Fluid Dynamics for Astrophysics
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Lecture - 05
Fluid Kinematics – Recap

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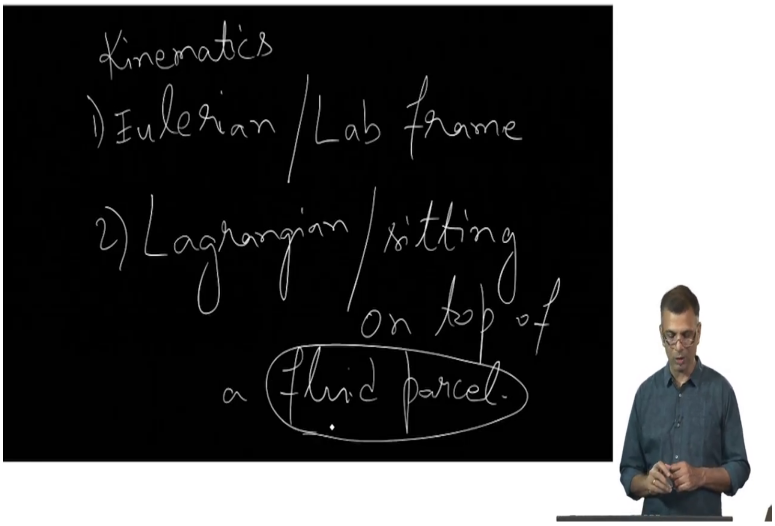

Hi. So, I figured, we would do a quick recap of what we did in the last session, where we talked about Fluid Kinematics. Which if you recall was the study of fluid parcels of the manner in which fluid parcels move, without being too concerned about the forces that are acting on them.

Yeah, characterizing the you know the flow of fluid parcels and in doing so, we you know we mostly concentrated our attention on how to define a streamline and so on so forth. But before that, even before that and this will be very useful for our subsequent work in terms of conservation equations and so on so forth.

We defined two ways of looking at the movement of fluids; one was what is called an Eulerian description or Lab frame description and the other one was a Lagrangian description or sitting which is one in which you are sitting on top of a fluid parcel, yeah. So, the Eulerian description would be the description for fluid as if you are standing outside of the actual fluid, say you are watching the fluid flow by in a pipe.

The Lagrangian description would be what? An observer would observe if he or she were sitting on top of fluid parcel and before we went on, we took a little bit of time to define what a fluid parcel is. It is a parcel of fluid that is small enough so that there can be many many many fluid parcels in a macroscopic volume.

But the same time, it is large enough so that there are many individual molecules or elementary constituents as the case might be, to justify the continuum hypothesis ok. So, this thing this definition of a fluid parcel is a little ambiguous, but nonetheless very important right.

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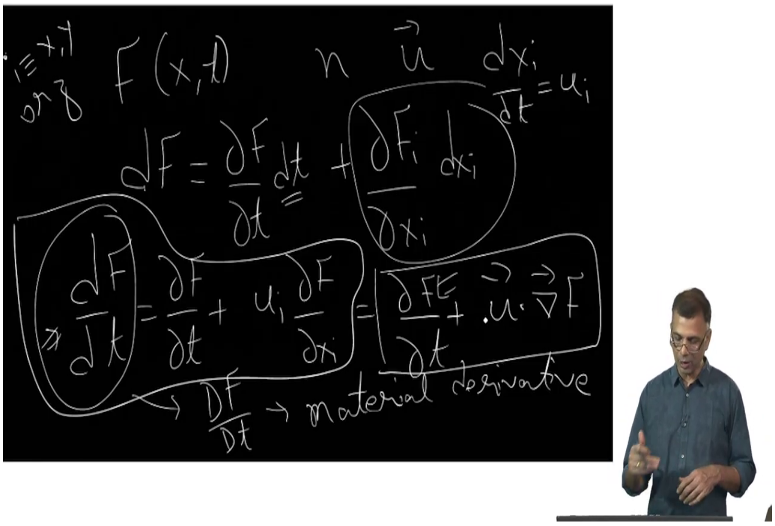

So, we talk in terms of some general function F of t. It could be a scalar function which represents a density or it could be a vector function which represents the velocity of the fluid. I mean in general in fact, I tend to write, I tend to represent velocity by u. So, we let us write it that way.

