Fluid Dynamics for Astrophysics

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Lecture - 45
Spherical blast waves: Sedov- Taylor solution

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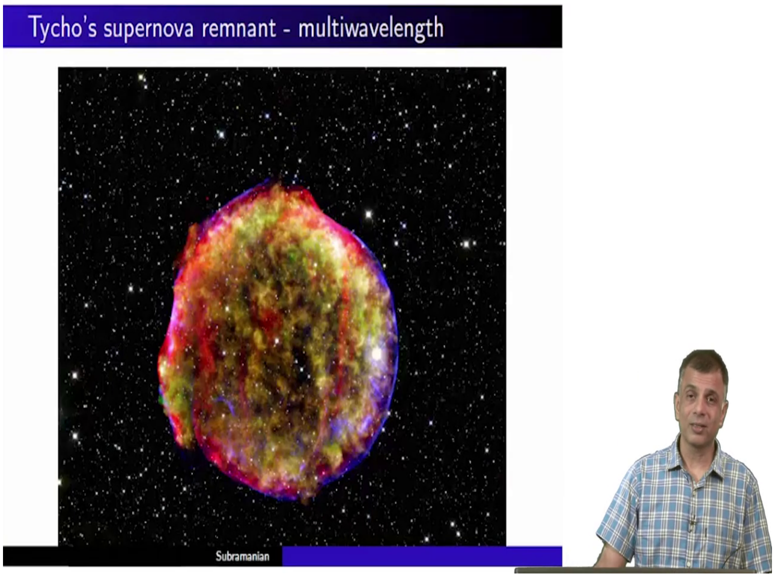

Yes. So, we are back, and we will resume our discussion of supernovae shocks, yeah. And it is always good to in astrophysics which gives us so many pretty pictures to restart our discussion from the last pretty picture that we saw. So, this is what we had seen some time ago, a multi-wavelength picture of Tycho supernova remnant one such spectacular.

So, this is not the supernova itself, ok. This is the supernova remnant. In other words, the supernova would have been somewhere in the center and it would have gone off, and it has passed. And for reasons that we will see it set off a blast wave and the blast wave propagated quasi spherically, ok in a roughly spherical manner, heating the material inside it to varying degrees, ok.

And so and the heated material emits at various wavelengths. So, what this is a collage of images at various wavelengths. None at visible wavelengths, but they I mean what you see here is a false color image, ok. This is a multi-wavelength image. You see, what happens is you have different instruments capturing different wavelengths, for visible you will have one kind of one kind of CCD, for X-rays you will have a completely different instrument, ok.

So, you will have an image pixels and these pixels are super posed together to give you this incredibly pretty picture. And the different colors you see here represent different wavelengths, not in the optical, that is all I want to say. So, the main thing you should take away from this picture is that you know there are different kind, there are different wavelengths present indicative of the fact that there are different temperatures present as the shock wave has passed through.

So, all of this would represent shocked material, material that has experienced a shock wave passing through it. The other most important thing is the incredibly quasi spherical, incredibly near spherical nature of this front here, ok. It is almost as if you can actually; this blue line represents. So, let me show you another picture which shows, yeah.

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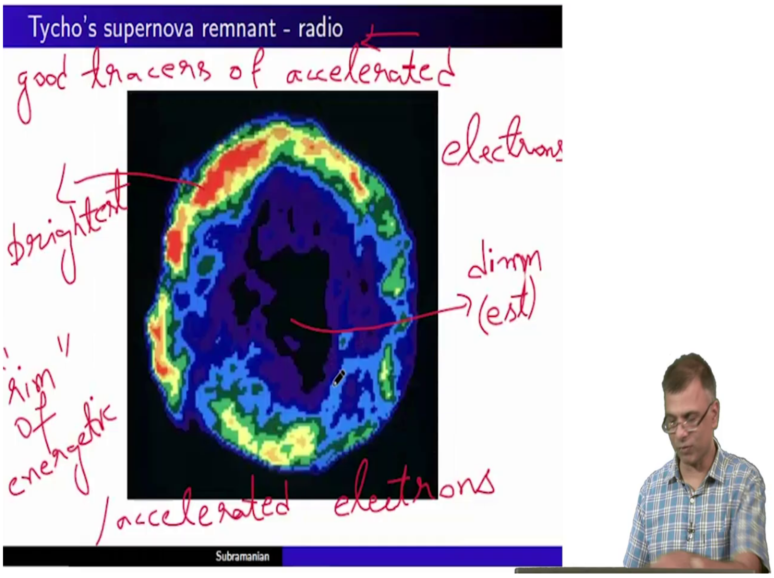

So, this is the same object in radio, at radio wavelengths. Now, radio wavelengths the specialty about radio wavelengths is that they are very good tracers of energetic electrons, of accelerated electrons, ok. So, they are good tracers of accelerated electrons that is what radio wavelengths are good for.

