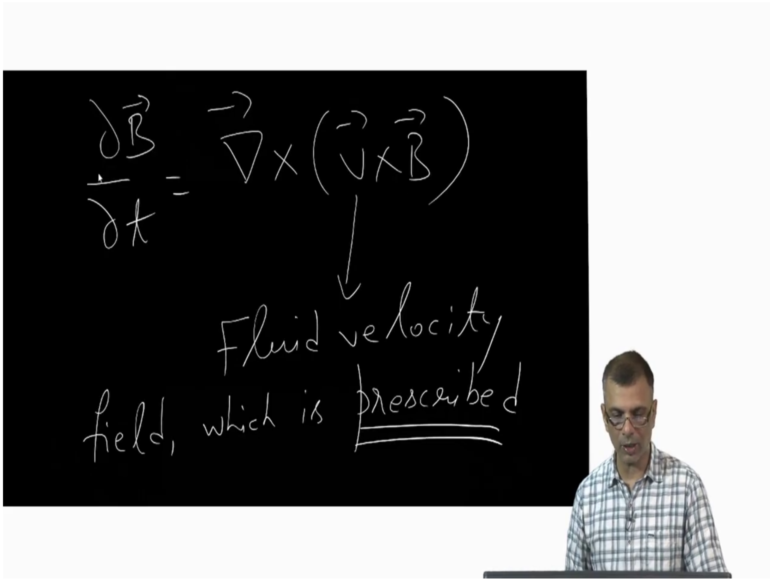

So, I put a negative sign on this yeah and I take it over to the other side right and so, dB dt minus curl of V cross B has to be equal to 0 and that is exactly what we have here. We have dB dt minus curl of V cross B and this is always equal to 0 right and so, since integral is equal to 0 the integral will always be equal to 0 and therefore, we have right, we have this equal to 0.

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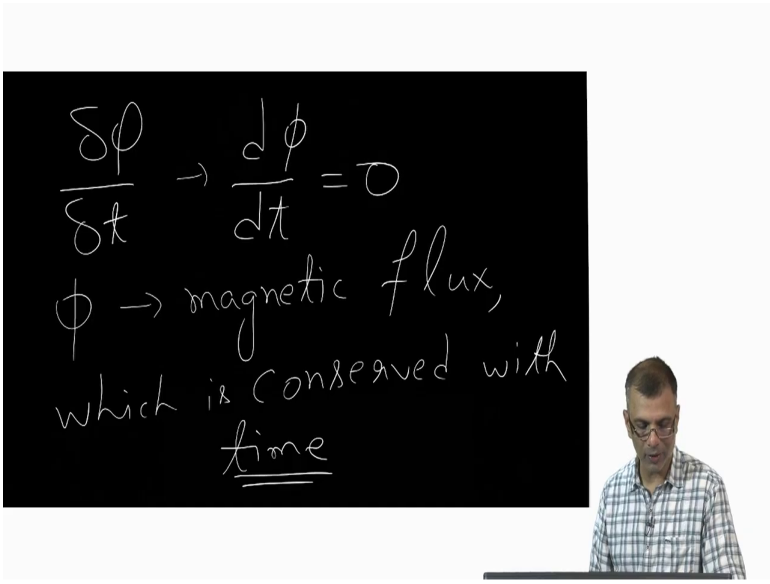

In other words, what we are saying here is that d phi delta phi delta t which in the in the limit goes as goes to d phi d t is equal to 0 and this is what we were have to prove ok. So, and phi is the magnetic flux is the magnetic flux which is conserved with time and this is a direct consequence of the induction equation, this is a direct consequence of the fact that the integrand here is equal to 0. In other words, the this is the induction equation and this is an integrand and it is equal to 0. So, there is no choice, the magnetic flux has no choice, it has to be conserved ok.

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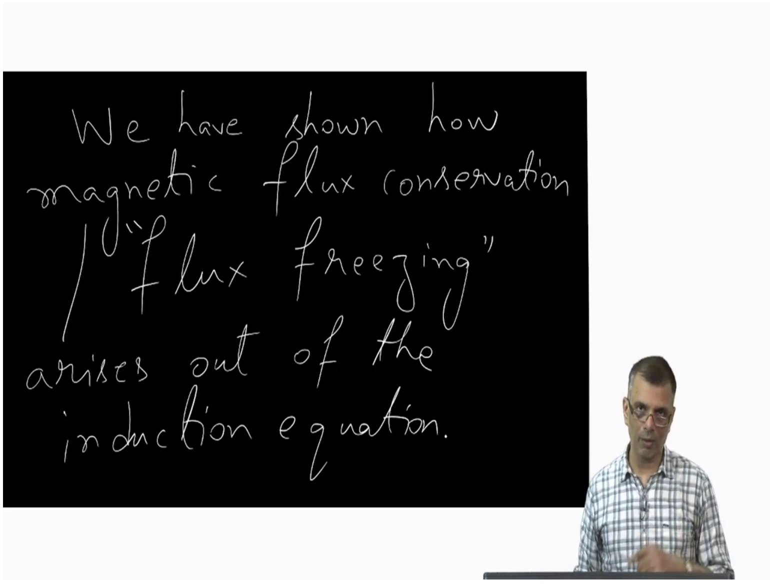

So, what we have done now is we have we have shown, we have shown how magnetic you know flux conservation or what is called flux freezing and I want to put this flux freezing in arises out of the induction equation.