So, in the two descriptions, the two the Eulerian and in the Lagrangian descriptions are typically related by Galilean frame transformation which involves the fluid velocity and we started out by writing in the Lagrangian frame. Suppose, you are considering a differential change in this quantity F, how is that related to what an a lab frame observer would see?

Well, it would be a true change in F as discerned by the lab frame, a true temporal change in F as discerned by the lab frame observer times of course, d t yeah plus a change in F which is due to the fluid motion yeah and that is what is expressed by this. Yeah. You divide all these you know a d F d t rate of change of F with respect to time as discerned by the Lagrangian observer d F d t.

So, you divide everything by d t. So, this d t goes away and this d x i d t is nothing but the velocity, yeah. Well, there is this really no since I am writing an i, there is no need to there is no need to assign a vector sign here yeah. So, and this i can be i is x, y or z as the case might be right. So, this is essentially or in vector language, u dot gradient of F ok, where in this case F is still a scalar function.

And sometimes people write this as d F d t and this is often called the material derivative. And to emphasize, this is the derivative; there is a rate of change of F as felt by a Lagrangian observer. So, this line represents, this equation represents the this or for that matter, this way of looking at it represents the relation between the Lagrangian rate of change and the Eulerian rate of change yeah.

So, the Eulerian observer can observe can discern a rate of change due to true temporal changes which is represented by this; but also temporal changes that are a by-product of the fact that the fluid is flowing and the fluid is of course flowing only for the lab observer.

When you are sitting on top of a parcel of fluid, it is not flowing right. As in there is no real u [vocalised-noise] for the Lagrangian observer. So, that was one the other important thing the Eulerian and the Lagrangian description that was the other important thing that we introduced.

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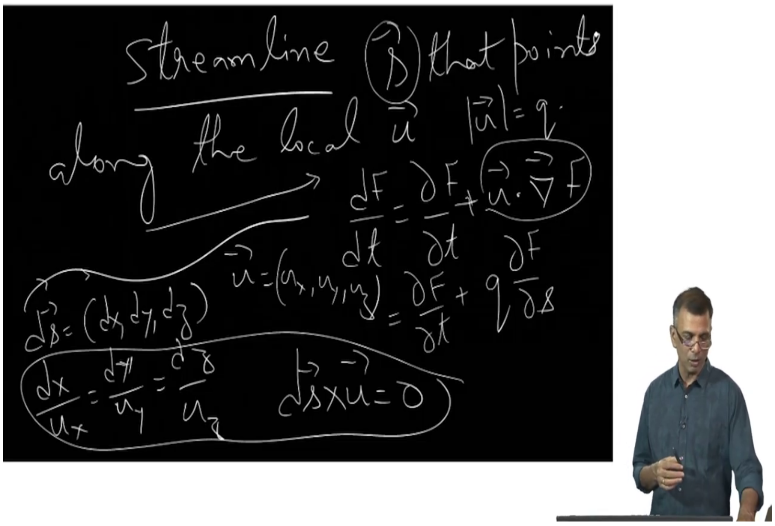

And then, we introduced the concept of a streamline. It is intuitively obvious, a streamline is something that you know you say a colourless fluid is flowing fast and you inject a little bit of red dye or blue dye in there. The path traced out by the by this dye that is a streamline and you can define a streamline coordinate say s that points along, the local direction of u like this, ok.

So, s is like so and the advantage, there is that this material derivative that we wrote down and this is the way you related to sorry yeah. This is greatly simplified when you talk about a streamline coordinate s is greatly simplified and can simply be written as right, where the like that ok.

So, you do not have to worry about this gradient anymore you can simply write it this way. The other thing we remarked was if d s is equal to d x, d y, d z. So, that is what you know this d s means and you have a u which is like so a streamline can be defined either as or equivalently. So, both of these are equivalent definitions of the streamline and I urge you to show this. I urge you to show that these are equivalent definitions of a streamline yeah.

So, this is one problem that you can solve, not terribly difficult. Once you understand that the definition of a streamline is essentially this, the s that points along the local u. We then, showed a couple of pretty pictures, which illustrated which made it obvious you know what a laminar flow is? What a turbulent flow is? A turbulent flow is one in which the streamlines are not nice and regular instead, they are tangled.

