Fluid Dynamics for Astrophysics
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Lecture - 51
Magnetohydrodynamics [MHD]: Plasma beta, force-free fields and potential Configurations

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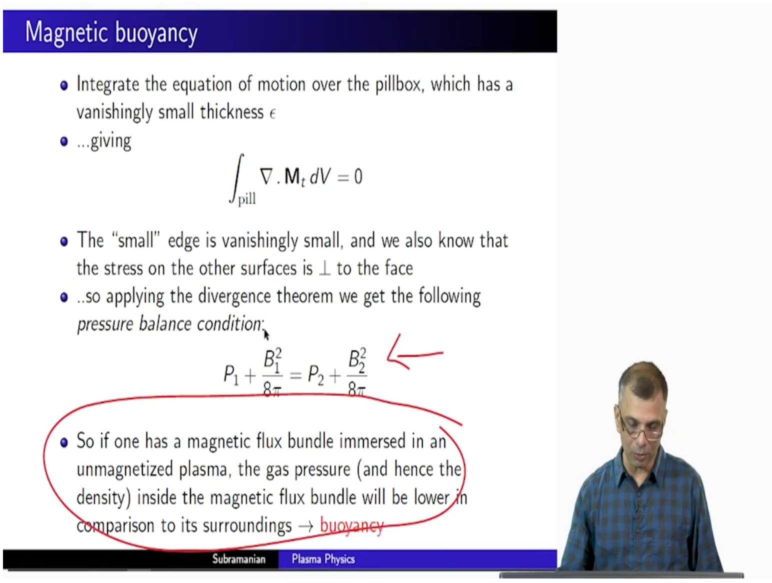

So, this is what we essentially said uh, this consequence of magnetic buoyancy. So, if you have and there is a direct consequence of this, the pressure balance condition right, the fact that you the magnetic the gas pressure plus the magnetic pressure has to be constant ok.

So, if you have a magnetic flux bundle immersed in an unmagnetized plasma uh, the gas pressure and hence, the density inside the magnetic flux bundle will be lower in comparison to its surroundings something like this.

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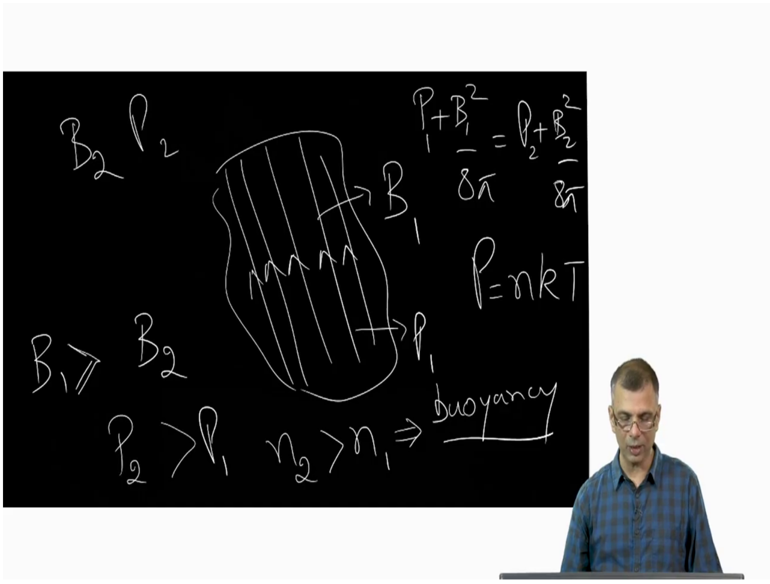

So, suppose you have a magnetic flux bundle in here yeah. So, you have some B 1 right and outside say the you know the medium in instead of being unmagnetized, let it have some you know let there be another magnetic field B 2 except this is a magnetic flux bundle.

Therefore, B 1 is say much much larger than B 2 or even I do not really have to say much much larger than there is really no need to do that let us say let us just say that is larger than B 2 like that B 1 so, this is a flux bundle therefore, there is a concentration of magnetic field lines.