And this picture is essentially showing you the color code is that this would be, the red would be brightest or yeah brightest essentially and here the deep blue would be diminished, ok. So, what this is telling you is that the largest concentrate of concentration indirectly, indirectly of course; what this picture is telling you is that the largest concentration of accelerated electrons are to be found roughly on the rim here.

It is not very, it is not terribly, spherically symmetric I mean you know this green is somewhere in between red and blue, right. So, here the concentration of accelerated electrons is somewhat lower than what it is here, but still it is definitely higher than it is in the center. So, what you have is a rim; what this picture is essentially showing you is a rim, ok, this kind of a rim of energetic or accelerated electrons that is what this picture is essentially telling you.

This is a rim of accelerated electrons. And as we have discussed many times, the utility of shocks in astrophysics, the reason we are interested in shocks in astrophysical context is that they are very good agents for accelerated electrons, for accelerating electrons, for taking electrons from a thermal pool and accelerating them into non-thermal electrons.

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And here you see an evidence, (Refer Time: 05:42) most likely refers to the constellation it was found in and so on so forth, ok. This again is a multi-colored image and the main thing is a multi-color meaning, multi-wavelength image and then the main thing to note here is the incredible spherical symmetry, ok. So, having seen these pretty pictures let us proceed to try to understand what is behind them, ok, right.

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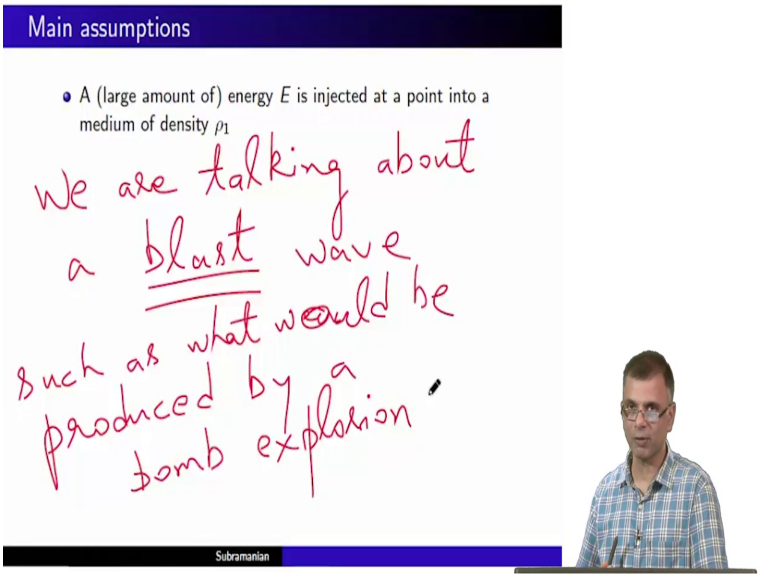

So, the main assumptions, like we said we are talking about a blast wave, ok. We are blast wave the kind that would be produced due to the injection of a large amount of energy, right, for large amount of energy at a point, such as what would be produced by a bomb and bomb explosions are very very similar to supernova explosions, ok.

So, let us consider and so this is different from a piston driven wave. A large amount of energy injected into a point, ok mathematically speaking an ideal point of course, you know a large amount of energy is never injected into a you know point with 0 dimensions is always. But still in practice what this means is that there is a large amount of energy injected into a very small region, ok. The density of the medium into which this energy is injected is say rho 1.

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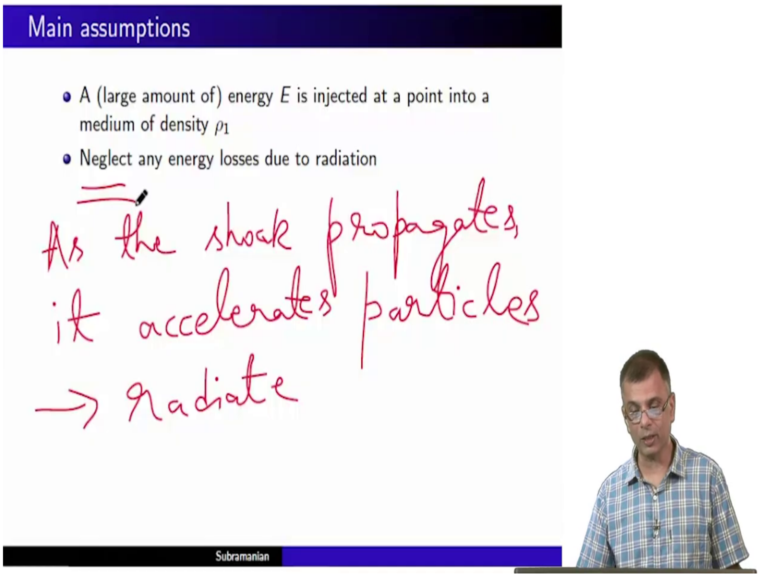

