Fluid Dynamics for Astrophysics
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Lecture - 47
Magnetohydrodynamics [MHD]; Introduction

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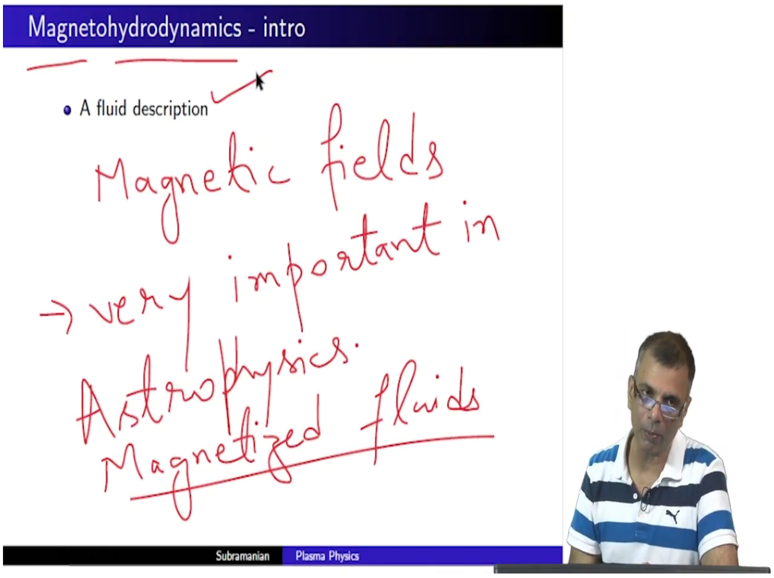

Now, we are we have covered a fair bit of discussion about fluids and we have also said a little bit about applications of Fluid Dynamics and Astrophysics. Now, as we said in the in the beginning of the course, magnetic fields are of great importance in astrophysics ok ah. So, and this is something that we have not really considered much magnetic fields are very important in astrophysics right.

So, now you see the thing is, but we have not considered we have not talked about magnetic fields at all right we have mostly we have almost exclusively been talking about neutral fluids so far right. So, now I think in any kind of discussion to do with fluid dynamics you know as applied to astrophysics I think it is important to consider magnetic fields.

Specifically, so from now on we will be starting to deal with magnetized fluids, fluids which are which have a magnetic field in it and it is important to consider these things a little carefully. So, I will list out what we are talking about and we will proceed systematically. So, essentially from now on for the remainder of the course we will be talking about magnetized fluids and once we establish the basics the basic equation, the basic approximations, and the limitations and so on so forth, we will then go on to starting to you know apply these concepts to astrophysical applications.

So, let us start with the basics first right, this field is called magnetohydrodynamics it is a bit of a mouthful, but you understand why you know what this means, is hydrodynamics right it is hydrodynamics, which is what we have been talking about so far with magnetic fields. Hence the name magnetohydrodynamics and so, the first thing to note is that it is a fluid description ok.

So, all the usual all the usual approximations that we made to define fluids they hold here too.

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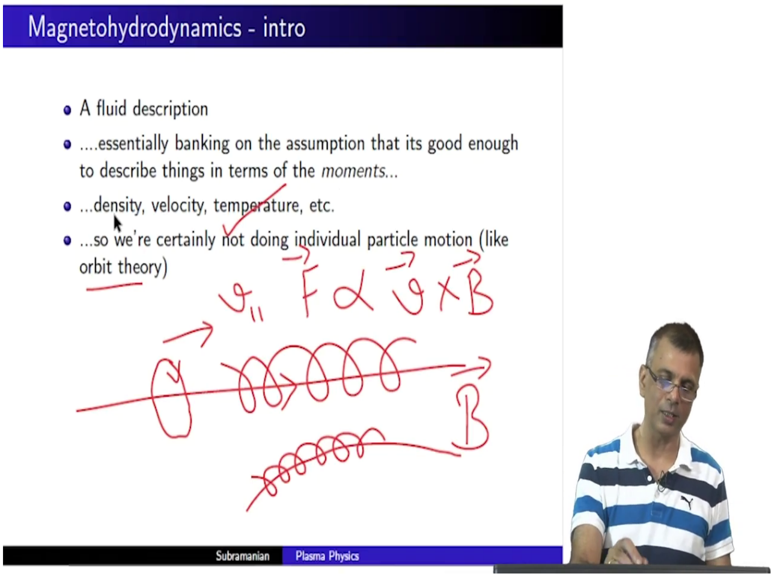

So, it is a fluid description first of all what that means, is that we are essentially banking on the assumption that is it is good enough to describe things in terms of the moments like density velocity temperature and so on so forth right.