So, the magnetic pressure inside, the magnetic field inside here, this volume is larger than the magnetic field outside ok. Now, we know that the gas pressure P 1 plus B I am sorry has to be equal to P 2 plus B 2 square over 8 pi where P is the so, the you know P 1 is the gas pressure inside and P 2 is the gas pressure outside right.

Now, since B 1 is greater than B 2 because of this equality, P 2 has to be greater than P 1. Since B 1 is greater than B 2, P 2 has to be greater than P 1 in order to maintain this equality isn't it? Now, P is equal to nkTt right where n is the density, the gas density. Now, if the T, if the temperature inside and outside are the same, then the only way P 1 can be greater than sorry, the only way P 2 can be greater than P 1 is if n 2 is greater than n 1.

In other words, the density inside, the gas density inside is lower than the gas density outside which leads so, what happens when you have a; when you have a bubble a low-density bubble inside a high density a relatively high density fluid? What happens is the low-density bubble is subject to buoyant forces, this leads to buoyancy.

So, this is what I meant, this leads to magnetic buoyancy. So, this is this was one you know important consequence of magnetic fields right being embedded in in fluids.

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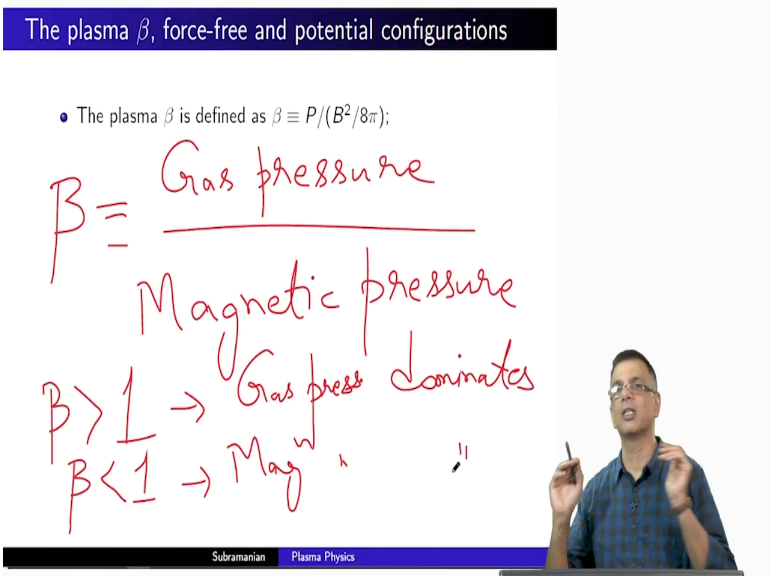

We have already started talking about P 1 P and B right magnetic pressure and gas pressure as though you know there were different kinds of so, I mean you know a P.

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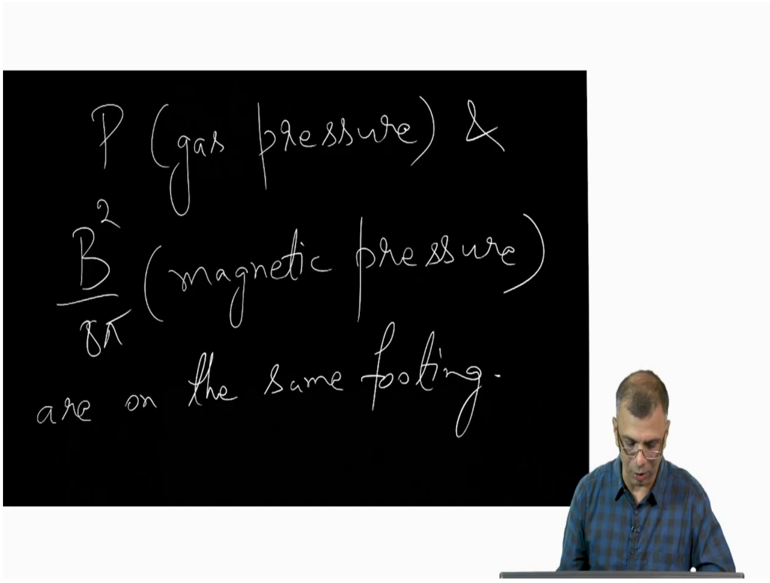

So, essentially, P, the gas pressure and B squared over 8 pi the magnetic pressure are on the same footing. So, it is natural to ask, it is natural to know you know assign a number, assign a dimensionless number you remember we all like dimensionless numbers in fluids and MHD and everything, is natural to assign a dimensionless number.