We neglect any energy losses due to radiation. In other words, as the shock propagates, ok, as the shock propagates it accelerates particles, right and these particles will radiate and they radiate; we neglect these accelerated particles will radiate, right. They radiate energy which is how we observe these things, right. What we do here is we neglect we neglect any energy losses due to radiation. In other words, the E, this E that we talk about here is conserved.

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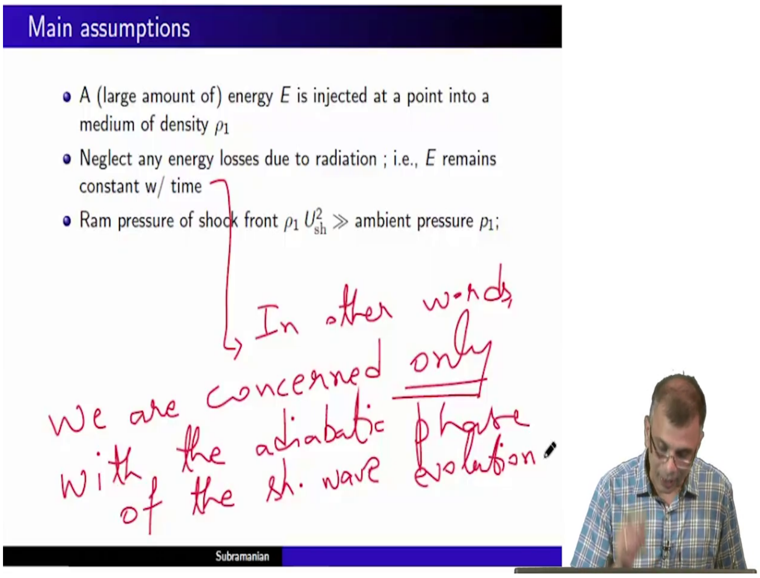

The E remains constant with time. This is called the adiabatic assumption, ok and so this is good for supernova shockwaves up for a certain phase of evolution, ok. In other words, we are investigating, we are concerned with only with the adiabatic or energy conserving the adiabatic phase of the shock wave evolution.

This is not always true. As the shock wave propagates it is like a snow plow, or you know, so it takes snow in front of it or it takes it is as if you are you know a snow plow or maybe one of these one of these mud, you know one of these machines that you see that that gathers mud and pushes it out.

So, more and more material, more and more interstellar material is accumulated in front of the shock wave. For a while the shock wave does not care, ok. It just carries on because the amount of interstellar material that is accumulated in front is very very small, ok. But, beyond the point that the amount of material that it accumulates becomes significant, and it slows the shock down, ok.

What we are investigating here does not concern that phase. It also and, also beyond a certain phase of evolution, beyond a certain distance, the energy losses due to radiate due to radiation from the accelerated particles starts to matter.

Again we are investigating the part of the evolution of the shock wave, which does not take either of these effects into consideration, which in the in other words we assume that if you inject an amount of energy E into a very small region that E remains constant, that E is conserved, ok. It does not decrease with time due to such effects, ok.

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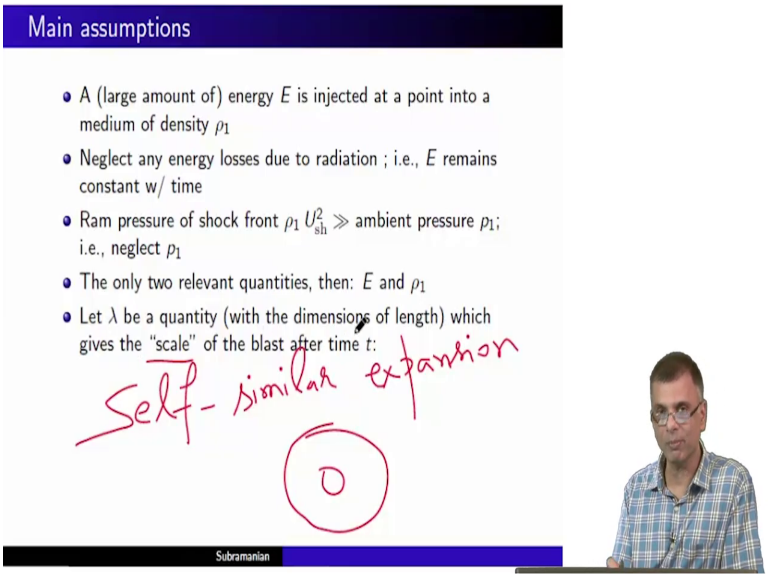