You remember this is what we did in dealing with fluids as well, we were considering only the moments of the distribution function such as density which would be the zeroth moment velocity would be the first moment temperature would be related to the second moment and so on so forth. So, it is the same thing here too right.

So, we are certainly not doing individual particle motion, no not at all for instance orbit theory what this means, now this this might be some somewhat unfamiliar this particular word; what this means is that consider a magnetic field yeah consider a magnetic field like so yeah, consider we will normally use the symbol B to denote magnetic fields. So, consider a magnetic field like so

and what does a charge particle do, when it is it encounters a magnetic field, a charged particle is subjected to what is called a Lorentz force. Which is proportional to the velocity of the particle as well as the magnetic field so, that is one thing, but more importantly the direction of the force is perpendicular to both the velocity as well as the magnetic field right.

So, so as a result the charge particle executes a circular motion, we know this right. So, it executes a circular motion with the plane of the circle perpendicular to you know to the magnetic field ok so, this that is one thing. Second thing is of course, if it already had a component of velocity that was parallel to the magnetic field, if it already had a v parallel then that is unaffected by the presence of the magnetic field. So, as a result the particle executes a helix ok.

So, this is what an individual particle would look like and hence the name and these are orbit is ok and so, you know there is an elaborate theory I mean one can go on trying to describe these orbit is for instance what would happen if the magnetic field was curved I mean you know would you need to follow the orbit is all throughout or what or so on so forth. And so this is the purview of orbit theory, but we are not doing orbit theory here we are doing fluids.

It is important to as we go along to recognize and realize what we are doing and what we are not doing. In this case it is important to realize that we are not looking at individual particles and therefore, we are not doing orbit theory that was obvious even from here. We are not doing individual particles we are actually not even doing distribution functions of particles, we are doing we are concentrating only on the moments of the distribution function like density velocity temperature and so on so forth ok.

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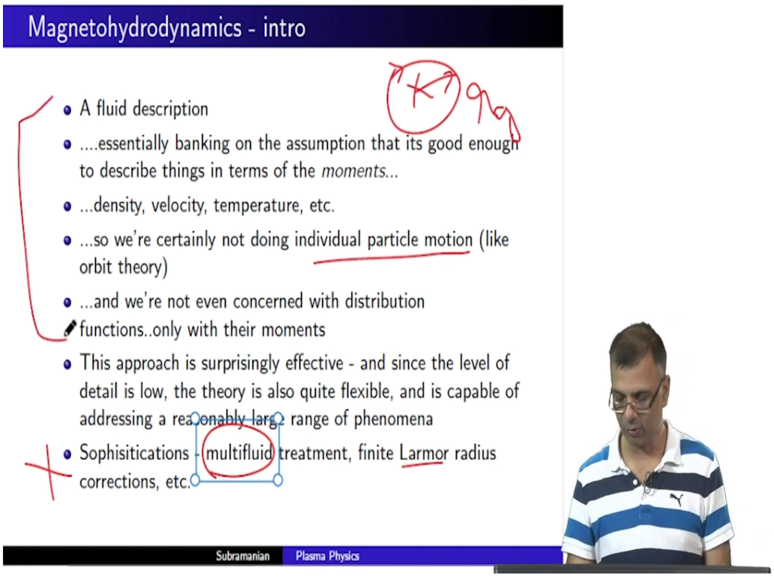

Yeah. So, the other thing is we are not even concerned with the distrib particle distribution functions. We are concerned only with the moments of the distribution function, such as density, velocity, temperature. All of this is I realize a repetition ok I just want you know just want to emphasize this once again because, we are now you know starting to a slightly new field of study that of magnetohydrodynamics. So, it is important to emphasize what is similar to fluids and what is not similar to fluids ok.

Now, although we are not considering individual particles, we are not even considering distribution functions and so on so forth. This approach that of magnetohydrodynamics is surprisingly effective and since the level of detail is low by way of say I mean what I really mean by low is that you know this this theory is incapable of addressing the distribution function it certainly cannot follow individual particles around.