That tells you which is dominant, is the gas pressure dominant or is the magnetic pressure dominant in a given situation and that dimensionless number is called the plasma beta which is defined as the ratio of the gas pressure to the magnetic pressure; magnetic field pressure right.

So, plasma beta sorry gas pressure over magnetic pressure that is what the magnet that the plasma beta is right. So, if the plasma beta is larger than 1, it is gas pressure dominates and if the; if the reverse is true, then magnetic pressure dominates right. So, if the if I make a statement of the plasma beta is less than 1.

That means, this is a magnetically dominated situation because the magnetic pressure dominates. If I make a statement that the plasma beta is larger than 1 that means, this is a gas pressure dominated situation, the magnetic fields do not matter that much. So, this is a nice dimensionless number which gives you an idea of who is dominating.

There are simplifications in on either sides if beta is much much larger than 1 or beta is much much less than 1, you know things are simplified. It is only in between that you have to retain all terms, the gas pressure term as well as the magnetic pressure term right yeah.

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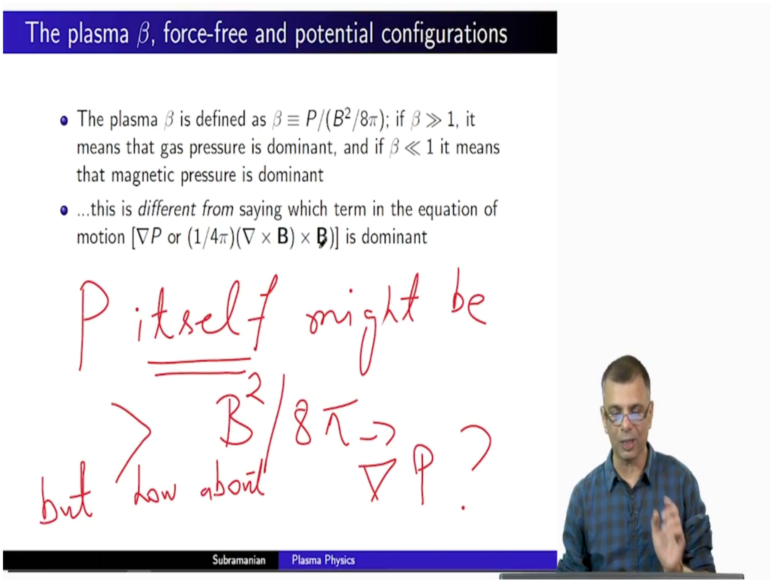

So, if beta is much much larger than 1 that means, the gas pressure is dominant and if beta is much much less than 1, it means that the magnetic pressure is dominant. However, this is different from saying which term in the equation of motion gradient of P or this is dominant.

You see P, P itself for instance, let us consider a situation where beta is much much larger than 1. P itself might be larger than B squared over 8 pi ok, but that is saying nothing about, but how about gradient of P? P itself might be larger than B squared over a pi, but the if for instance, the P is not varying much with distance, if the P at a certain x is the same as the P at another x, then the gradient of P is very small so, P itself might be large.

But if the gradient of P is small that means, in the force equation, you remember its gradient of P which appears not P right. So, simply because the plasma beta is large that does not mean that you can neglect the Lorentz force term in the force equation ok. In the force equation, what matters is gradient of P and this thing curl B cross B the.

So, therefore, the plasma beta is a different statement from saying that you know this for this term or that term in the force equation can be considered or neglected, this is something that one has to. It is not P itself that matters in in in the in the momentum equation, it is a gradient of P that matters. So, this is something that needs to be kept in mind very clearly ok.