So, the other important thing is that the ram pressure of the shock front rho 1 U shock squared this thing is much larger than the ambient pressure. In other words, you know here is the shock front, it is got a certain amount of rho v squared, rho 1 rho is just rho 1 because that is a density of the medium through, which the shock wave is propagating.

So, the rho v squared is much much larger than the pressure outside. It really does not care about the pressure outside. It is almost as if there is a vacuum outside on the other side of the shock, ok. So, that is what; that is what this means, ok. You just neglect the ambient pressure on the other side, yeah.

So, well actually the ambient pressure of the medium through, which the shock is propagating more, so the density of the medium is rho 1 and the pressure of the medium is p 1, but the p 1 really does not matter at all, ok. It is the kinetic energy of the shock front which dominates, right.

Therefore, there are only two relevant quantities then E and p 1, E and rho 1; p 1 does not matter and E does not decrease with time. So, that we are left only with two and this is very very important, ok. This is one of the main building blocks on which this brilliantly simple theory is constructed.

So, there are only two relevant quantities E and rho 1. And now, remember that we made use of this word, we mentioned this word self-similar expansion. In other words, you have a circle at time say t equals t 1 and at t equals t 2 you have the circle expanding like this. It is as if the shape remains the same with time, ok. It is simply as if this fellow has, at a later time it is simply an enlarged version of what it used to be at a at an earlier time. So, what you; what.

So, all that matters is really the scale of the blast, ok, some kind of some kind of a multiplying factor, ok. So, all you need to know is after time, say after a certain time is elapsed 10 seconds or 10 years or whatever, as long as I know that the expansion is self-similar, ok I do not need to worry about the change in the shape of the of the shock front. I know that the shape of the shock front is the same as what it was at t equals say t 1, ok.

All I need to know is how much larger is it. In other words, what scale should I multiply it with, ok is it 10 times larger, is it is it hundred times larger, that is the only thing I need to know. So, and that is a consequence of this assumption, the assumption that the blast wave expands self-similarly, ok. So, therefore, let us now try to construct a quantity lambda with the dimensions of length, which gives the scale of the blast after time t, ok.

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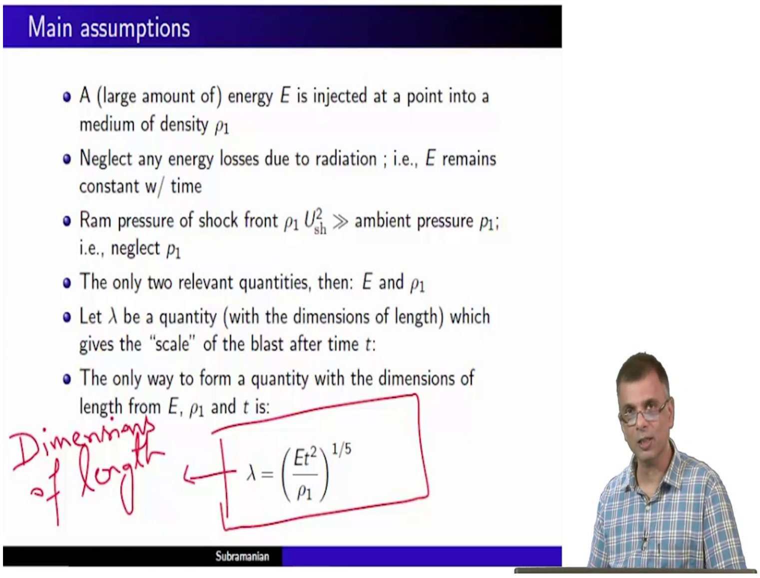

Now, I have only; so we said that we only have two relevant quantities E and rho 1 that is not entirely true, we also have at time t. So, we are left with the task of constructing a quantity lambda with dimensions of length from E, rho 1, and t. So, what we need to do is we need to play around with this, should it be E raised to 2 times rho raised to one-third times t raised to whatever, what combination of these three quantities will give me something with the dimensions of length.