So, it is it is kind of a course it is a coarse grained theory, but because of that the theory is quite flexible and it is capable of addressing a reasonably large range of phenomena is very very useful which is why we study ok.

Hm, there are apart from the basic magnetohydrodynamics you know framework that we will lay out there are sophistications, such as a multifluid treatment for instance one word one might want to you know consider electrons and protons or electrons and ions as two separate fluids ok; an electron fluid and ion fluid that would be a multifluid treatment.

The other thing is even though particle individual particle you know motion is not being considered you know this is what is called a Larmor orbit by the way I should explain this this adjective a little. So, consider a magnetic field that is you know into the plane into the plane of the screen and you know and a charged particle that is gyrating like so, this is called Larmor motion and so, the radius of gyration is called the Larmor radius this would be the Larmor radius.

So, even though we are not considering individual particle motion in it is full glory one can modify a fluid treatment to account for the presence of to account for the fact that the Larmor radius is indeed finite ok; in as such in principle in magnetohydrodynamics there is no Larmor radius at all ok. So, the Larmor radius is technically zero, when you are doing pure magnetohydrodynamics there can be a higher level of sophistication

where we take into account the fact that the Larmor radius is non zero and so, these are called finite Larmor radius corrections so, but these are all you know higher level treatments I just wanted you to be aware of it we will not be doing any of these ok; we will only be concentrating on this much that is all we will be concentrating on ok right.

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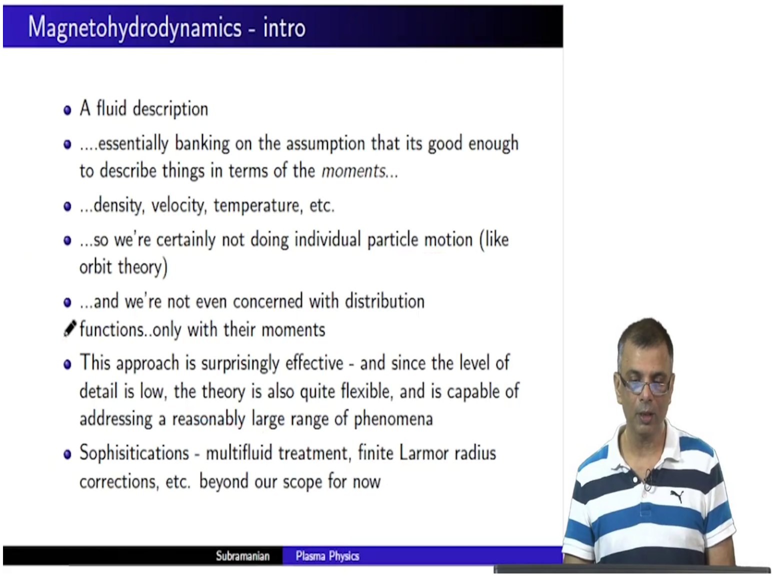

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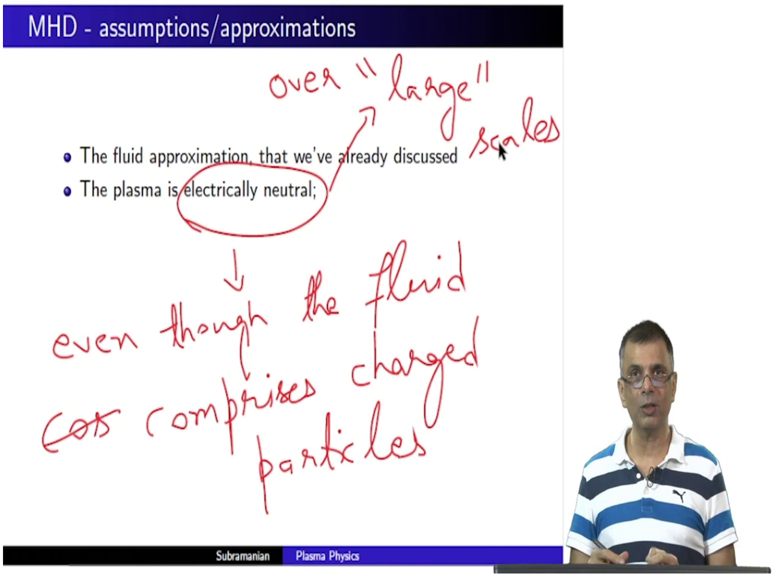

So, now what are the basic assumptions and what are the basic approximations that are made in this approach very very important right, we should we should be very well aware of the basic assumptions and approximations before going ahead. Well one of the main pro assumptions is the fluid approximation that we have already discussed.