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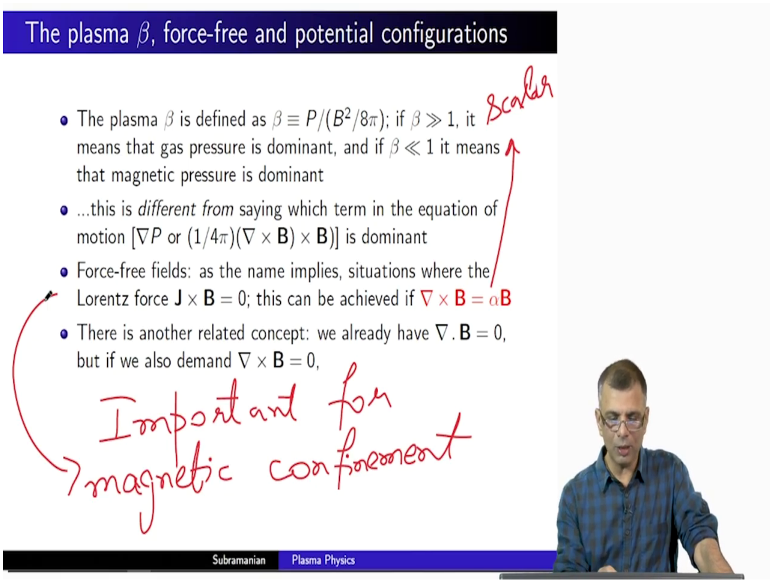

So, this was one in interesting thing that I wanted to you know emphasize.

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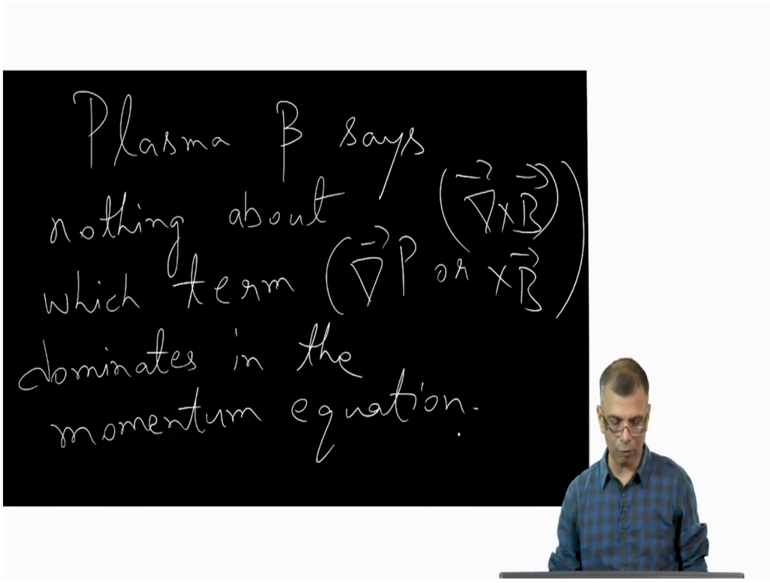

You will see discussions of plasma beta all over the place, everybody talks about plasma beta which is fine , but it is important to recognize that the plasma beta says nothing about which term gradient which term gradient of P or curl B cross B dominates in the momentum equation. So, this is something that needs to be kept in mind quite clearly.

The plasma beta is a very useful parameter, do not get me wrong ok, it is a very useful parameter, but simply knowing the plasma beta does not mean that you can you know which term to neglect or consider in the force equation, in the momentum equation. So, there is something that that is what I meant. Now, here is another very interesting concept that of force free fields ok.