And turns out that there is only one combination and that is this. This is the only way. You can play around as much as you want, this is the only way you can construct you know a quantity with dimensions of length, this has dimensions of length physical length, centimeter or whatever, ok.

This is the only way you can construct a quantity with dimensions of length from energy, time, and density, ok. So, that is very very important, and this is to be kept in mind, ok.

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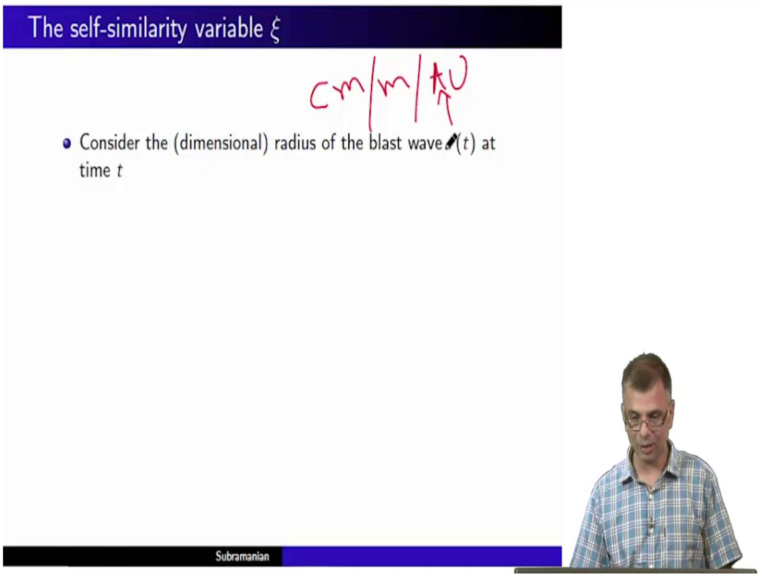

Therefore, now how about a non-dimensionalized distance, right. So, here consider the dimensional radius of the blast wave r t at time t say you know this would be so many you know centimeters or meter or you know astronomical units or whatever, ok. So, this has units of length; so many centimeters or so many meters or so many astronomical units, at time t. So, this you know this would be a physical you know radius.

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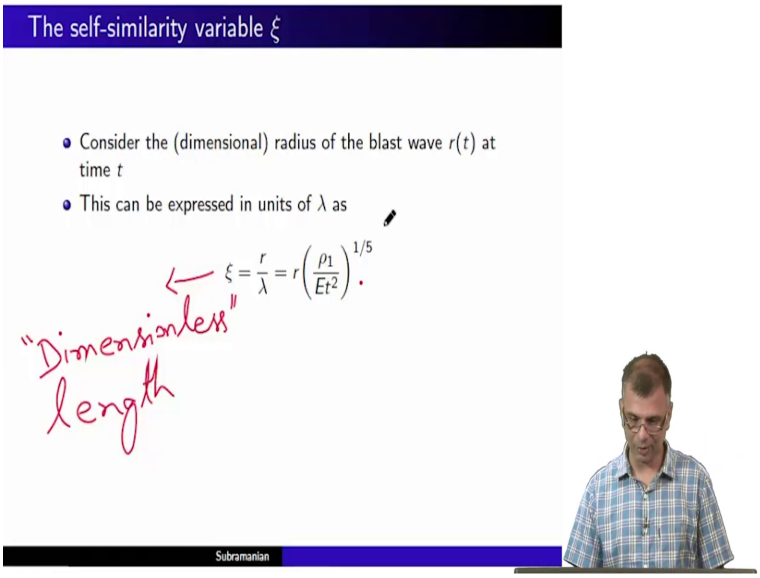

And now how about if I did not want to want to express it in terms of centimeters or whatever it is a huge number, I would like to express it in units of this lambda of this lambda, ok. So, I simply write, I construct a new dimensionless variable this is essentially a dimensionless length, this is psi, ok clearly, right.

So, r has dimensions of length and lambda by definition has dimensions of length. So, you divide the two and it has no dimensions. Nonetheless, it is telling us something about the scale, right. So, I simply take the definition of lambda that was in the previous slide and I multiply r, so r over lambda is r times rho 1 over E t squared raised to one-fifth, ok. So, this is extremely simple. And let us see how far this can take us, right.