This is what we have shown and how did the induction equation arise? Well, the induction equation simply arise arose from Maxwell's equation with the assumption of infinite conductivity ok and once you have in infinite conductivity you can never have an electric field inside the fluid and so, any electric field has to be only due to the fact that an observer is not inside the fluid at all.

So, you have that curious e equals minus V cross B over c kind of thing and you substitute that into amperes law and you get the induction equation.

So, once you have the induction equation which is I would say you know one could go so far as to say that this is a natural consequence of infinite conductivity, then we know that what we have shown here is magnetic flux is conserved. So, flux is frozen you know in a situation where the conductivity is infinite ok. Now, what about this? Why are we making such a big deal about it right?

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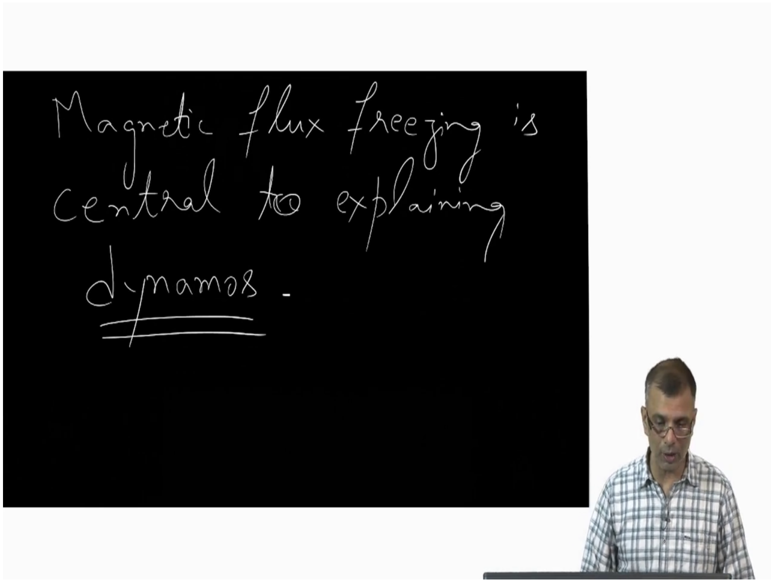

So, the point is flux freezing, magnetic flux freezing is an important ah, magnetic flux freezing is central to explaining dynamos, magnetic dynamos in astrophysical situations. So, the thing is why what is this thing called dynamo?

Well, a dynamo is as you know you want to sustain magnetic field that is what I mean dynamo does and that is what you would do in a bicycle dynamo for instance, these days you just have LED batteries and you turn the turn the light on.

You know when you want a light in front of a bicycle, but in the olden days what you would have is you would have a dynamo that you would it would be mechanical rotating object that you would that you would tilt onto your onto your bicycle tire and so you would be rotating, you would be mechanically rotating the spindle by your muscle power and that would generate a current ok.

Now, we know in MHD the current is nothing, but the curl of the magnetic field. So, in our context a dynamo would simply mean device that helps us sustain magnetic fields ok. Now, why is this important? Because in astrophysics you know magnetic fields are quite ubiquitous. They are everywhere.

In many cases they are measured directly as in the solar wind, the immediate environment of the earth and so on so forth. We know that the earth has a magnetic field and we have indirect evidence of the fact that the sun also has a magnetic field ok and we have even more indirect evidence.

So, the fact that the galaxy and is permeated by a magnetic field fairly exotic objects like pulsars, they have very high, very strong magnetic fields. These are all indirect evidences, but nonetheless magnetic fields are everywhere in astrophysics.

This is kind of well known, but the same time no plasma is infinitely conducting ok and there is always a finite resistivity in any plasma and as we know this kind of so, this integrand would never be exactly equal to 0, there will always be an additional term here. Some kind of a lambda times nabla square B which would be a resistive term.

So, essentially what we are trying to say is that in the pres presence of finite resistivity magnetic field simply cannot sustain. They will always decay away and if you put in the numbers you will find that for expected values of resistivity really there is the magnetic fields that we observed have no right to be there ok. They should have really have decayed away long ago. How are they there? That is the question, at the heart of dynamos ok.