It is the kind of streamline that you expect from your faucet, when you turn it on you know high essentially. When you turn it on low, you expect the water to flow out in a nice laminar manner and the streamlines are nice and well-ordered. And if you turn it on high you expect that some kind of instability will set in and the streamlines will get all tangled.

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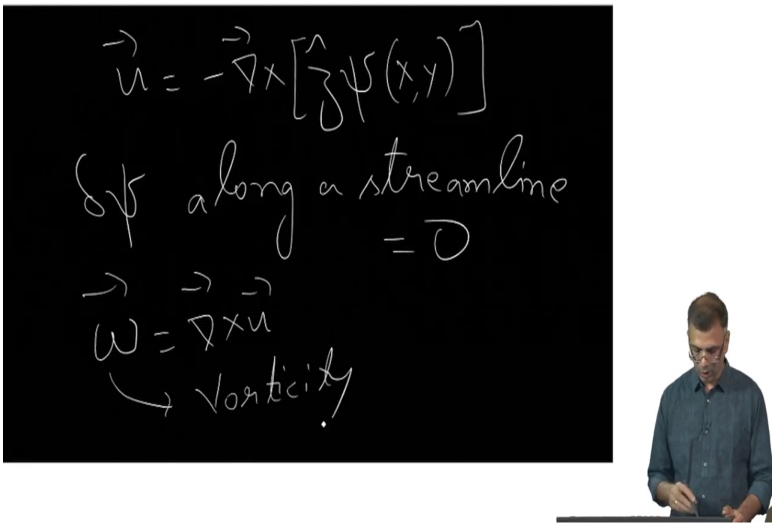

Yeah, and we also introduced something called a stream function which is defined as a something the negative is a matter of definition. So, the stream function is a scalar function psi and this is how it is defined. And the advantage of this definition is that the delta psi along a streamline equals 0 psi is conserved ok. So, that is the merit of this function psi ok. We also define another quantity called the Vorticity, which is simply the curl of the velocity. This is called.

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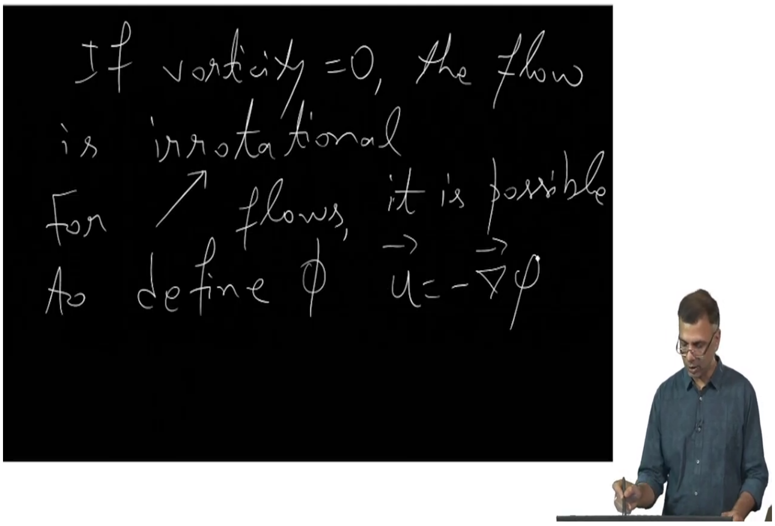

And for a flow for which if the flow is irrotational and by and for irrotational flows, for irrotational flows, one can write it is possible to define a scalar potential phi such that like so. Again, the minus is a matter of convention; and there is a nice little and this is because you know the curl of because of vorticity is essentially the curl of u and the curl of any gradient is always 0.

Especially, the curl of the gradient of a scalar function is 0 by definition and there is a nice relation between the phi, the scalar potential and the psi which is the stream function.