The other thing is that the plasma the medium is electrically neutral this is very very important to remember ok; even though even though the fluid comprises charged particles, this is very important to keep in mind. This is something that many students make mistakes in understanding, you see the fluid

now, we are talking about is you know a charged fluid in that it has electrons and protons or electrons and other heavier ions ok. So, the particles are charged ok this is the examples are for instance you know your tube light, the way the tube light works is you have an arc discharge and essentially you have a very high voltage applied across the tube across the neon inside the tube and that creates an arc and that ionizes the gas in there ok.

So, in other words it split is the gas up into electrons and protons or electrons and heavier ions. So, the gas it is self is comprised of charged particles however, the plasma is still electrically neutral on large scales I must say over quote unquote large scales ok. So, on the whole in the bulk the plasma is electrically neutral very important to keep in mind even though the constituents of the plasma are charged particles ok.

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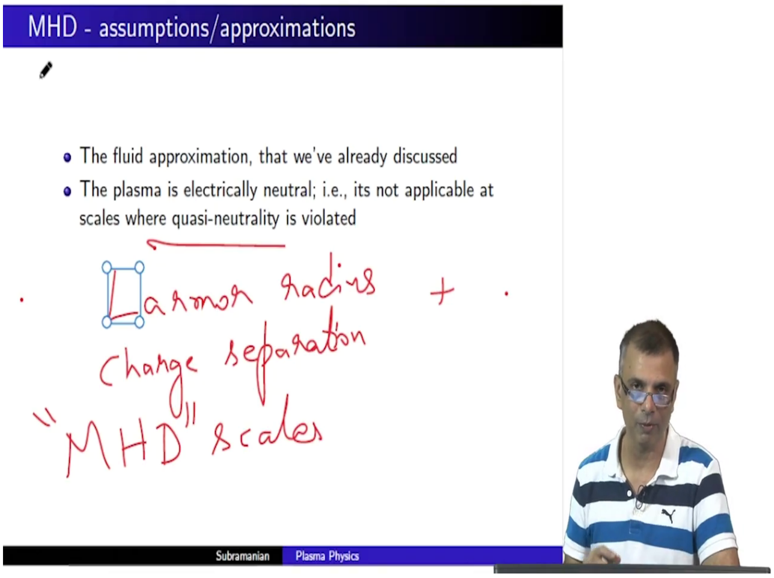

Important to remember, in other words it is not applicable at scales where this quasi neutrality is violated.

So, you know it is, it is, it is, it is like this, you have you know scale say the Larmor radius scale a scale of Larmor radius you would have a scale of charge separation for instance. In other words the kind of scales, where you would say that a positive charge and negative you would be able to make out that a positive charge and negative charge are separated by this amount ok.

And then, you would have the largest of these would be the "MHD" scales. Which is much larger than the Larmor radius even larger than the charge separation scale ok it would be much larger say from here to here or so, very large. So, that you are not able to figure out the fact that there are individual charges in the fluid however, the fact that there are individual charges in the fluids is very important because, that is what leads to this to the fact that you know magnetic fields can be sustained.

So, it is a bit of a you have to keep this slight contradictions in mind ok. So, it is applicable only at large scales at smaller scales where this quasi -neutrality is violated ok; MHD that the theory of magnetohydrodynamics is not applicable very important to keep in mind right.