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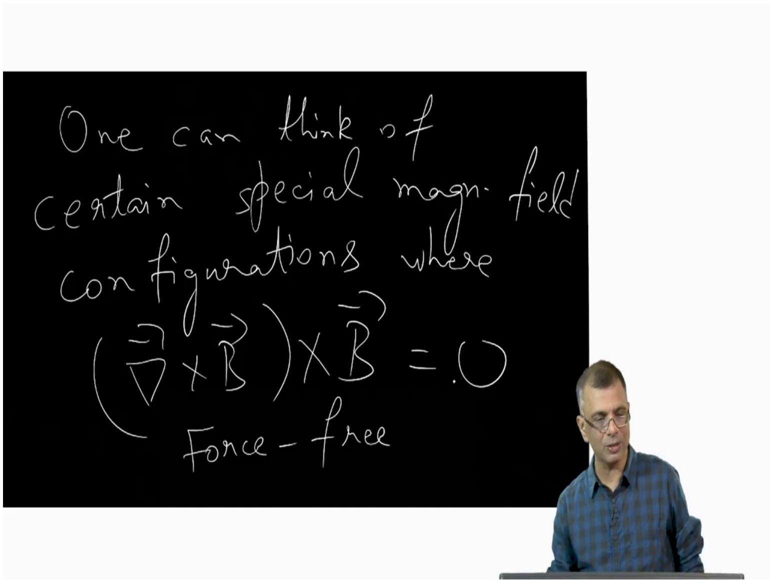

So, you see here is the Lorentz force J cross B or curl B cross B. Now, there are certain configurations where you can; you can; you can manufacture certain configurations. One can think of certain special magnetic field configurations which are arranged such that where curl of B is perpendicular rather is parallel to B so that curl B cross B is 0 and these are called, these kinds of configurations are called force free configurations.

What kind of force free? Lorentz force free configurations because this is g; this is J and J cross B is the Lorentz force. So, if this condition is satisfied that means, the Lorentz force is 0 and so, yeah and the Lorentz force is 0 and therefore, you know the Lorentz force and that on that volume of fluid which contains this special kind of force, this special kind of magnetic field configuration is absent ok.

And this can happen for a instance, if you insist that the curl of B is cross B is equal to 0 that means, the curl of B is in the same direction as B right which is to say if this is true, if the curl of B is some scalar parameter some scalar constant alpha times B, then you know curl of B is in the same direction as B.

Therefore, curl of B cross B is equal to 0 right. So, this alpha has to be just a simple scalar, a scalar constant. If that is the case, then you have you know force free fields. So, if you can manufacture and these are very important not merely as a theoretical curiosity, but also in important for magnetic confinement in lab plasmas. See you see in the lab, plasmas are manufactured to facilitate magnetic fusion.

So, you want to confine the plasma to a certain region for long enough such that fusion can occur ok. Now, in such situations, Lorentz forces are actually nuisance because they make what Lorentz forces do is that they distort the volume, they distort the element of fluid ok.

So, you would like to engineer the magnetic field in ins this this volume in such a fashion that the Lorentz forces are 0 in other words, you have force free fields, if that is the case, the fellow your little you know volume element remains stable at least for long enough such that fusion can occur.

So, force free fields are a subject of a lot of curiosity in lab situations ok. So, it is not merely a theoretical concept, it is a you know it acts as a very practical thing um, ideally force free fields might never be possible to achieve, but you know one strives towards it ok. So, these are the these kinds of configurations are important for magnetic confinement.

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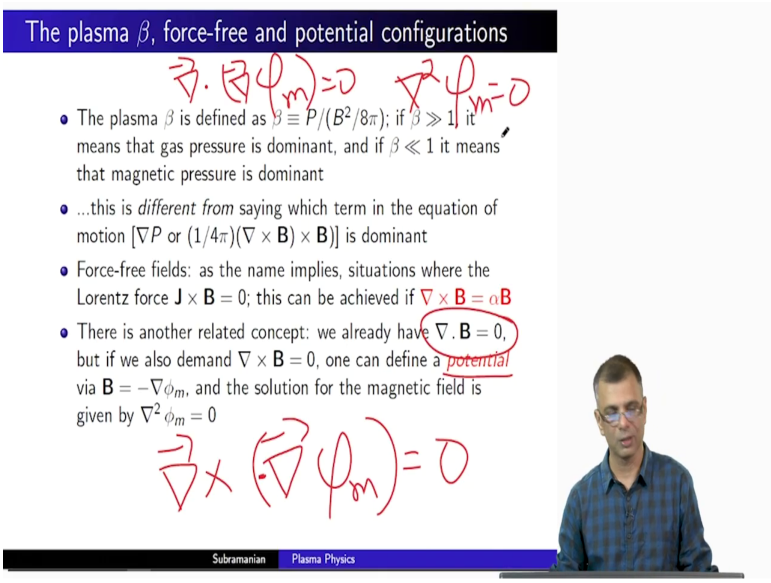