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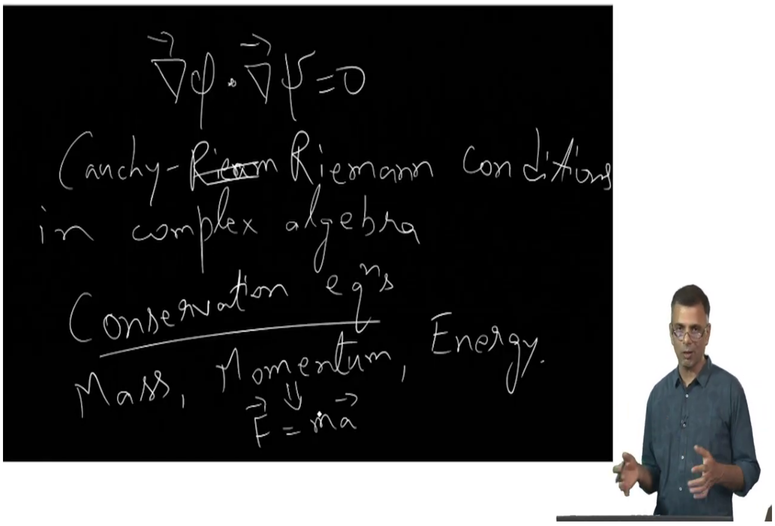

And that is as follows. This is equal to 0 and the advantage is and the advantage I am writing it in introducing functions these ways are that you know the Cauchy Riemann conditions. I am simply saying this, we will see this later. Sorry. This is parallels to the Cauchy Riemann condition in complex algebra. And this is especially useful in describing two dimensional flows, that do not have that are inviscid and also irrotational ok.

So, this is pretty much all we covered up until now in talking about fluid kinematics and from now on, we will start talking about conservation equations. This is what we will start talking about; conservation equations, mass conservation, momentum conservation and to some extent energy conservation.

We will not concern ourselves too much with energy conservation; but you know and just to remind you momentum conservation is nothing but F equals m a. This is just that. That is nothing more. Mass conservation is even more basic is simply says that mass cannot be you know created or destroyed.

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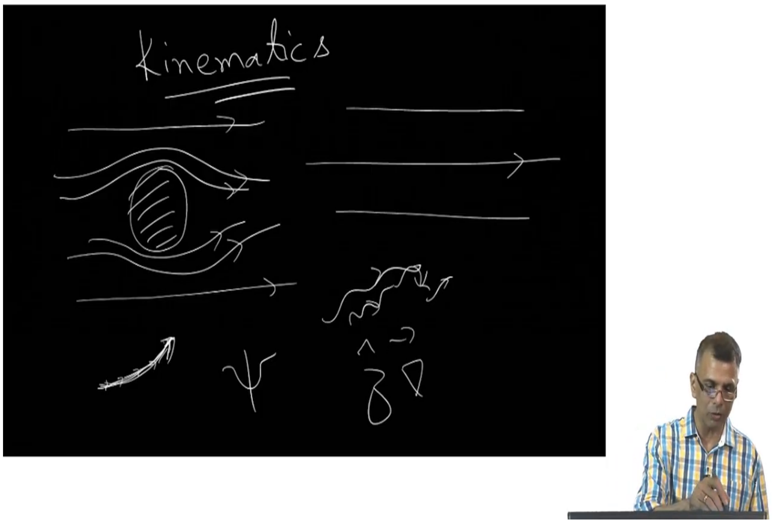

Just recall what we have been doing so far is a study of kinematics, which is in short, it is describing the character of the flow, without being too bothered about what is causing the flow, without being too bothered about the forces acting on a fluid element right. And in doing so, we mainly focused on this thing called a streamline which intuitively you know here is a here is some fluid flowing along a pipe and you say here is a streamline ok.

This is something that is intuitively obvious, here is a sphere embedded in a flow and the stream lines look like that, like this, like this, like this. In other words, the radius of curvature gets progressively smaller as you know you move away from the sphere. So, this is something that is intuitively obvious and when you move very far from the sphere, it is as if the sphere was not there at all.

So, this is what one calls a streamline and the streamline for a turbulent flow is one in which the you know the flow is all tangled and so on so forth yeah. So, essentially you can think of it as a collection of velocity vectors, tiny little velocity vectors like this and like this and like this and like this and you join them together and you get a streamline right.

So, you join the velocity vectors together to get a stream line. So, this is something that we defined mathematically and then, we also introduced the concept of a stream function psi. Something that is there is a function, that is constant. If I recall correctly, it was well. Let me not leave you to go back through the definitions and not bother with it right now.