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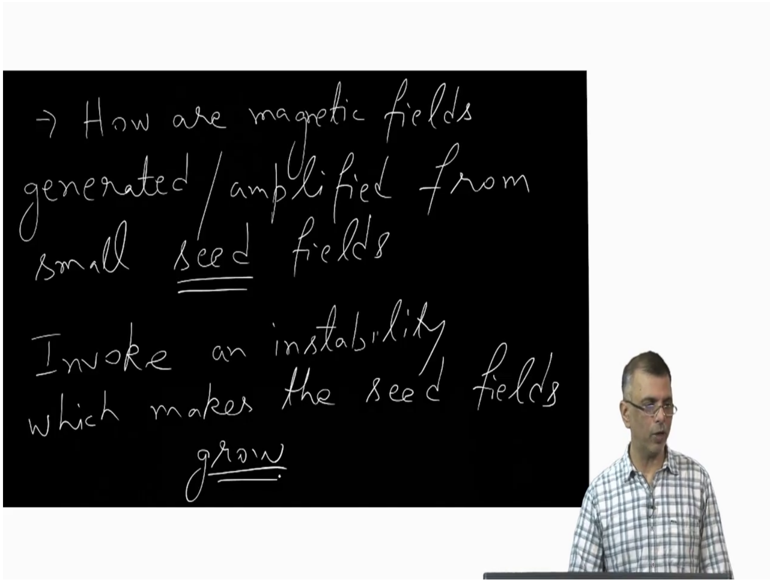

So, the question we are asking is what is a mechanism that generates a how to so, the question really that an astrophysical dynamo theory should address is a how are magnetic fields generated or amplified or amplified from small seed fields. So, now, here is the thing you always have to assume that there was some existing seed field. The magnetic field that we are observing today is not that seed field ok.

It is a it is a very matured field it is a tree. So, to speak that has grown from the seed, but you cannot get away from the fact, you cannot generate a magnetic field from 0. You always need to have a small seed field. So, you have to given picture of, given understanding of a magnetic field dynamo has first got to address this question ah which is that how are magnetic fields generated or amplified from small seed fields? So, this is one question.

Slightly more elaborate way of considering this question is we have to show that the presence of a seed small seed magnetic field renders a system of MHD equations unstable with time so that you show and invoke an instability which make the seed fields which makes the seed fields grow. So, this is one thing, this is one ingredient of bona fide dynamo theory.

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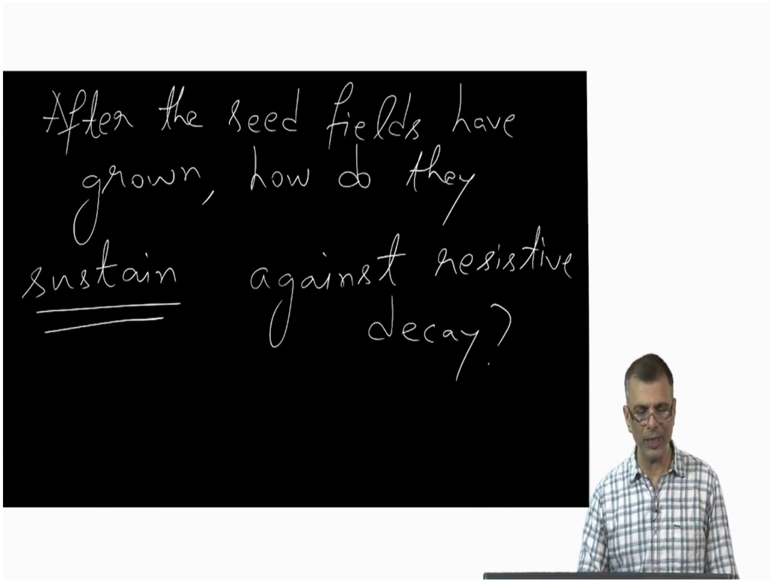

The other ingredient is that after the seed fields, the seed fields have grown, how do they sustain how do they sustain themselves against resistive decay? Because now, we are talking practical stuff, we are not talking an ideal MHD kind of situation.

We are talking about you know a situation where there is always a resistive term and that resistive term we have seen it leads to the decay of magnetic fields, decay with time and so, if you wait for long enough and in astrophysical situations.

It is it is always one can put plug in the numbers and show that we have indeed a greater for long enough in many-many situations and if there was no mechanism for the seed fields for the fields to have grown from the seeds and attained a certain value, if there was no mechanism for these fields to be to sustain themselves in some manner then there was an instability which enabled these seed fields to grow.

These grown fields would die due to resistive decay unless there is a mechanism to sustain them. So, this leads us to the next question that a dynamo scenario should address which is that of sustaining ok and so, so, these are the two basic things about a magnetic dynamo.