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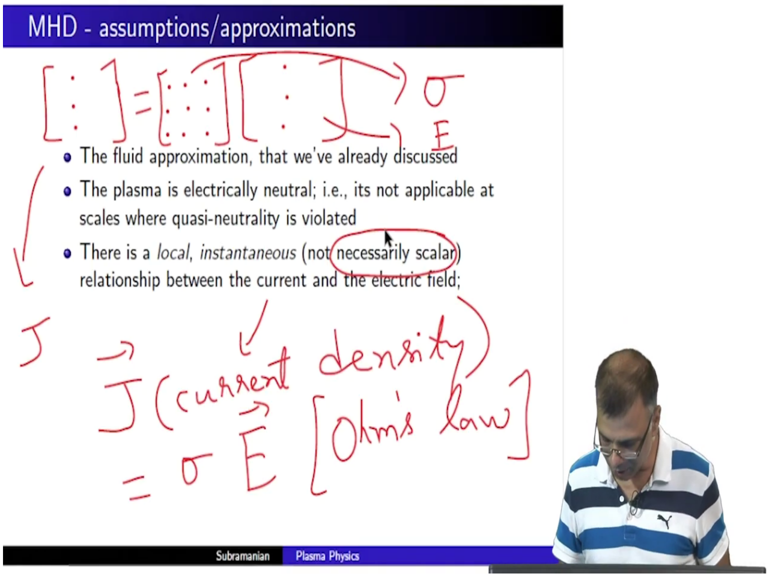

And then there is a local and instantaneous relationship between the current and that and the electric field, technically we are really talking about the current density J which is the current density is equal to some kind of sigma times the electric field.

So, this is, this is the conductivity. So, this is essentially like an ohm's law, except you have seen ohm's law being applied to discrete circuit is where you know v equals i r kind of thing there is an ohm's law in the sense that this is relating J to E ok. So, there is a local and instantaneous relationship between the current and the electric field technically the current density and the electric field and in between you have the sigma ok.

Now, the sigma can be a scalar, but it need not be. You see you have a vector on this side so, J would be a vector and so is the electric field the electric field is also a vector. So, in between what do you have? You could possibly have a matrix.

So, this could be sigma right; this would be E and this would be J J x, J y, J z, E x, E y, E z and in between in principle you can have a matrix right. So, it is, it is possible that that the off diagonal terms of the matrix are all zero and you just have the diagonal terms and they are all equal in which case sigma is just a scalar, but not necessarily ok.

So, so that is what I mean by this so, sigma as such in other words what we are saying is the sigma in one direction is not the same as this one can allow for the fact that the sigma is anisotropic, one can allow for the fact that the conductivity is anisotropic, but we would not really bother with these complications we will just go ahead.

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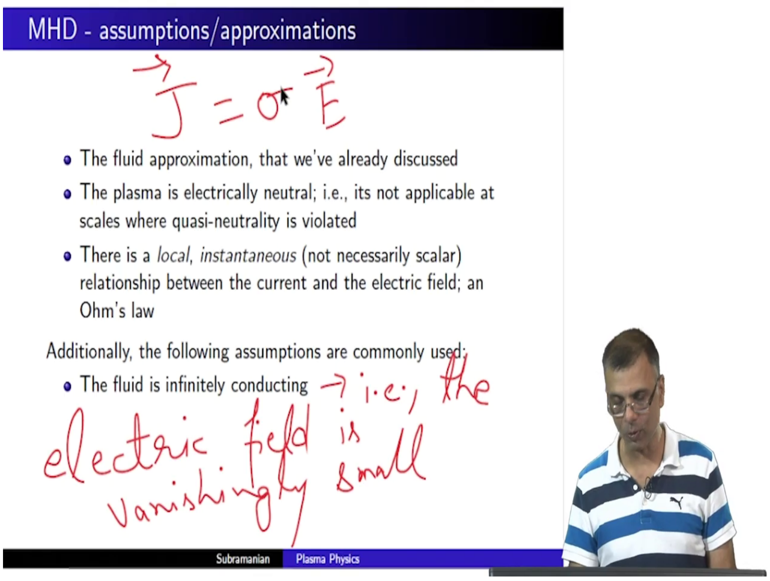

Hm with simply thinking that you know the sigma the conductivity is just a scalar, additionally the following assumptions are commonly used. The fluid is infinitely conducting in other words you see this J equals sigma E this guy this thing sigma is technically infinite.

Now, you might say oh wait a minute there is a problem here if sigma is technically infinite, in which case you know that would that would either mean an infinite current or a vanishingly small electric field yes there are some problems here ok; there are some issues here, but keep this in mind and we will come back to it technically what it means is that the electric field i e, the electric field is vanishingly small.

So, you see this is one way you can satisfy the requirement that sigma is infinite right. If the electric field is vanishingly small even with a finite current density you can have you know an infinite or a arbitrarily large conductivity you can you can manage this right. So, so this is another important assumption in magnetohydrodynamics that this fluid is infinitely con conducting equivalently the electric field is vanishingly small essentially zero.