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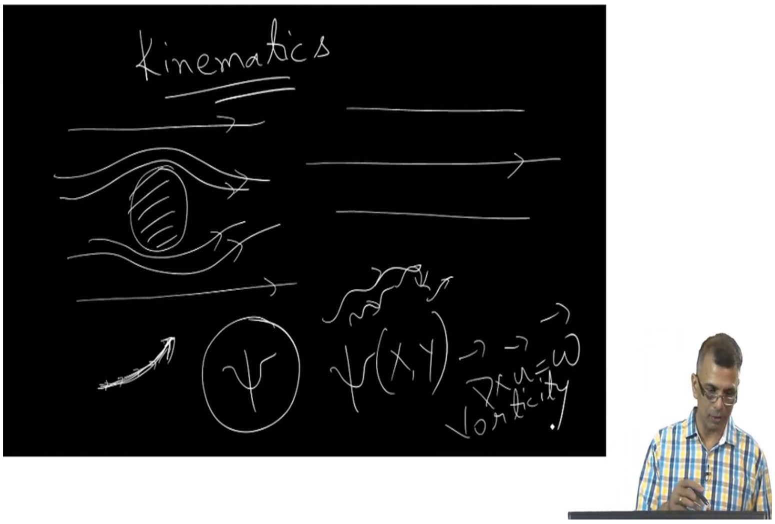

But essentially, we introduce a concept of a stream function. This is a scalar function and it is useful mostly for two-dimensional flows for which you know the flow is confined just to x and y. And so, that the merit of this function is it is something that is conserved along a stream line.

One streamline, one stream function; another streamline, another stream function; it almost labels the stream lines yeah. And then, we also introduce the concept of vorticity which is this is a curl of the velocity vector, the vorticity right. And this tells you how rotational or irrotational the flow is.

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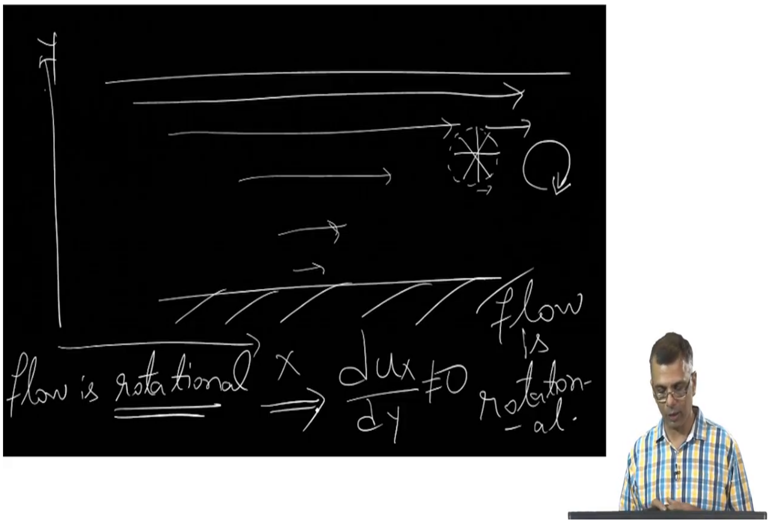

And to just to demonstrate what that means, if for instance, you have a viscous flow right; in other words, here is the bounding surface and here is a free surface of the fluid, yeah and this is a viscous fluid. So, that this would be the x axis and that would be the y axis and there is a viscous fluid so that the x directed velocity at this y would be something like this for instance and as you move down in y, it decreases. That is an example of a flow right.

Now, imagine you placed a little paddle wheel in here, a little wheel yeah. Now with this wheel rotate in this flow, I urge you to think about it and the answer is yes, that is because you know the velocity here is larger than the velocity here. So, this would rotate in this way ok. So, because the because this is true the flow is rotational ok. So, let me write this again here and that is because this is true ok.

In other words, if the flow is rotational, there is a non-zero vorticity ok. We will see that these concepts have immense practical value, when it comes to describing fluids; especially, the flow of inviscid fluids.