There is also another related concept that of potential fields ok. We know that the divergence of B is always equal to 0 that that is you know that is sacred, you cannot violate that, you cannot have magnetic monopoles and therefore, the you know magnetic field lines always have to close in on themselves so, the divergence of B is always equal to 0.

But now, if additionally, we also demand that the curl of B is 0 in other words, if we can come up with magnetic field configure and these are all you know, these are more applicable to lab plasmas right, both these concepts,, but suppose we can engineer the magnetic field such that the curl of B in other words the current yeah is also equal to 0.

Then one can define a potential field via B equals gradient of some scalar potential and this arises directly from here because the curl of a gradient is always equal to 0, we know this from basic vector calculus. So, this would be the B and that is what this is and so, I introduce a negative sign here for convention purposes. So, this is this right.

So, now, what happens is the solution for the magnetic field is given by from here and so, since you know the divergence of B is 0 that means, I am asking that the divergence of the gradient of this one1 is equal to 0. In other words, the Laplacian of this is equal to 0 and you remember that mathematically, we have lots and lots of situations where we have solutions to this ok.

So, this is the concept of a potential field and this is a scalar potential, it is not the vector potential, there is not the magnetic vector potential that you saw in electrodynamics, this is different ok, this is a scalar potential and the scalar potential is applicable only to very very specific situations only to a specific situation where we can ensure that the curl of B is 0.

These are very local situations, it is not global ok. So, in that case, we can write down this kind of an equation and we know lots of solutions to this right.

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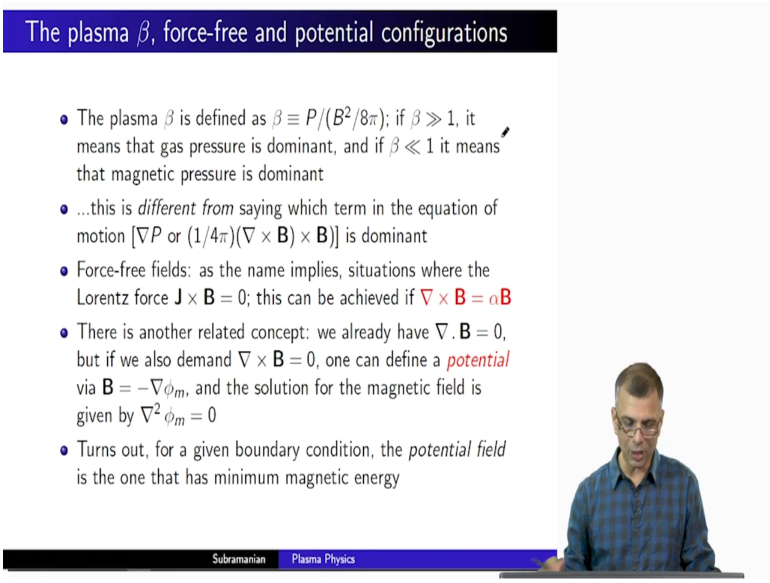

Now, what is the significance of a potential field?