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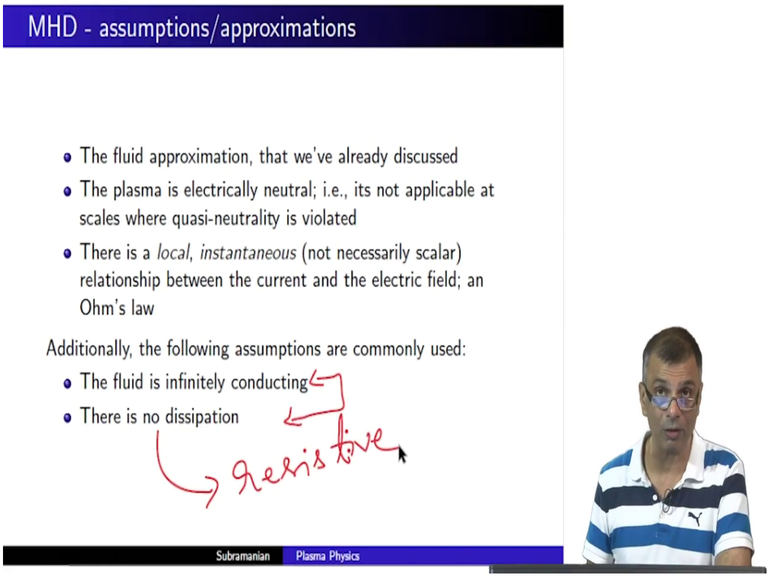

There is no dissipation, in other words it ties with this infinite conductivity means almost zero resistivity so, there is no resistive dissipation ok. So, I should probably say there is no resistive dissipation ok there is no resistive dissipation so, this is another important thing to note.

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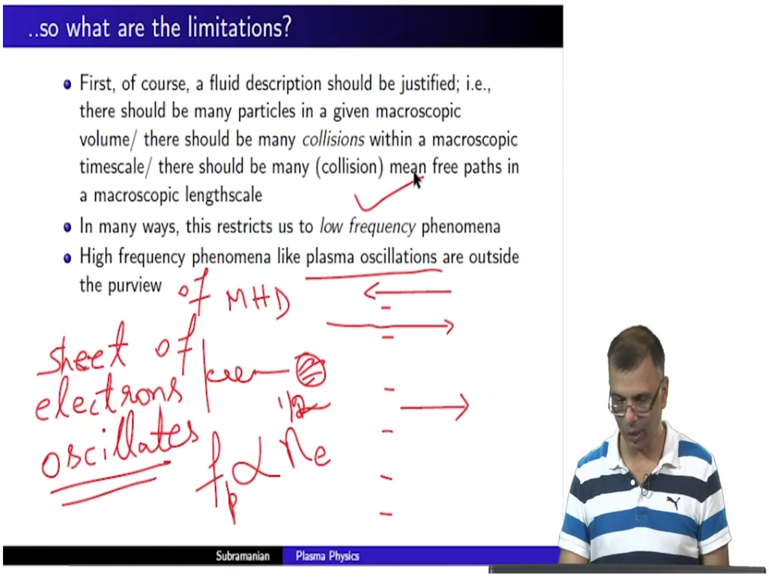

So, ok so these are the assumptions of magnetichydrodynamics so within these assumptions what are the limitations? So, this this is I mean we said that this is what we are going to consider ok we are not going to consider a theory that is any more sophisticated than this. So now, it is important to ask before launching into the details of the theory it is important to ask what are the limitations of this theory? Well first of all obviously

a fluid description should be justified in that there should be many particles within the within a given macroscopic volume, you know the usual fluid thing equivalently there should be many collisions between a given macroscopic time scale right.

In other words, whatever time scale of variation you are considering it should be long enough. So, that there are lots and lots of inter particle collisions ok or there should be many collision mean free paths within a macroscopic time scale. These are all the familiar fluid you know restrictions that that we know this is nothing new we elaborated on this towards the beginning of the course so, that holds now too ok, so that is number one.