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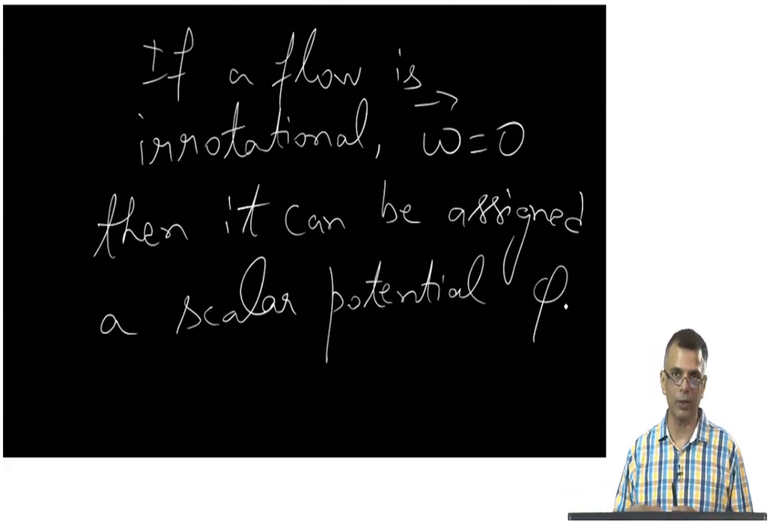

And we also discuss the fact that you know if a flow is irrotational; in other words, i a equals 0, then it can be assigned a scalar potential and that is simply because you know that simply follows from basic vector calculus ok. And we also discussed a few interesting relations between the scalar potential and just one interesting relation between the scalar potential and the stream function.

And we hinted at the fact that this can be I mean the theory of complex variables can be brought to bear; but we did not say why and we will defer it to a little later and I will show you why.

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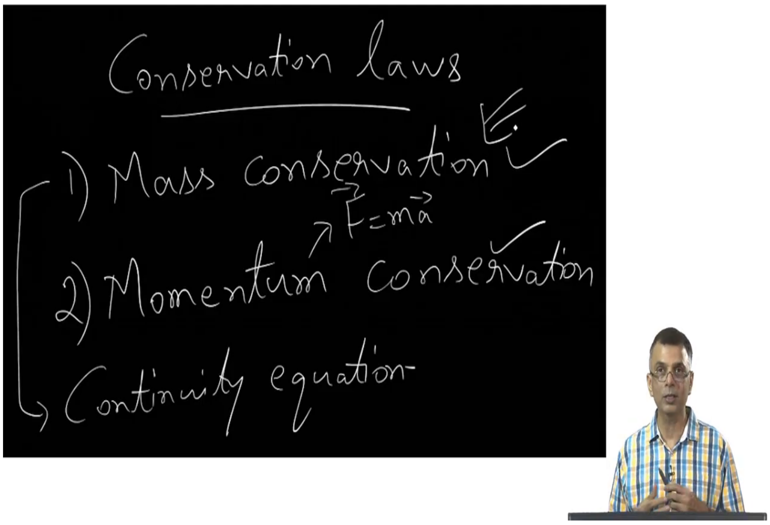

Now, what I want to do from here on is start the discussion of Conservation laws. The first thing, we will do is Mass conservation. Mass is neither created nor destroyed yeah. Second thing, we will do is Momentum conservation ok. So, these are the two major concepts that we will cover going forward. And mass conservation leads to essentially it leads to what is normally referred to as a continuity equation.

Momentum conservation is nothing but F equals m a, that is all momentum conservation is. You are doing a continuity equation in particular F equals m a, you might be familiar with in the context of point particles or you know rigid bodies. Here, you are really writing down F equals m a for a parcel of fluid ok.

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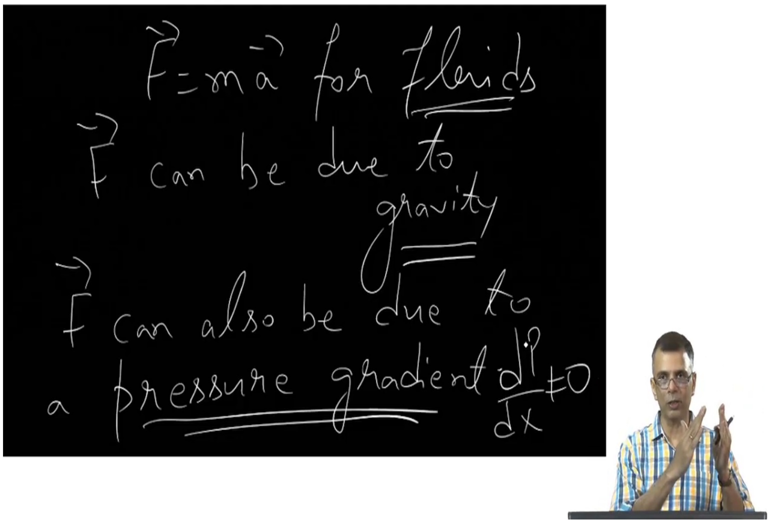