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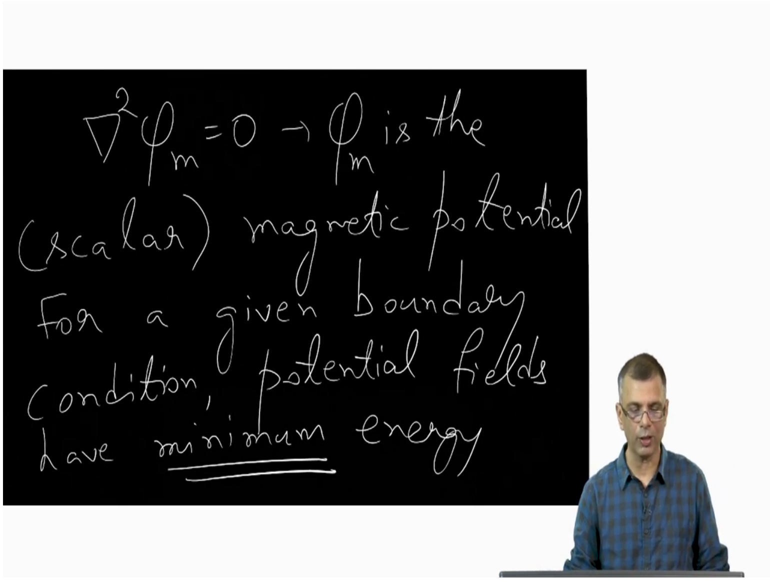

We were talking about a situation where magnetic potential isn't it? And so, what is the significance of this? The significance of this is that the potential fields are the ones which have the minimum possible energy of all given of all possible configurations for a given boundary condition.

So, for a given boundary condition; condition, potential fields have minimum energy and this is very important for astrophysics and I will tell you how have is the minimum energy configuration right ok.

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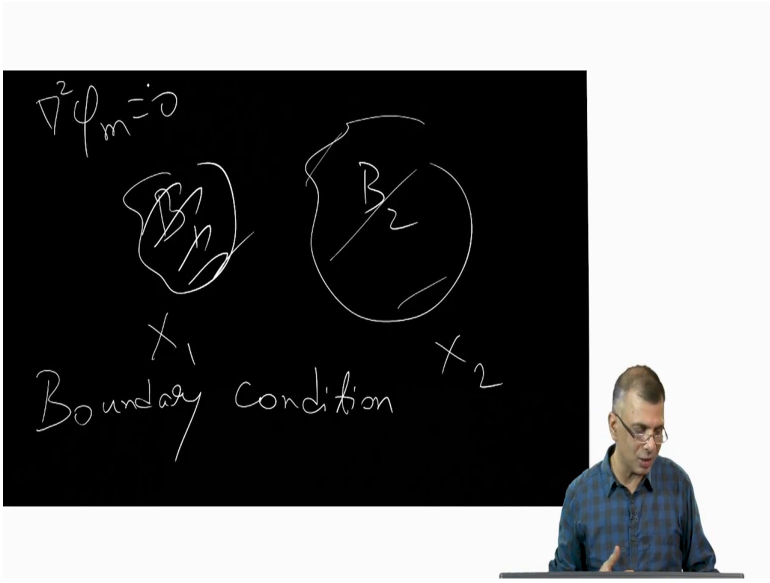

So, it is important in the following sense. Consider the solar surface where you know the surface of the sun, the photosphere, this is where you know is the last visible surface, you cannot see beneath the photosphere ok.

Now, there are instruments these days called magnetometers where you can the sun you can you can measure the magnetic fields on the surface of the sun in other words, on the photosphere and you can say, you can figure out that the longitudinal magnetic field is say B 1 here and B 2 here and so on so forth.

So, you would have a patch of large magnetic field and you would have a patch of low magnetic field and so on so forth. So, this would be say the solar photosphere where you would, this would be X 1 and this would be X 2 for instance and this would be the boundary conditions.

So, you have the what this is telling you is for and these are the longitudinal magnetic fields that are sticking out at you, the these measurements are made using the Zeeman effect and longitudinal Zeeman effect and the Zeeman effect can only tell you something about the magnetic fields that are pointing towards you not in the transverse direction ok. So, this is boundary condition at X 1 you have B 1 and X 2 you have B 2. So, this would be a boundary condition.

Now, given the boundary condition, you know that there can be many many solutions. So, the question is given these boundary conditions, what are the, what is the magnetic field configuration in the entire volume say in the corona above the photosphere yeah, what is the magnetic field configuration? This is the question.