In many ways, this restricts us to low frequency phenomena ok. So, this is something to in other words high frequency phenomena like plasma oscillations are outside the purview of MHD. Now, it is important since we have not you know encountered this term before what are plasma oscillations right; it is like this consider for instance we are talking about you know fluid which comprises of charged particles ok, but then you know the fluid is infinitely conducting and so on so forth.

So, essentially you see in in any system let us simply consider the fluid to be comprised of you know electrons and protons. Protons are very heavy they are 2000 times more massive than the electrons right. So, it is essentially any mobility should be you know assigned only to the electrons ok. So, electrons are almost infinitely mobile whereas, the protons are just sitting in their place , consider in you know and we said that the plasma is large scale electrically neutral right.

Now, consider a sheet of electrons like this ok there are may many sheets many such sheets, but consider a sheet of electrons that is displaced to the side for some reason ok; for some reason I decide to displace the sheet of electrons, now what happens ok.

There is another sheet of electrons at it is original place so, that will repel it, but there is also you know there are also protons here there are electrons and protons right. So, you displace this sheet of electrons and that is there is an attractive force that brings it back to it is that tries to bring it back to it is original position ok.

And what is more, it might overshoot it is original position right and then it is subject to the attractive force yet again ok and so, what happens is there is an oscillation. So, the sheet of electrons oscillates much like much like you know mass attached to spring ok; the spring is essentially the coulomb attraction ok. So, you have what you do is you displace the sheet of electron, either to the right or to the left either way ok.

It is attracted back by the positive charges that are always there, but they are kind of immobile ok because, they the protons are so heavy that they are kind of immobile right. It swings back, but it does not stop at it is original equilibrium position it overshoots and then it swings back again and so on so forth, it is essentially simple harmonic motion and these are what are called plasma oscillations ok.

And the frequency of plasma oscillations the frequency of plasma oscillations is proportional to the square root one half power of the electron density ok and these are what are plasma oscillations, but these are high frequency phenomena. MHD cannot deal with these plasma oscillations it cannot resolve this plasma oscillations. The frequencies at which the temporal frequencies at which magnetohydrodynamics is applicable is well above the plasma oscillation frequency.

So, plasma oscillations the MHD theory the magnetohydrodynamic theory is blind to plasma oscillations it cannot resolve ok. So, it is we are really restricted only to low frequency phenomena frequencies that are in physics you always whenever you say you use words like high or low you should always you know say with respect to what well low frequency with respect to the frequency of plasma oscillations for instance ok.

Plasma oscillations are considered to be high frequency phenomena so, so plasma oscillations are very interesting phenomena, but this is something that cannot be captured by this theory ok.

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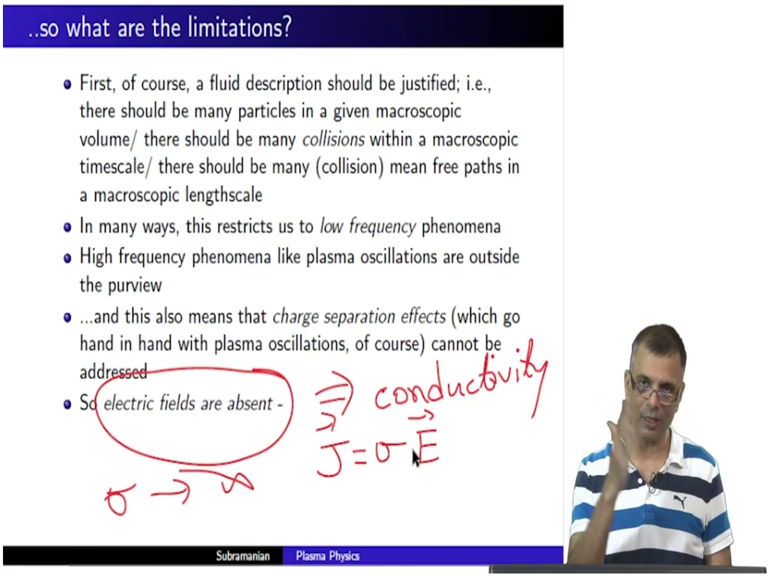

This also means the charge separation effects, which go hand in hand with plasma oscillations of course, you know like we said you know the whole point of plasma oscillations is that charges are being separated from the from their equilibrium positions you know displace a sheet of negative charges from the equilibrium position and because, of the fact that it is separated from it is equilibrium position there is an electrostatic attraction towards it is equilibrium position and then it comes back and then oscillates.