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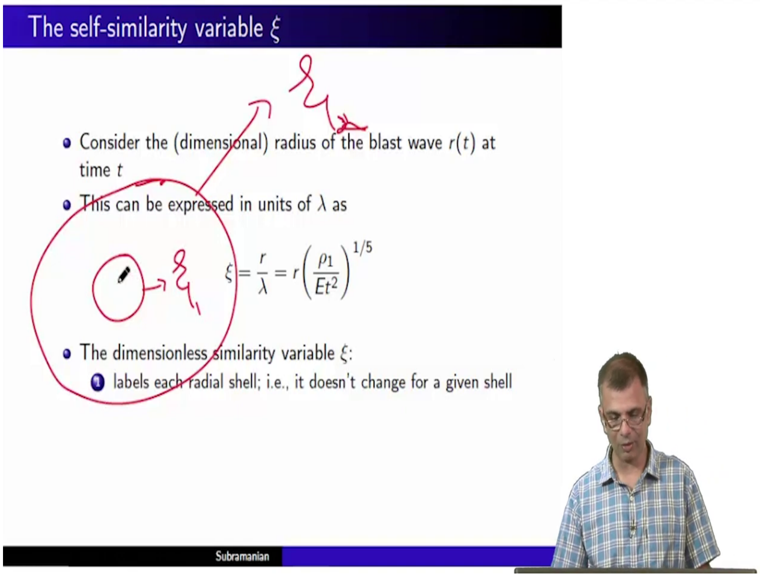

So, the dimensionless similarity variable psi what it does is it labels each radial shell, ok. So, each shell, this has a certain label. This is labeled by sum psi say psi 1, ok psi 1. After some time it is expanded like this and this has another label psi 2, this would be 1, and so on so forth.

So, as long as I know psi 1, I know everything about the shape of the shock at a certain time and that is it. So, psi essentially it labels each radial shell. It does not change for a given shell, all over the shell you know I have the same value of psi 1. Psi changes only when the shell changes, ok, right.

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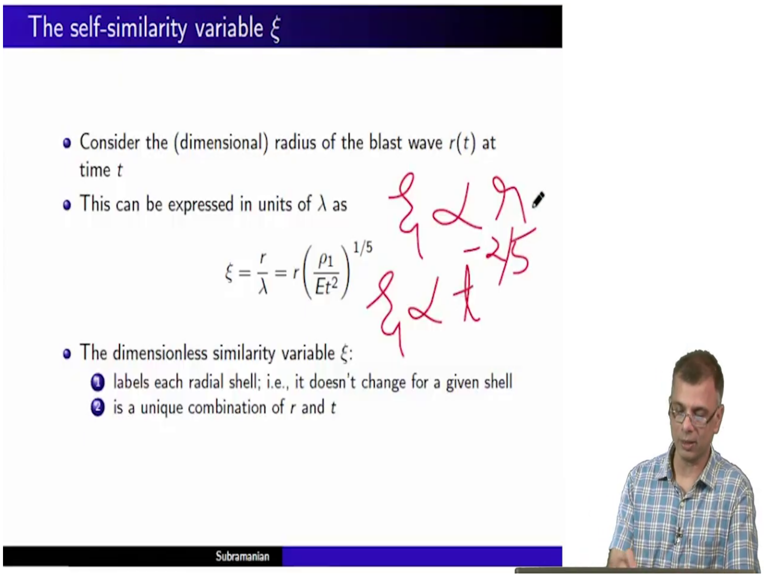

It is a unique combination of r and t as you can see. It is a unique combination of r and t. Psi is directly proportional to r and also psi is proportional to t raised to minus two-fifth, right that just follows from here, ok.

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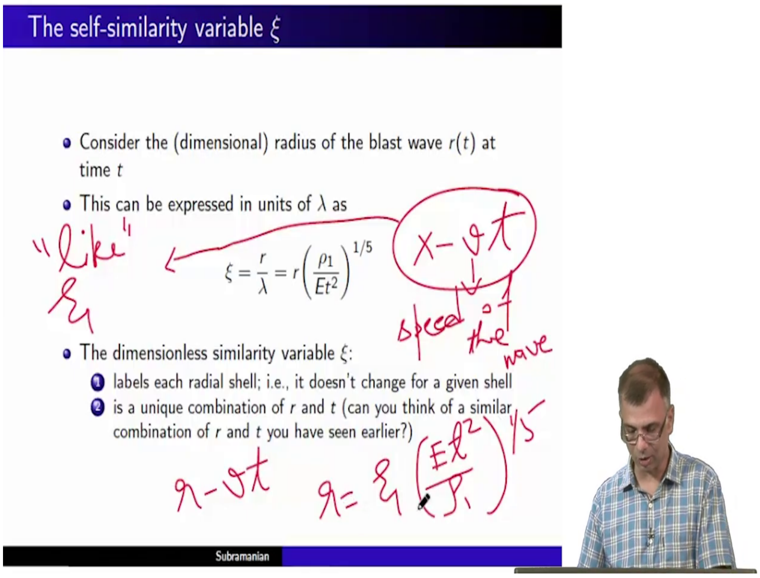

Can you think of a similar combination of r and t that you have seen earlier? How about r minus vt? Ok. When you write the wave equation you know this is also another self-similar say you know, you remember I mean one-dimensionless of this, solution of a one-dimensional wave equation is expressed as a function of x minus vt, where v is the speed of the wave. Everything the entire wave solution is a function of this variable x minus vt.