Now, turns out that if you solve for a potential field, if you solve the condition; the equation with these boundary conditions, with for the same boundary conditions if I solve this one, then the solution that this phi at the magnetic field solution that this phi m will eventually give you is the one that will have the minimum possible energy of all, of any solution.

You can have many solutions with these boundary conditions, but if you solve the Laplace's equation with these boundary conditions, you are guaranteed that the solution that it gives will have the minimum possible energy and minimum energy is something that is always sacrosanct in physics as you know.

You know all fields and everything in nature always likes to come back to the minimum energy configuration, anything else is an unstable configuration ok. You perturb the unstable configuration; it will always like to come back to the minimum energy configuration which is the most stable configuration.

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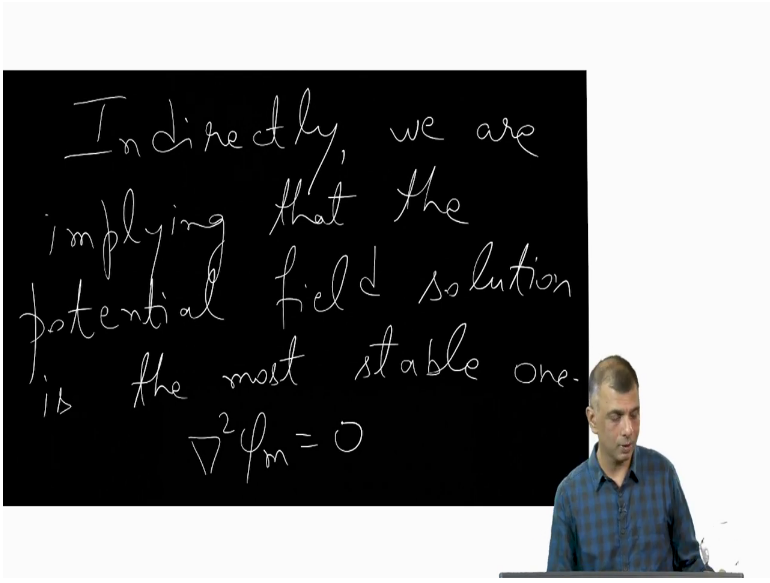

So, therefore, you are essentially kind of indirectly making the statement that the potential. So, indirectly or indirectly, we are implying that the potential field solution is the most stable; is the most stable one of the various solutions that can satisfy a given set of boundary conditions.

Given a certain set of boundary conditions of the various solutions that can satisfy those boundary conditions, the potential field solution which is a solution to this is the most stable one, this is the statement where we are indirectly making.

In some sense, you can see why we talked about the force free solution and the potential solution in the same breath, this is also very closely related to concept of a force free solution because a force free solution is also one where you have no extra Lorentz forces acting on them and so, that is also you would tend to think of that as also an equilibrium solution or a stable solution.

The potential field solution is another kind of equilibrium solution ok. An equilibrium solution in that it is the lowest energy configuration so, it is the configuration that all other configurations will try to come down to, you perturb any other configuration, it will always try to come down to the potential field solution why?

Because the potential field solution is the one which has the minimum energy ok, it is a minimum energy solution and so, so, these are useful concepts to keep in mind and all of these arise from the fact that is curious way in which magnetized plasmas behave ok.

So, when we meet next, we will take up another very important aspect of magnetized fluids. In particular magnetized fluids which are also infinitely conducting, this aspect follows directly from the induction equation that we have already seen, the name of the of the phenomenon that we will discuss is what is called the frozen field approximation ok.

The fact that the field in an infinitely conducting plasma seems to be frozen into the fluid, the fluid flows one-way, the field also flows the same way the fluid you know so, so, the velocity field in some sense tells the magnetic field what is; what to do that is what the frozen field you know concept is all about its also often called Alfven flux freezing theorem and so, it bears a little bit of detailed analysis.

And once we have done that, we will go on to the to an astrophysical application of that to considering astrophysics the phenomenon of dynamos which as the name implies, dynamos are a situations where a small small seed magnetic field can be amplified ok. So, for the time being, we will stop here.

Thank you.