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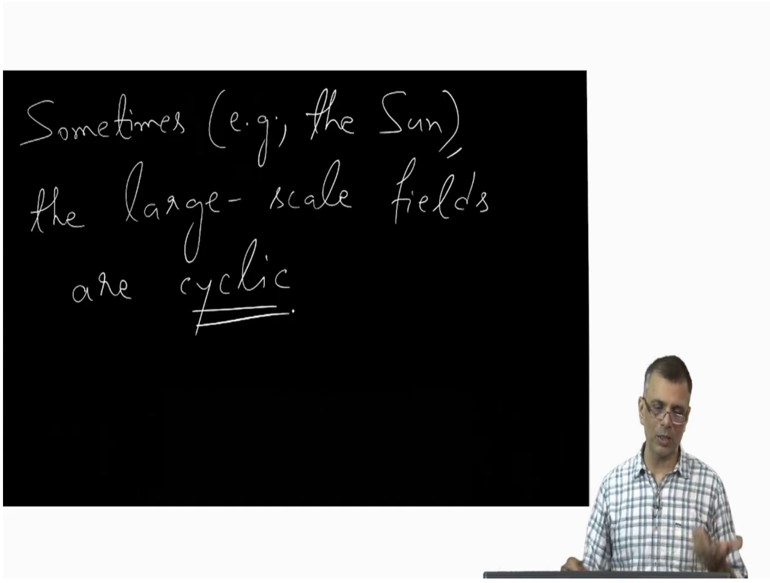

There are some other complications that sometimes for instance in the context of the sun the large scale large scale magnetic fields which have been generated and somehow you can show that the large scale generate magnetic fields which have been generated are able to sustain ok they are also cyclic.

In other words the large scale fields are cyclic in time. In other words they grow, then they die, they grow again and they die again ok that is another complication, but so that is another complication and we will not get so much into that, but you have to wonder. Now, why did I suddenly start talking about dynamos and so on so forth evidently, it has some connection with the alpha influx phrasing theorem right.

So, the most common approach to these two uh questions to solving these two questions, how are magnetic fields generated or amplified from the small seed fields and having amplified them how do they sustain against you know magnetic decay the most common approach is what is called a kinetic dynamo.

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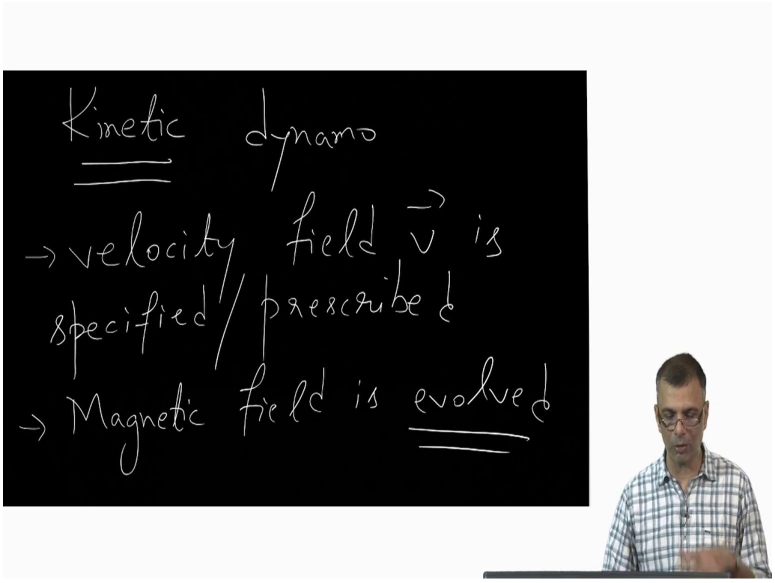

Ah rather this was the historically the first kind of approach where the velocity field the velocity field V is specified, is prescribed or specified, I do not bother about how this came about it is given to me from observer from certain observations which we will discuss ok and the magnetic field is evolved the magnetic field is evolved in time and we know we know very well this is what we have been talking about for a long time now.

What is the one equation that tells you how the magnetic field, you know evolves in time in response to a given a prescribed velocity field? This, the magnetic induction equation, you prescribe a certain velocity field and then you see how the magnetic field is evolved.

So, this is at the heart, this is the heart of this approach called a kinetic dynamo ok and so, we what we will see when we meet next is that this particular approach has been quite successful in explaining at least some elements, not all elements.

There are still very-very big unanswered questions in this in this field, but still there has been some remarkable success in explaining at least some elements of these two questions pertaining astrophysical dynamos; one is how do seed fields grow and having grown how do they sustain in the face of resistive decay. And we will mostly address the second question how these large scale fields sustain using this kinetic dynamo picture. So, we will discuss this when we meet next.

Thank you.