And so this is an essentially a similarity variable. So, I just wanted to bring this up because you have actually seen such a thing before. You have, actually this is essentially, this is the equivalent of psi. This is the same thing this is like psi this is x minus vt. Here in this case it is not x minus vt it is this is as good as x as r.

It is not r minus vt its r times you know t raised to minus two-fifths. That is ok, but still it is in the same spirit, ok. So, this is a similarity variable and it is very similar to this other slightly simpler similarity variable that you have seen before. I just wanted to emphasize this, ok, right.

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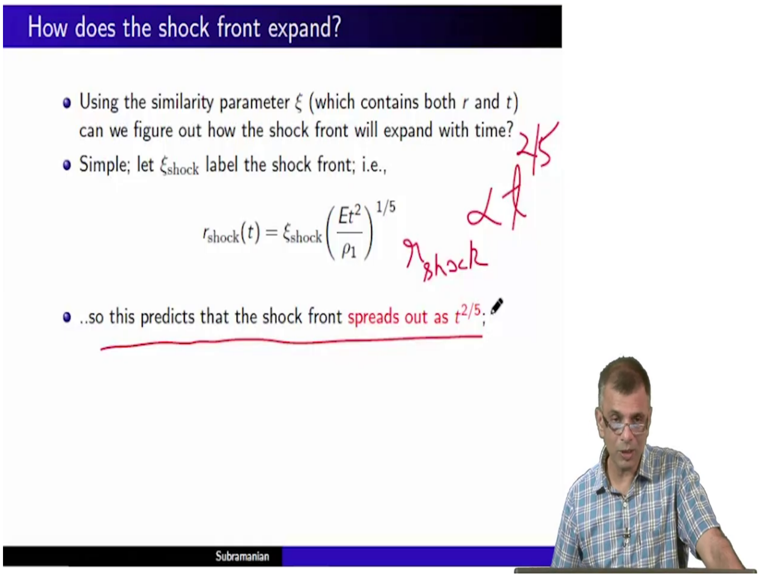

So, how does the shock front expand? We use the similarity parameter, which contains both r and t. And simply using this can we now figure out how the shock front will expand with time, right. So, it is quite simple. Let psi shock, let psi underscore shock label the shock front in other words, yeah.

So, this simply comes from, so r shock is psi shock times E t squared over rho 1, this simply comes from comes from this. This is just a rewrite of this. So, what we do is here we rewrite this to say r is equal to psi times E t squared over rho 1 raised to one-fifth and we just put a put a under a shock subscript on both of them and that is all this is, yeah, so right, ok.

What this is saying is that the radius the physical location of the shock, what this is saying is that r shock is proportional to t raised to two-fifth that is a prediction, ok. So, just look at the beauty of this. Simply from dimension analysis, simply from the fact that you have that; well essentially simply from the fact that; we that the energy is constant, ok it does not change with time.

And the only relevant variables now are the energy, the density and of course, the time yeah and the fact and the assumption that the shock front spreads out self-similarly. Just with these 3 assumptions, ok we are able to say something quite profound about the manner in which this shock front spreads out with time. This is the you know this is the prediction, ok.

We really did not have to resort to too much shock physics or anything, ok. However, we did make use of some very important physical you know assumptions and we have detailed them earlier. And what it also means is that if any one of these assumptions are violated then you know this nice this nice result cannot be obtained. It also means that, right, ok.