And the one new thing that will arise because we are doing F equals m a for fluids, F the forces acting on a parcel of fluid can be F can be due to for instance gravity. This is something that you are familiar with from the study of point objects. The new thing is the fact that F can also be due to a pressure gradient.

This is new this is something that you would not have encountered while studying point particles. Well, while doing F equals m a for discrete bodies, this is a consequence of; well, this arises only while studying continua in particular fluids right. And as such again, as with pretty much everything to do with fluids, it is intuitively obvious right.

You know for instance, if you have high pressure here and low pressure here, fluid will tend to flow from here to here. What am I saying? High pressure here, low pressure here and there is a I mean these areas are separated by some. In other words, there is a non-zero; in other words, there is a pressure gradient ok.

The fluid is flowing because there is high pressure here and low pressure here right. So, the pressure gradient is causing the fluid to flow. In other words, there is a force on a fluid element due to the pressure gradient, yeah that is causing it to flow; before you know formally writing down the momentum equation and the two guises of the momentum equation for fluids. 1 is called; it is essentially, although I am writing this down as two different things for inviscid fluids.

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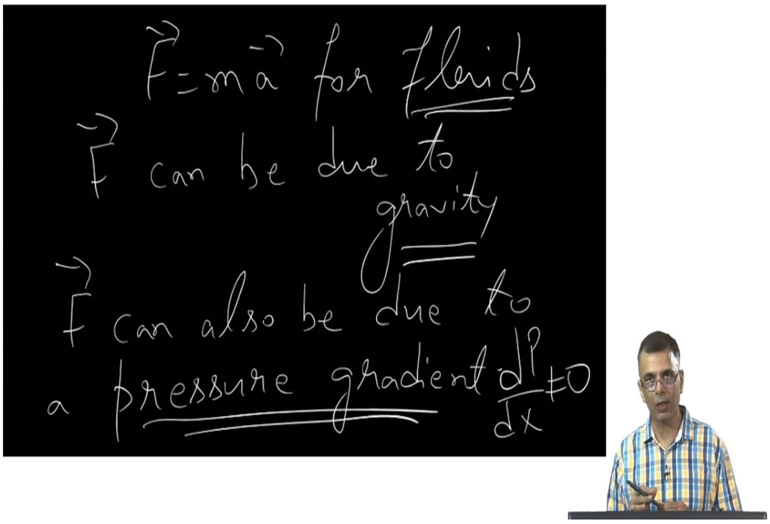

In other words, fluids which for which the viscosity is technically 0 and the other one is the Navier-Stokes equation, where they are both the same. They are both momentum equation except the Euler, it is simply called the Euler equation, when viscosity is not included; it is called the Navier-Stokes equation, when viscosity is included. It is just historical. We will get to that.

But one of the main things I wanted to bring across here is both of these are simply F F equals m a. The major new thing that you will encounter while studying F equals m a for a fluid parcel is that the role that pressure plays ok. So, pressure gradient manifests itself as a force on fluid element and that is something new, that is something you would not have encountered while studying F equals m a as applied to rigid bodies.

So, we will sort of stop here. When we continue next, we will start with conservation laws and we will start with mass conservation. We will see how this leads to you know the continuity equation. So, this is basically the first conservation law, we will encounter. The other thing, I forgot to tell you was that we will you might wonder I mean you know normally one talks about mass conservation, momentum conservation and energy conservation right.

So, why are not we talking so much about energy conservation? The answer is yes; we will talk about energy conservations. It just gets a little hairy, but we will talk a little bit about energy conservation as well. It is important, all these three conservation laws are the cornerstones of physics and so, they apply equally well in this situation. So, we will talk a little bit about energy conservation; but most of our energy, most of our focus will be on the mass and momentum conservation equations.

Thank you.