So, charge separation effects cannot be dealt with in magnetohydrodynamics so, this is another thing right yeah they cannot be addressed.

The other thing is that very very important electric fields are absent ok, this is a direct consequence of the fact that the conductivity sigma tends to infinity right; and the conductivity is where you get it is like this this is the ohms law that we discussed earlier. So, this conductivity tends to infinity. So, that is why electric fields are absent, you can look at it in this in the following way which we have already discussed we can say, well

if the fluid is infinitely conducting and you insist on a finite current density in that case there is only one way out the electric fields have to be very they have to be vanishingly small that is one way of looking at it. The other way of looking at it is that the way you get infinite conductivity is to you know assign infinite mobility to the electrons, in other words the electrons are just like in a metal in fact, plasma you know much of this applies to metals as well. Just like in a metal you have a sea of free charges and they are very very mobile right.

So, now what happens if you introduce an electric field in between what does an electric field do it actually charges and it makes them move right; charges move in response to an electric field charges obey the directive of an electric field that is what charges do right. Now, what happens is the moment you since charges are infinitely mobile somewhat like you know an assemble assemblage of marbles with almost zero friction right.

So, you move one of the marbles a little bit that is what an electric field does right; well immediately all the other marbles from everywhere will rush in ok so, so as to neutralize this electric field. The electric field will be immediately shorted by shorted I mean it will be made to go to zero ok; why this is a direct consequence of the almost infinite mobility of the electrons which goes hand in hand with the infinite conductivity.

So, electric fields cannot exist inside of an infinitely conducting fluid. So, electric fields are essentially absent.

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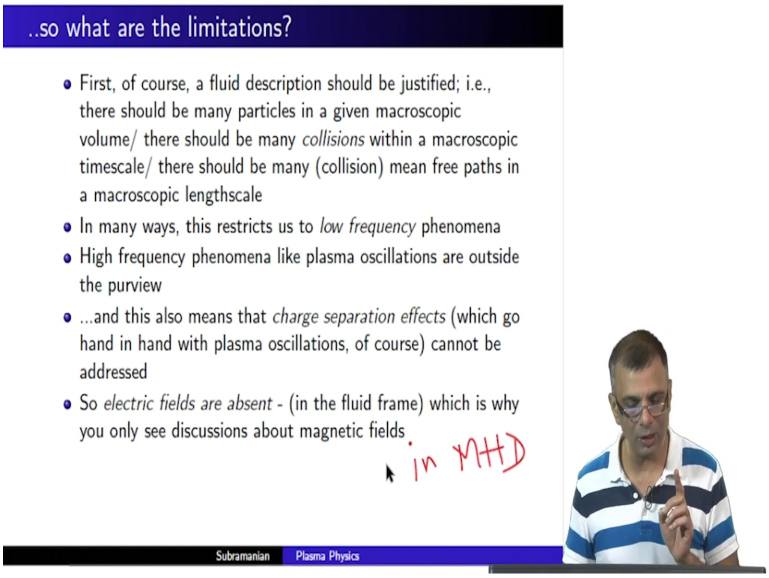

In the fluid frame very very important the frame is important ok, as long as you are inside you are immersed in the fluid as long as you are in the fluid frame you cannot observe any electric fields ok. But, you if you are standing outside of the fluid frame if you are a lab observer and you are watching this charge fluid go by you will you know discern an electric field we will come to that in a minute.

But right now, we are talking about being inside of the fluid inside the fluid inside this highly charged fluid because, the fluid is infinitely conducting you cannot have any electric fields.

Which is also why in MHD I should say in MHD you see only discussions about magnetic fields in magnetohydrodynamics, this this is a question that many people sort of you know miss this is a point that many people miss you go on you look at the equations of MHD so on so forth, you see that only magnetic fields are being considered and you forget about these basic facts.

After everything is done someone asks you, but wait a minute how about electric fields, why did you why did electric fields never make an appearance in your equations this is the reason. Because, electric fields are by definition equal to zero in in the frame of the infinitely conducting fluid therefore, in magnetohydrodynamics since the whole basis of magnetohydrodynamics is about infinite conductivity you only see discussions about magnetic fields.

So, these are the various limitations of the theory and; when we proceed next we will lay down the starting equations and we will go from there. So, that is all for the time being

Thank you.