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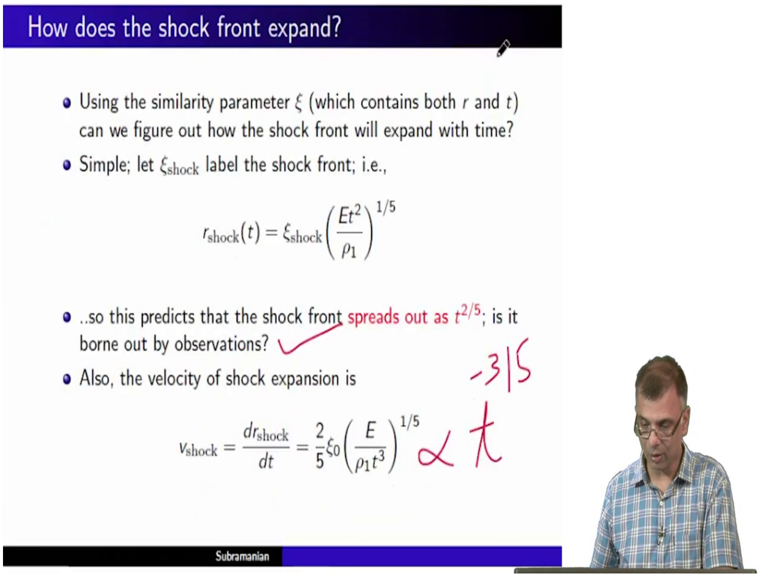

So, right is it borne out by observations. If it is not it is no good. You can make all kinds of fancy predictions, but it has to be borne out by observations obviously, right. We will you know examine this. As you can; as you can sort of anticipate, yes there are regimes where observations you know support this very well and I will show you one such observation.

And also, the velocity of shock expansion, you just differentiate r with respect to t and so the v shock which is dr shock dt goes as t raised to minus three-fifths, proportional to t raised to minus three-fifths. What is this saying? As time progresses, the velocity decreases in this manner, right, ok.

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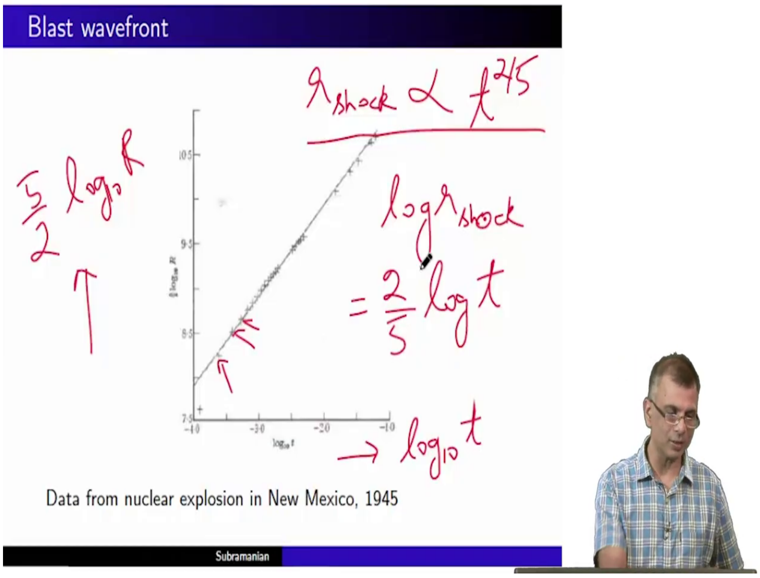

Now, let us so, this is an answer to this, is an answer to this question. And this is data very declassified data from a nuclear explosion in New Mexico in 1945. Marked the date, this is around the time that the First World War ended. So, these crosses, crosses here, ok here, here, these are all crosses, so on so forth, there are many crosses here, ok.

What this is? You cannot read it very well, but what this is log base of 10 time. And what this is log base of 10 R, and it is 5 halves. That is on this axis and this is plotted on this axis. Now, why are we doing this? Remember that the prediction was that r shock would go as t raised to two-fifths, right.

So, you take the, you take the log on both sides, so log r shock equals two-fifths log t, you agree. And you multiply 5 halves on both sides, so 5 halves log r shock would be 1 times log t, right. So, you are plotting log t here, and 5 halves log R shock here or shocker R for that matter.

And it should be a straight line with slope 1. And these crosses that you see, the cross here, across here, all these crosses lie amazingly well. This is a fitted line, this straight line is a fitted line and the data agrees amazingly well with this prediction. It is almost scary how well the data agrees, ok.

So, this is an amazing vindication of our simple prediction, of the prediction that involved only these simple assumptions. The fact that the energy remains unchanged, the ram pressure of the shock front is much much larger than the ambient pressure and therefore, there are two you know there are only two relevant quantities.

And of course, the other thing is that the shock front evolves self-similarly. So, this I mean. So, at least by way of terrestrial shocks, terrestrial you know shocks produced by a bomb blast the data agrees amazingly well with the predictions. So, this gives us some confidence that the way we are proceeding is indeed correct.

So, we will stop for the time being, and thank you.